MIT 18.335, Fall 2006: Homework 3

Due October 19

1. Trefethen/Bau 13.1

2. Trefethen/Bau 13.2

3. Suppose that x and y are floating point vectors (no rounding necessary) and the standard dot product $f(x,y) = \sum_{i=1}^{n} x_i y_i$ is computed using floating point arithmetic. Show that the error in the computed result $\tilde{f}(x,y)$ is

$$|\tilde{f}(x,y) - f(x,y)| \leq n\epsilon_{\text{machine}} |x|^T |y| + O(\epsilon_{\text{machine}}^2).$$

Use this to show that if the matrix product AB is computed by dot products, the error is

$$|\widetilde{AB} - AB| \leq n\epsilon_{\text{machine}} |A||B| + O(\epsilon_{\text{machine}}^2).$$

where A is $m \times n$, B is $n \times p$, and the absolute values and the inequality are meant componentwise.

4. Consider evaluating the function

$$y_1(x) = \frac{\log(1+x)}{x}$$

for $x \approx 0$ using double precision floating point arithmetic. Plot the function on the inverval $x \in [-10^{-15}, 10^{-15}]$ and explain the result. Repeat the experiment but evaluate the function using the following expression:

$$y_2(x) = \begin{cases} \frac{\log(1+x)}{(1+x)-1} & \text{if } 1+x \neq 1, \\ 1 & \text{if } 1+x = 1. \end{cases}$$

Prove that this expression always produces an accurate result. The log function computes an accurate answer for any argument.

5. Trefethen/Bau 14.1 (f)-(g)

6. Trefethen/Bau 15.1 (a),(b),(c),(d),(g)

7. Trefethen/Bau 16.2