18.700 STUDY GUIDE FOR EXAM 2

Exam 2 will be in lecture on Friday, October 28. It will be closed book, closed notes, and calculators will not be allowed. You will have approximately 50 minutes for the exam. You should show all work, unless instructed otherwise; partial credit will be given only for work shown.

This guide contains a checklist of important skills, definitions and theorems to learn before Exam 2. You do not need to memorize the proofs of the theorems, but you should learn the statements and be able to use the theorems.

Skills checklist:

1. Given two ordered bases of a vector space, find the coordinate change matrix, 1.5.
2. Compute the determinant of a matrix by using elementary row operations and by using cofactor expansion/Laplace expansion, 2.1 and 2.3.
3. Determine whether an $n \times n$ matrix is invertible by computing its determinant, 2.1.
4. Find the solution of a square system of linear equations using Cramer's rule, 2.3.
5. Given the determinant of $n \times n$ matrices A and B, find the determinant of AB, find the determinant of A^t and find the determinant of A^{-1} (if it exists).
6. Given a permutation of $\{1, \dots, n\}$, compute the sign $sg(\sigma)$ by counting inversions and by computing a determinant, 2.2.
7. Given a function $T : V \to W$, check if it is a linear transformation and and compute its $m \times n$ matrix representation, 3.1.

Definition checklist: Learn each of the following definitions. n-linear, alternating, determinant function (Def. 2.1.1); Upper triangular matrix, Lower triangular matrix, Diagonal matrix (Def. 2.1.6); Permutation of n, transposition, symmetric group S_n (Def. 2.2.1); inversions of a permutation, sign of a permutation (Def. 2.2.2); determinant of a matrix (Def. 2.2.3); ijth maximal submatrix of a matrix (Def. 2.3.1); ith row cofactor expansion, jth column cofactor expansion (Def. 2.3.3); adjoint matrix of a matrix (Def. 2.3.6); linear transformation (Def. 3.1.1); associated linear transformation of an $m \times n$ matrix (Example 3.1.2(iii)); (standard) matrix representation of a linear transformation (page 175)

Theorem checklist: Learn the statements of the following theorems (you do not need to learn them verbatim, but learn their meaning). You do not need to memorize the proofs, but be able to use the theorems.

Theorem 2.1.4 The determinant of a matrix changes in a prescribed way upon performing an elementary row operation.

Theorem 2.1.13 The determinant of a product of $n \times n$ matrices is the product of the determinants.

Theorem 2.1.14 The determinant of the transpose of a square matrix equals the determinant of the matrix.

Theorem 2.2.9 A determinant function is given by $det(A) = \sum_{\sigma \in S_n} sg(\sigma) a_{1\sigma_1} \dots a_{n\sigma_n}$.

Theorem 2.3.5 Cofactor expansion along any row or any column gives a determinant function.

Theorem 2.3.8 Cramer's rule gives a solution of a nonsingular $n \times n$ system of linear equations.

Theorem 3.1.5 There is a unique matrix A associated to a linear transformation $G : \mathbb{F}^n \to \mathbb{F}^m$.