

Lecture 1 Introduction, Basic Linear Algebra

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Introduction to Numerical Methods

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Matrix-Vector Multiplication

- Matrix-vector product $b = Ax$

$$b_i = \sum_{j=1}^n a_{ij}x_j, \quad i = 1, \dots, m$$

- Linear mapping $x \mapsto Ax$

$$A(x + y) = Ax + Ay$$

$$A(\alpha x) = \alpha Ax$$

- Every linear map can be expressed as a matrix-vector product

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Linear Combination of Columns

- Columns a_1, a_2, \dots, a_n of A :

$$A = \left[a_1 \mid a_2 \mid \cdots \mid a_n \right]$$

- Alternative view of matrix-vector product:

$$b = Ax = \sum_{j=1}^n x_j a_j = x_1 \begin{bmatrix} a_1 \\ \vdots \\ a_1 \end{bmatrix} + x_2 \begin{bmatrix} a_1 \\ \vdots \\ a_2 \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$

- b is a linear combination of the columns of A

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Matrix-Matrix Multiplication

- Matrix-matrix product $B = AC$

$$b_{ij} = \sum_{k=1}^m a_{ik}c_{kj}$$

- Matrix-vector product for each column of C
- Each column of B is a linear combination of the columns of A

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Range, Nullspace, and Rank

- The *range* or *column space* of A :

$\text{range}(A)$ = All linear combinations of the columns of A
= The space spanned by the columns of A
= All vectors that can be expressed as Ax

- The *nullspace* of A :

$\text{null}(A)$ = All solutions to $Ax = 0$

- Dimension of space = number of vectors in a basis
- The *column rank* of A is the dimension of the column space $\text{range}(A)$
- column rank = row rank = rank

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Matrix Inverse

- Nonsingular* or *invertible* matrix = square matrix with full rank
- The *inverse* A^{-1} of A satisfies

$$AA^{-1} = A^{-1}A = I$$

- Change of basis:

$$x = A^{-1}b = \text{solution to } Ax = b$$

= the vector of coefficients of the expansion of b
in the basis of columns of A

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