18.700 STUDY GUIDE FOR CHAPTERS 0–1

This guide contains a checklist of important skills, definitions and theorems to learn before Exam 1. You do not need to memorize the proofs of the theorems, but you should learn the statements and be able to use the theorems.

Skills checklist:

1. Put a linear system or matrix in row echelon form and in reduced row echelon form.
2. Determine when 2 linear system or matrices are row equivalent.
3. Determine whether a linear system is consistent, and if so, determine all solutions.
4. Determine if 2 matrices can be multiplied in a particular order, and if so, compute their product.
5. If it exists, compute the left inverse, resp. right inverse, left-right inverse, of a matrix.
6. Determine if a vector $\mathbf{w}$ is a linear combination of a set of vectors $(\mathbf{v}_1, \ldots, \mathbf{v}_m)$, and if so, find coefficients $c_1, \ldots, c_m$ such that $\mathbf{w} = c_1\mathbf{v}_1 + \cdots + c_m\mathbf{w}_m$.
7. Determine whether a set of vectors is linearly independent or linearly dependent. If linearly dependent, find a linearly independent subset spanning the same vector space.
8. Find the rank of a matrix, a basis for the row space, a basis for the column space, and a basis for the nullspace.
9. Given 2 ordered bases for a vector space, compute the change-of-basis matrix.

Definition checklist: Learn each of the following definitions.

System of linear equations = **Linear system of equations** (Def. 0.1.1); **Set of solutions** = **Solution set** (Def. 0.1.2); **Elementary row operations** (Def. 0.1.3, Def. 0.4.1); **Row equivalent** (Def. 0.1.6, Def. 0.4.1); **Echelon form** (Def. 0.2.4, Def. 0.4.3); **Leading variable** ≈ **Determined variable** (Def. 0.2.4); **Free variable** (Def. 0.2.4); **Reduced echelon form** (Def. 0.2.6, Def. 0.4.3); **Matrix** (Def. 0.3.1); **Row vector**, **Column vector** (p. 24); **Upper triangular matrix**, **Lower triangular matrix**, **Diagonal matrix** (Def. 0.3.8); **Block matrix** (Def. 0.3.11); **Coefficient matrix**, **Value matrix** (p. 39); **Augmented matrix** (p. 40); **Elementary matrix** (Def. 0.4.8); **Homogeneous system**, **Associated homogeneous system** (Def. 0.5.1); **Rank** (Def. 0.5.6); **Left inverse**, **Right inverse**, **Inverse** (Def. 0.6.2); **Transpose** (Def. 0.6.13); **Symmetric matrix** (Def. 0.6.13);

Field (Def. 1.1.1); **Linear combination**, **Span** (Def. 1.1.2); **Vector subspace** (Def. 1.1.5, Def. 1.2.5); **Row space**, **Column space**, **Nullspace** (Def. 1.2.1); **Vector space** (Def. 1.2.4); **Linearly independent**, **Linearly dependent** (Def. 1.3.1); **Linear relation**, **Trivial linear relation**, **Nontrivial linear relation** (in lecture); **Basis** (Def. 1.4.1); **Dimension** (Def. 1.4.3); **Nullity** (p. 121); **Ordered basis** (Def. 1.5.2); **Coordinates** (Def. 1.5.2); **Change-of-basis matrix** (in lecture)

Theorem checklist: Learn the statements of the following theorems (you do not need to learn them verbatim, but learn their meaning). You do not need to memorize the proofs, but be able to use the theorems.

Theorem 0.1.8 Row equivalent linear systems have the same set of solutions.

Theorem The result of Gaussian elimination is a row equivalent linear system or matrix in row-echelon form. The result of Gauss-Jordan elimination is a equivalent linear system or matrix in reduced row-echelon form.

Theorem 0.3.3 Addition, scalar multiplication, and multiplication of matrices satisfy familiar arithmetic identities, but matrix multiplication is *not necessarily* commutative, i.e., $AB \neq BA$.

Theorem 0.4.10, Theorem 0.6.8 Row equivalent matrices differ by left multiplication by an invertible matrix.

Theorem 0.5.5 Row equivalent matrices have equal reduced row-echelon matrices.

Corollary Consistent $m \times n$ linear systems with the same set of solutions are row equivalent.

Theorem 0.5.7 The linear system $AX = B$ is consistent iff $\text{rank}(A) = \text{rank}(A|B)$. There is a unique solution iff $\text{rank}(A) = n = \text{rank}(A|B)$ where n is the number of variables.

Theorem 0.6.6 A product of invertible matrices is invertible, and the inverse is the product of the inverses in reverse order.

Theorem 0.6.9 A square matrix A is invertible iff $AX = B$ is consistent for every B iff $\text{rank}(A)$ is the number of rows iff A is a product of elementary matrices.

Corollary Matrices differing by left multiplication by an invertible matrix are row equivalent.

Theorem 1.1.4 The p-vector $\mathbf{w}$ is a linear combination of the p-vectors $\mathbf{v}_1, \ldots, \mathbf{v}_m$ iff the system $AX = \mathbf{w}$ is consistent, where $A = (\mathbf{v}_1|\ldots|\mathbf{v}_m)$.

Theorem 1.3.4 The p-vectors $\mathbf{v}_1, \ldots, \mathbf{v}_m$ are linearly independent iff the matrix $A = (\mathbf{v}_1|\ldots|\mathbf{v}_m)$ has rank m.

Theorem 1.4.6 The rank, row space, column space and nullspace of a matrix can be simply computed given the reduced row-echelon form.