

MIT 18.335, Fall 2006: Homework 1

Due September 21

1. Trefethen/Bau 2.1
2. Trefethen/Bau 2.3
3. Trefethen/Bau 2.6
4. Is the function $f(x) : \mathbb{R}^2 \rightarrow \mathbb{R}$ shown below a valid norm? Prove or give a counter example.

$$f(x) = \left(\sqrt{|x_1|} + \sqrt{|x_2|} \right)^2$$

5. Verify the following inequalities for the p -norms, with $x \in \mathbb{R}^m$ and $A \in \mathbb{R}^{m \times n}$, and give examples that achieve equality.

$$\|x\|_2 \leq \|x\|_1 \leq \sqrt{m}\|x\|_2$$
$$\frac{1}{\sqrt{m}}\|A\|_1 \leq \|A\|_2 \leq \sqrt{n}\|A\|_1$$

6. Work out the details of Example 3.4 on page 21 in Trefethen/Bau.
7. Trefethen/Bau 4.5
8. Trefethen/Bau 5.3
9. Trefethen/Bau 9.3 (b),(c). Below is a solution to 9.3 (a).

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m=15; n=40;
A=zeros(m,n);

A(2:9, [2,3,6,7])=1;
A(5:6,4:5)=1;
A(3:10,10:11)=1;
A(9:10,12:15)=1;
A(4:11,18:23)=A(3:10,10:15);
A(5:12,26:31)=A(3:10,10:15);
A(6:13,34:39)=A(3:10,10:15);
A(6:11,36:39)=A(10:-1:5,13:-1:10);
A([3,4,6,7],12:15)=1;
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