18.085 Quiz 3 December 8, 2006 Professor Strang

Your PRINTED name is: ____________________

Grading **1** **2** **3**

1) **(30 pts.)** (a) Suppose $f(x)$ is a *periodic* function:

$$f(x) = \begin{cases} 0 & \text{for } -\pi < x < 0 \\ e^{-x} & \text{for } 0 \le x \le \pi \\ f(x + 2\pi n) & \text{for every integer } n \end{cases}$$

Find the coefficients c_k in the complex Fourier series $f(x) = \sum c_k e^{ikx}$. What is c_0? What is $\sum_{-\infty}^{\infty} |c_k|^2$?

(b) Draw a graph of $f(x)$ from -2π to 2π. Also draw a careful graph of df/dx. How quickly do the coefficients of $f(x)$ decay as $k \to \infty$ and why?

(c) Find the Fourier coefficients d_k of df/dx. Do they approach a constant (or what pattern do they approach) as $k \to \infty$? Explain the pattern from your graphs.

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2) **(33 pts.)** (a) Can you complete this 4-step MATLAB code to compute the cyclic convolution $f \circledast g = h$? I suggest `fhat`, `ghat`, `hhat` for their transforms.

```
1.  fhat = fft(f)
2.
3.  hhat =
4.  h =
```

(It is equally possible to start with the inverse discrete transform `ifft`. The only difference will be a factor of N somewhere, which I forgive! If you don't know MATLAB notation for commands 2, 3, 4 you can use words. MATLAB's `fft(f)` and `ifft(f)` automatically determine the length of f.)

(b) Suppose each of your quiz grades is a random variable (don't know how I thought of this). The probability of grade j on each quiz ($j = 0, \ldots, 100$) is p_j. The "generating function" for that quiz is $P(z) = \sum p_j z^j$. What is the probability s_k that the sum of your grades on 2 quizzes is k? Give a nice formula for $S(z) = \sum s_k z^k$.

(c) The chance of grade $j = (70, 80, 90, 100)$ on one quiz is $p = (.3, .4, .2, .1)$. What is the expected value (mean m) for the grade on that quiz?

Show that this quiz average m agrees with dP/dz at $z = 1$. What are the probabilities s_k for the sum of two grades? Give numbers or a MATLAB code for the s_k.

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3) **(37 pts.)**

(a) The hat function $H(x) = 1 - |x|$ for $-1 \le x \le 1$ has height 1 and area 1 and integral transform $\widehat{H}(k) = (2 - 2\cos k)/k^2$. Find the transform $\widehat{R}(k)$ of the roof function $R(x)$:

$$R(x) = \textbf{box} + \textbf{hat} = 2 - |x| \quad \text{for } -1 \le x \le 1, \;\; 0 \text{ else.}$$

(b) What is the value of $\widehat{R}(k)$ at $k = 0$ and how does this connect to the graph of the roof?

(c) Suppose $R(x)$ is the response of a sensor to a point source $\delta(x)$ at $x = 0$. The sensor is shift-invariant (shifted response when source is shifted). The output F from a distributed source $U(x)$ is the convolution $F = R * U$. Describe how to find $U(x)$ if you know $F(x)$.

(d) There could be a difficulty with your solution method in part (c). That would arise if ______ $= 0$. For 1 point, does this difficulty appear in this example?

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