18.700. Exam 1. Fall 2005.

Name: ____________________ October 7, 2005

Problem 1: ______ /40

Problem 2: ______ /32

Problem 3: ______ /28

Total: ______ /100

Instructions: The exam is closed book, closed notes and calculators are not allowed. You will have 50 minutes for this exam. The point value of each problem is written next to the problem - use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. Good luck!

Problem 1(40 points) Let A be the 3×5 matrix

$$A = \begin{pmatrix} 2 & 1 & -1 & 1 & 3 \\ 1 & 0 & 1 & 2 & -1 \\ 3 & 1 & 2 & 5 & -2 \end{pmatrix}.$$

a)(20 points) Find all its right inverses, if they exist.

(b)(10 points) Find a basis for the column space of A and write the fifth column in coordinates with respect to this basis.

(c)(10 points) Find a basis for the nullspace of A.

Problem 2(32 points)

(a)(10 points) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$. First compute AB, then BA.

(b)(10 points) Assume A and B are some 2×2 matrices such that $AB - BA = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$. Find $A^t B^t - B^t A^t$.

(c)(12 points) For the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, determine all 3×3 matrices B such that $AB = BA$.

Problem 3(28 points) Answer the following questions "True" or "False". *Give* clear and concise explanations of your answers or *show* counterexamples.

(a)(7 points) If a vector space V is n-dimensional, then every subset of V with more than n elements is a spanning set for V.

(b)(7 points) The set of polynomials $\{f_1(X), f_2(X), \ldots, f_r(X)\}$ is linearly independent *if* the set of polynomials $\{Xf_1(X), Xf_2(X), \ldots, Xf_r(X)\}$ is linearly independent.

(c)(7 points) Let v be a fixed vector in $\mathbb{R}^n$. The set of $m \times n$ real matrices A with the property that $Av = 0$ form a real vector space.

(d)(7 points) (Recall that $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z} = \{\bar{0}, \bar{1}\}$ is the field with two elements) There exists a vector space V over $\mathbb{F}_2$ with exactly 18 elements.