

18.310 Homework # 4

1: Write a one to two page essay explaining Shannon's first and second theorems (coding for efficiency and coding for error correction). What do they have in common? There should be a brief introduction, and it should be typeset or word processed. Equations should either be typeset in LaTeX or composed using an equation editor (one is included with Word and with OpenOffice) or with other mathematical typesetting software.

2a: Suppose you have a coin which has probability p of heads and probability $1 - p$ of tails. You toss it N times. Give a formula for the probability that there are exactly Nq heads and $N(1 - q)$ tails.

2b: Take the above expression and use Stirling's formula to approximate it in the form

$$2^{-Nf(p,q)}$$

where f is a function involving terms composed of p , q , $1 - p$, $1 - q$ and the logarithms of these quantities.

2c: For $N = 100$, plug $p = 0.6$ and $q = 0.6, 0.65, 0.7, 0.75$, into the above expression (or estimate their values from a plot). Do the same thing for $N = 10,000$, and the values $p = 0.6$ and $q = 0.6, 0.605, 0.61, 0.615$. What is the expectation and mean square deviation of the number of heads for a coin with probability of heads 0.6 and for the number of coin flips $N = 100$ and 10,000?

2d: If you had kept track of the $\sqrt{2\pi n}$ terms in Stirling's approximation, you would have needed to multiply the answer obtained in (2b) by some factor. What is that factor?

3a: Using a spreadsheet, construct a Hamming code that encodes 11 bits into 15 and corrects one error.

3b: On the spreadsheet, construct an encoder for your code.

3c: On the spreadsheet, construct a decoder for your code. This should allow you to input any message with a single error, and output the message with the error corrected. A hint for how to do this will be posted later this week.