18.085 Quiz 1 Professor Strang October 4, 2002

Your name is: _____

Grading
1
2
3
Total

1) **(30 pts.)** This question is about the closest straight line $b = C + Dt$ to the measurements b_1, b_2, b_3 at three different times t_1, t_2, t_3.

What system $Ax = b$ (especially find A) would we like to solve for C and D but probably can't? Is $A^{\mathrm{T}}A$ positive definite or semidefinite?

Suppose the "center of mass" is at $(0, 0)$, meaning $\sum t_i = 0$ and $\sum b_i = 0$. Find the best least squares line $C + Dt$ (so $\widehat{x} = (C, D)$). Show that this line goes through the origin $(0, 0)$.

What is the condition on t_1, t_2, t_3 and b_1, b_2, b_3 for the three points to actually **lie on a line** (so $Ax = b$ is solvable without going to least squares)? Various ways to answer, the more specific the better.

xxx

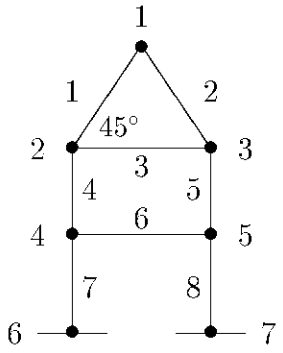
2) **(30 pts.)** We have a line of two equal masses m connected by three springs with spring constants $c_1 = 1, c_2 = 1, c_3 = S$. Spring 1 is fixed at the top and spring 3 at the bottom, so $x_0 = x_3 = 0$.

Find the stiffness matrix K in the equation $Kx = f$ for the mass displacements. Solve for the displacements x_1 and x_2.

If the third spring constant S becomes very large or very small ($S \to \infty$ and $S \to 0$) what are the limiting values of the displacements x? What are the limiting values of the spring forces y_1, y_2, y_3?

xxx

3) **(40 pts.)** The figure shows a network with 8 edges. It is also a plane truss with 8 bars. The lowest two nodes are grounded in the network ($x_6 = x_7 = 0$) and they are supported in the truss. The upper 5 nodes are free, the edges are numbered.



grounded/supported

Find the incidence matrix A (5 columns) for the network. What are all solutions to $Ax = 0$? *How many independent solutions to* $A^{\mathrm{T}}y = 0$ (Current Law)? (not required to find y's).

If $f_1 = 1$ amp enters node 1 and travels to ground, what equations would you solve for the potentials x and currents y? *Assume all* $c_i = 1$. What are the 8 currents (you can answer from the picture without solving equations)?

What shape is the matrix A for the truss problem? Describe a complete set of mechanisms (solutions to $Ax = 0$). Draw a picture of each mechanism.

For the truss stiffness matrix $K = A^{\mathrm{T}}A$,

(a) Is K positive definite? Is it positive semidefinite? Why or why not?

(b) Find a set of horizontal and vertical forces $\boldsymbol{f}$, not all zero, so that $K\boldsymbol{x} = \boldsymbol{f}$ can be solved.

xxx