

Problem Set 2 due at lecture on Monday, March 6, 2006

I. Problems on **traffic flow** from H2 – 57.6, 60.2, 61.1, 63.8, 67.3, 69.1, 71.2, 73.6, 74.1 and from H1 – 12.2.6.

II. Additional problems on **expansion fans**.

1. Consider the dynamics of initially stopped cars behind a red light at $x = 0$,

$$\rho(x, 0) = \begin{cases} \rho_j & \text{for } x < 0 \\ 0 & \text{for } x > 0 \end{cases} \quad (1)$$

when the light turns green at $t = 0$, assuming the general velocity-density relation, $u(\rho) = u_m(1 - \rho/\rho_j)^\alpha$, some constant $\alpha > 0$.

- (a) What is the capacity of the road, $q_m = \max\{\rho u(\rho)\}$?
- (b) What is the car density at the light, $\rho(0, t)$, for $t > 0$?
- (c) Does the flow rate ever reach the road capacity, and, if so, when and where does it occur?

2. Now consider the velocity-density relation, $u(\rho) = u_m/(1 + \rho/\rho_j)$ for $\rho > 0$. Since there is no maximum density, this may not be the best model for traffic flow, but it could describe other nonlinear wave phenomena.

- (a) Plot the flow rate $q(\rho)$ and wave velocity $c(\rho)$ versus density.
- (b) For the same initial condition (1), solve for $\rho(x, t)$, plot some density profiles for $t > 0$, and draw a characteristic diagram.