18.700. Exam 2. Fall 2005.

Name: ____________________ October 28, 2005

Problem 1: _____ /33

Problem 2: _____ /30

Problem 3: _____ /27

Problem 4: _____ /10

Total: _____ /100

Instructions: The exam is closed book, closed notes and calculators are not allowed. You will have 50 minutes for this exam. The point value of each problem is written next to the problem - use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. Good luck!

Problem 1(33 points) For the following matrices, compute the determinant, *in (a) and (b), by the method indicated*. Show work.

(a)(11 points) Using cofactor expansion:

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 9 & 0 & 1 \\ 1 & 0 & 9 & 5 \end{pmatrix}.$$

(b)(11 points) Using row-reduction:

$$B = \begin{pmatrix} 1 & 0 & 0 & 9 & 8 \\ 2 & 1 & 0 & 7 & 6 \\ 3 & 2 & 1 & 5 & 3 \\ 0 & 0 & 0 & 7 & 4 \\ 0 & 0 & 0 & 5 & 3 \end{pmatrix}.$$

(c)(11 points)

$$M_n = \begin{pmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 2 & 2 & \dots & 2 \\ 1 & 2 & 3 & 3 & \dots & 3 \\ 1 & 2 & 3 & 4 & \dots & 4 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & 2 & 3 & 4 & \dots & n \end{pmatrix}$$

(the ijth entry of M_n is the minimum of i and j, n is arbitrary, positive integer).

Problem 2 (30 points)

(a)(15 points) Using the formula for the inverse in terms of the adjoint, find the inverse of the matrix

$$A = \begin{pmatrix} \cos(\theta) & 1 & -\sin(\theta) \\ 0 & 2 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix}.$$

(b) (15 points) Using Cramer's rule, find the solution of the system

$$\begin{cases} 2X_1 & + & X_2 & & & = & 1 \\ X_1 & + & 2X_2 & + & X_3 & = & 0 \\ & & X_2 & + & 2X_3 & = & 0 \end{cases}$$

(the determinant of the matrix associated to the system is 4)

Problem 3(27 points) Let P^n be the real vector space of polynomials in the variable X of degree $\leq n$, with real coefficients.

(a) (15 points) Consider the following two bases of P^2: $\mathcal{B} = \{1, X, X^2\}$ and $\mathcal{C} = \{X+1, X-1, X^2+2X\}$. Find the change of basis matrix A such that

$$(f)_{\mathcal{C}} = A \cdot (f)_{\mathcal{B}},$$

for any polynomial $f \in P^2$.

(b) (12 points) Let $T : P^n \to P^n$ be the linear transformation given by

$$T(f(X)) = Xf'(X) + f(X).$$

Find the matrix associated to T with respect to the standard basis of P^n.

Problem 4(10 points) Let A be an $n \times n$ matrix, with odd integer entries on the diagonal and even integer entries everywhere else. Show that A must be invertible.