18.310 Homework # 10

1a. On a spreadsheet, implement the simplex algorithm for the linear program

$$\text{maximize} \quad 2x + y + 4z$$

subject to

$$\begin{array}{rcl} x + 2y - \ z & \leq & 4 \\ x \qquad + \ z & \leq & 3 \\ 2x + \ y + 3z & \leq & 5 \end{array}$$

and $x > 0$, $y > 0$ and $z > 0$. Perform pivots until you find the optimum.

1b. From the tableau obtained at the end of the simplex algorithm, give the values of the variables which result in the optimum, as well as the values of the dual variables that result in the optimum for the dual LP.

1c. Write down the tableau for the dual linear program for 1a. Is the origin feasible? If not, write down the tableau you would use to find a feasible origin.

2a: Remember the Lempel-Ziv algorithm. One way to analyze this is to use a linear program. Suppose you have n_0 0's and n_1 1's, and you would like to group them into the maximum number of distinct phrases, where no two phrases are equal. Explain why this number is approximated by the solution to the following linear program:

$$\begin{array}{ll} \text{maximize} & \sum_{i,j} x_{i,j} \\ \text{subject to} & \sum_{i,j} i x_{i,j} \leq n_0 \\ & \sum_{i,j} j x_{i,j} \leq n_1 \\ & \qquad 0 \leq x_{i,j} \leq \binom{i+j}{i} \end{array}$$

2b: Find the dual to this linear program.

2c: Use the relationship between the inequality being tight in the primal linear program and the slack variable being 0 in the dual program (or maybe vice versa) to explain why the optimal solution to the primal can be obtained by choosing a weight w_0 for bit 0, a weight w_1 for bit 1, and then taking phrases in order of increasing weight (for the right weights, of course).

2d: (Extra credit) Give an intuitive explanation for what the dual linear program really means in this case.