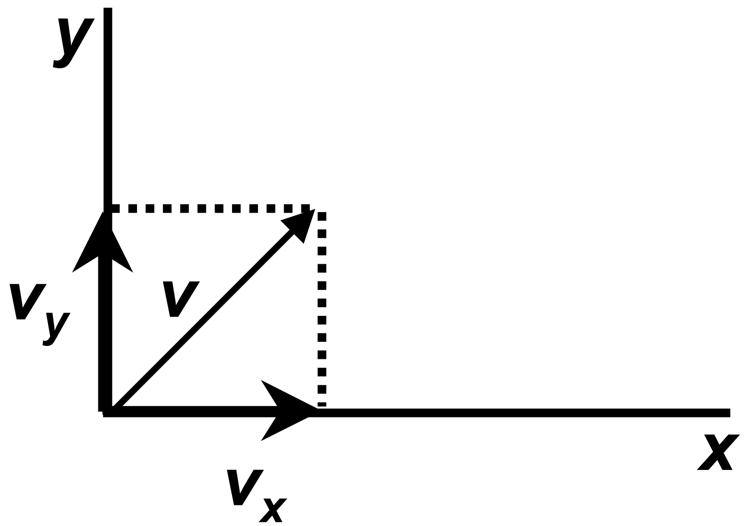
Course 18.327 and 1.130
Wavelets and Filter Banks

Orthogonal wavelet bases: connection to orthogonal filters; orthogonality in the frequency domain. Biorthogonal wavelet bases.

Orthogonal Wavelets

2D Vector Space:

Basis vectors are *i*, *j* $\rightarrow$ **orthonormal basis**



v_x **and** v_y **are the projections of** v **onto the** x **and** y **axes:**

$$v_x = \langle v , i \rangle \; i$$

$$v_y = \langle v , j \rangle \; j$$

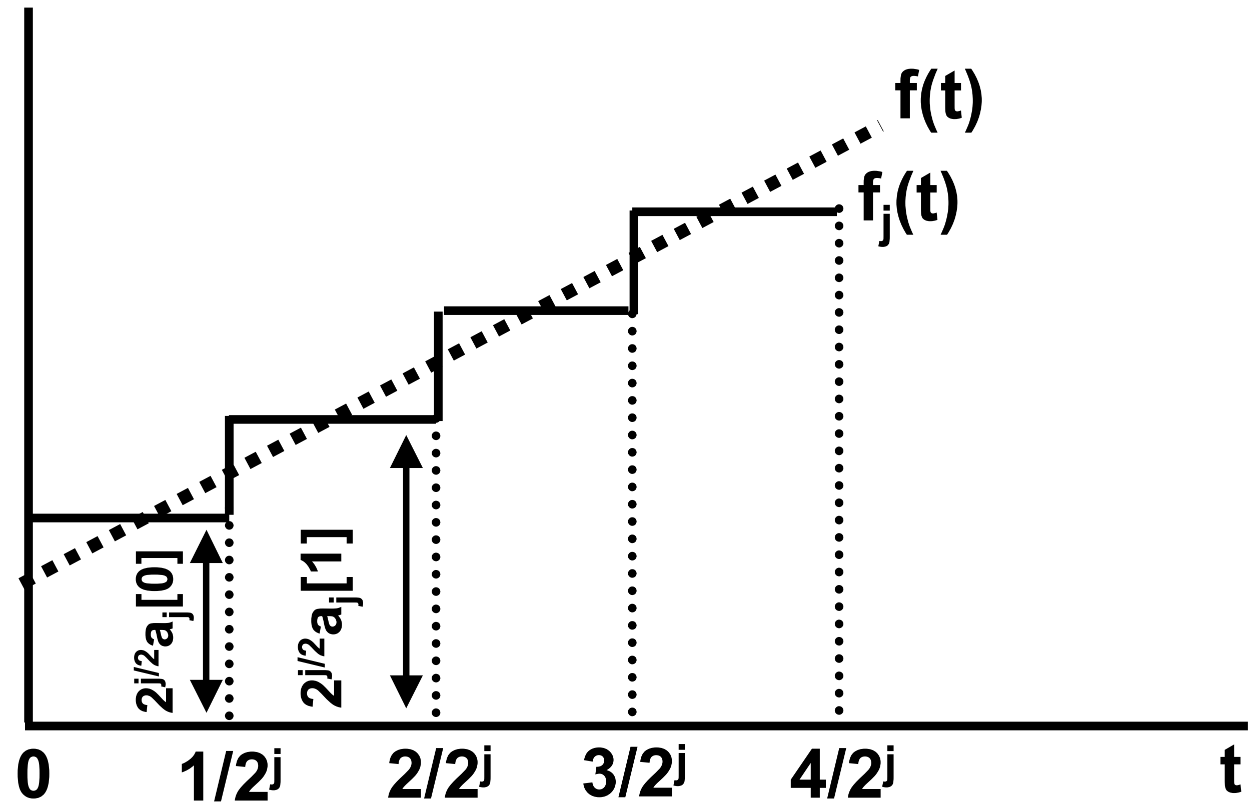
Projection of an L^2 function, f(t), onto V_j:

$$f_j(t) = \sum_k a_j[k]\, \phi_{j,k}(t)$$

with

$$a_j[k] = \langle f(t), \phi_{j,k}(t) \rangle$$

Haar example



Similarly, we can represent f(t) in the dual basis

$$f(t) = \sum_k \tilde{c}_k \tilde{\phi}(t - k) + \sum_{j=0}^{\infty} \sum_k \tilde{d}_{j,k} 2^{j/2} \tilde{w}(2^j t - k)$$

$$\tilde{c}_k = \int f(t) \phi(t - k) \, dt$$

$$\tilde{d}_{j,k} = 2^{j/2} \int f(t) w(2^j t - k) \, dt$$

Note: When $f_0[k] = h_0[-k]$ and $f_1[k] = h_1[-k]$, we have

$$\phi(t) = \tilde{\phi}(t) \Rightarrow V_j = \tilde{V}_j$$

$$w(t) = \tilde{w}(t) \Rightarrow W_j = \tilde{W}_j$$

i.e. we have orthogonal wavelets!