18.700. Problem Set 3

Due date: October 3 (Monday) in lecture or in my office before noon on due date. Late homeworks will be accepted only with a medical note or for some other MIT approved reason. You may work with others, but the final write-up should be entirely your own and based on your own understanding.

Each problem is worth 5 points.

Problem 1: (a) Find all right or left inverses, if they exist, for the matrices $\begin{pmatrix} 1 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 & 1 \\ 2 & 3 & 5 \\ 1 & 2 & 3 \\ 2 & 1 & 4 \end{pmatrix}$.

(b) Is $\mathit{span}\left\{ \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} \right\} = \mathit{span}\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \right\}$?

Problem 2: Define $\mathbb{F}_9 = \{a + bi : \ a, b \in \mathbb{Z}/3\mathbb{Z}\}$ (i denotes the solution the equation $x^2 + 1 = 0$). Show that $\mathbb{F}_9$ is a (9-element) field (special attention to multiplicative inverses).

Problem 3: 1.2.6.

Problem 4: 1.2.13.

Problem 5: Which of the following sets of vectors are linearly independent?

(a) $\left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ 3 \end{pmatrix} \right\}$ in $\mathbb{R}^3$.

(b) $\{X, X^2 + 1, X^3 + X, X^3 - 2X^2 - 2X - 2\}$ in P^4.

Problem 6: 1.3.10.

Problem 7: 1.4.2.(a)

The following problems are recommended for additional practice. They should *not* be turned in with the homework and they will not count towards the homework score. Section 1.1: 5,7,8,12. Section 1.2: 1(c), 2(b),7,9,12 . Section 1.3: 9,12,16,17.