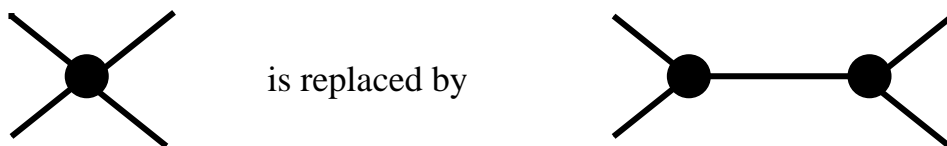


18.310 Homework # 7

- 1:** Write an essay giving Kempe's false proof of the four-color theorem, and explaining what the flaw in it is. Equations should either be typeset in LaTeX or composed using an equation editor or with other mathematical typesetting software. (Figures may be hand-drawn if it is easier).
- 2:** Write a spreadsheet that implements the tortoise-and-the-hare factoring algorithm described in class (and the notes). Make sure the input and output are clearly marked and easy to find.
- 3:** Use your spreadsheet to factor the number 294409 into primes. This is a Carmichael number, that is $x^{294408} = 1 \pmod{294409}$ for all x relatively prime to 294409. Use the Chinese remainder theorem to explain why any number whose prime factorization is $(6k + 1) \cdot (12k + 1) \cdot (18k + 1)$ is a Carmichael number.
- 4:** Consider a graph obtained when you replace every node of K_5 with two vertices in the following manner.



- 4a:** Show that there are several (≥ 2) possible such graphs. (One way to do this might be to construct one graph which has some property and another which doesn't have this property.)
- 4b:** Explicitly give a construction that finds a subdivision of a $K_{3,3}$ in any such graph. (It's easy to prove this exists using Kuratowski's theorem, but we want a more explicit answer than this.)
- 5:** Prove that every planar graph can be drawn in the plane so that all its edges are straight lines. I will outline a proof by induction. This can be quite difficult to prove unless you pick the right induction hypothesis.

First, show that it is enough to prove it for graphs whose only face which is not a triangle is the outside face. Next, prove (using induction) that if you have such a graph G with k vertices on the outside face, and are given any k -sided convex polygon in the plane, then there is a straight-line drawing of G with the outside face being this k -sided polygon. You might want to break the proof up into two cases, depending on whether one of the edges of G is a chord of the outside face.