

## Problem Set 1 due at lecture on Tuesday, February 21, 2006

I. Problems on **linear waves** from H1 - 12.2: 1, 3, 4, 5; 12.3: 6; 12.4: 4.

II. Additional problems.

1. **Implicit Differentiation.** In each case compute  $\frac{dy}{dx}$  as a function of  $y$  and  $x$ , given that  $y = y(x)$  satisfies:

(a)  $x^3 + xy + 2 = 0$ .

(b)  $y = \sin(y + x)$ .

(c)  $\ln(y) = x$ .

(d)  $\cos^2(y) = x$ , for  $x > 0$ .

(e)  $y = f(c - yx)$ , where  $f$  is an arbitrary function and  $c$  is a constant.

(f)  $y = f(x - cy)$ , where  $f$  is an arbitrary function and  $c$  is a constant.

2. **Dense Granular Flow.** The vertical velocity,  $v(x, z)$ , in a wide, quasi-two-dimensional, draining silo approximately satisfies a diffusion equation,

$$v_z = b v_{xx}$$

where the vertical distance,  $z > 0$ , plays the role of “time”. The velocity far from the orifice vanishes,  $v(\pm\infty, z) = 0$ , and the flow rate just above the orifice is specified,  $Q(0^+) = Q_0$ .

(a) Show that the flow rate is the same at all heights,

$$Q(z) = \int_{-\infty}^{\infty} v(x, z) dx = Q_0.$$

(b) Seek a similarity solution of the form,

$$v(x, z) = \frac{1}{z^\alpha} F\left(\frac{x}{z^\beta}\right)$$

for some constant  $\alpha$ . Determine the unique values of  $\alpha$  and  $\beta$  for which  $F(\zeta)$  satisfies an ordinary differential equation (ODE) in  $\zeta = x/z^\alpha$ , which does not depend explicitly on  $x$  or  $z$ . What is the shape of the flow region?

(c) Solve the ODE for the scaling function,  $F(\zeta)$ , and thus obtain the solution given in class. Explain why this solution corresponds to a small “point orifice”.