

18.310 Assignment 1:

1) Verify that one can have a good non-adaptive 3 weighing scheme with no good coin for each of 9 through 13 coins; by producing a three-row matrix having each number of columns, obeying the conditions that each row sums to 0, and no 2 columns are the same or negative of one another; for each of these values of n .

hint: You can without great effort produce 4 groups of 3 columns each, so that each group of 3 sums to 0; (Try it and see) and also the all 0 column; these handle 9, 10, 12 and 13 coins. To get the 11 case you may take the 12 solution, add a column that is not one of these or its negative, and omit two columns that add up to it. There is a way to do this.

(You can do it any other way you want if you can find one.)

2) Construct a 4 weighing optimum scheme with the maximum number of coins, including one good coin (which you know), using the procedure described in the notes. This can be done on a spreadsheet or on paper. It is easy on a spreadsheet. Verify that this scheme is optimal. Explain why one good coin is needed for this optimal scheme.

3) Build a 4 weighing automatic bad coin finder on a spreadsheet. It should have matrix that for each coin indicates on which weighings it goes on which side of the scale, with one column for each potentially bad coin, that also includes the number of the coin corresponding to it.

Also you enter a column of length 4 each entry consisting of 0, 1 or -1, it should tell you which coin is bad and indicate whether it is heavy or light when that is known.

(I find it useful to assign a ternary number to each column, and use the "if", and "sum" functions on the spreadsheet to do this.)

4) Suppose the balance only had two outcomes: balance or unbalance, so that you do not know which side is heavier. Find a lower bound on the number of weighings for n coins. Find an explicit general solution that meets this bound.

5) Suppose we are given k weighings, no good coin, and we want to find the one bad coin and determine whether it is light or heavy. Find a good scheme for $(3^k)/2-5/2$ coins.

6) Suppose there are two bad coins. Find a pigeonhole principle lower bound on the number of weighings need to find them given there are n coins to begin with, and no good coins.