PROBLEM 4

Following the hint given, you get the PDE

$$v_\tau - \left(r - \frac{v}{r}\right) v_r = 0$$

$$v(r,0) = 0$$

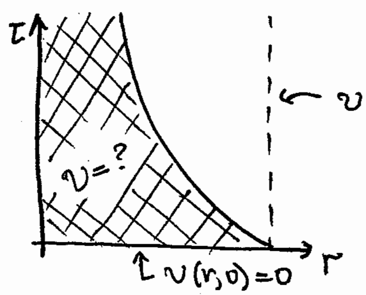
$$v(a,\tau) = a^2$$

You solve this PDE by the method of characteristics. In the end you ~~may~~ get something that looks like

$$\left.\begin{aligned} v &= K_1 \\ r &= F(\tau; K_1, K_2) \end{aligned}\right\} \text{①}$$

Apply the I.C. $v(r,0)=0$

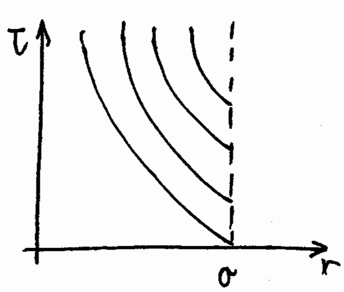
You will be able to 'fill' with characteristics the region shown below (Question: What is v in this region?)



In order to fill with characteristics the unshaded region you use the B.C. $v(a,\tau)=a^2$

Apply the B.C.

Try to define characteristics that originate along the line $r=a$



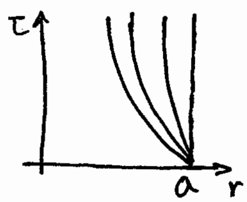
Parametrizing the B.C $v(r=a, \tau=s) = a^2$ gives

$$\text{①} \Rightarrow \left\{\begin{aligned} v &= a^2 \\ a &= F(s; a^2, K_2) \quad \text{②} \end{aligned}\right.$$

see Note

You'll see that equation ② has no solution for K_2^*. This suggests that $r=a$ is a characteristic!

$\therefore$ Try a fan of characteristics through $r=a$ by allowing v to vary continuously from 0 to a^2



<u>Note</u>. You may get a soln for K_2, depending how you define K_1 & K_2. In either case you will be able to conclude that $r=a$ is a CHAR

PROBLEM 5

$$\begin{pmatrix} u\rho_x + \rho u_x + v\rho_y + \rho v_y \\ c^2\rho_x + u\rho u_x + \rho v u_y \\ \rho u v_x + \rho v v_y + c^2\rho_y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad (*)$$

$$\Rightarrow \underbrace{\begin{bmatrix} u & \rho & 0 \\ c^2 & u\rho & 0 \\ 0 & 0 & \rho u \end{bmatrix}}_{A} \begin{pmatrix} \rho \\ u \\ v \end{pmatrix}_x + \underbrace{\begin{bmatrix} v & 0 & \rho \\ 0 & \rho v & 0 \\ c^2 & 0 & \rho v \end{bmatrix}}_{a} \begin{pmatrix} \rho \\ u \\ v \end{pmatrix}_y = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

As seen in class along the characteristics we have

$\frac{dy}{dx} = K_i$ where the K_i satisfy $\det|\underbrace{a - K_i A}_{B_i}| = 0$

For each V_i find the vectors that satisfy $V_i^T B_i = \underline{0}$

To get the variations multiply $(*)$ by V_i^T eliminating all partial derivatives using the fact that along characteristics we have

$$\frac{d}{dx} = \frac{\partial}{\partial x} + K_i \frac{\partial}{\partial y}$$

PROBLEM 6

The wave eqn is linear in u $\Longrightarrow$ Use the principle of superposition splitting the original problem into 2 :

<u>Problem (A)</u>
$$u_{1tt} - c^2 u_{1xx} = 0 \qquad x>0\ ,\ t>0$$
$$u_1(x,0) = q(x)$$
$$u_1(0,t) = 0$$
$$u_{1t}(x,0) = 0$$

<u>Problem (B)</u>
$$u_{2tt} - c^2 u_{2xx} = 0 \qquad x>0\ ,\ t>0$$
$$u_2(x,0) = 0$$
$$u_2(0,t) = p(t)$$
$$u_{2t}(x,0) = 0$$

Then the solution is given by $u = u_1 + u_2$

<u>Hints</u>:

- For problem (A)

  This reminds us of D'Alembert solution but this solution was for x on the whole real line

  $\longrightarrow$ Try to solve a problem for u_1 over $-\infty < x < +\infty$ that agrees with problem (A) for $x>0$

$$u_{1tt} - c^2 u_{1xx} = 0$$
$$u_1(x,0) = Q(x)$$
$$u_{1t}(x,0) = 0$$
$$u_1(0,t) = 0$$

  where $Q(x) = \begin{cases} q(x) & x>0 \\ \tilde{q}(x) & x<0 \end{cases}$ with $\tilde{q}$ to be found by the B.C.

- For problem (B). Try the general soln $u_2(x,t) = F(x-ct) + G(x+ct)$

PROBLEM 7

Hints:

- Obtain a single integral expression for ϕ in terms of f (not $\hat{f}$) using contour integration (see Handout #6)
- $\epsilon \ll \Omega$. Can we approximate the soln by setting $\epsilon = 0$? Why?