

18.701 Problem Set 1

As a general rule, you are expected to prove your assertions in these problem sets.

1. Chapter 1, Problem 5.3.

2. Chapter 1, Miscellaneous Problem 4.

This is a hard problem, because you must unlearn something that you have been taught in school. Make some experiments to understand that there is a difficulty. Then explain the sequence of steps logically, without reference to matrices or linear equations. Do the same for the sequence that attempts to check the work. Both sequences are interesting.

Keep in mind that the meaning of an equation may depend on context. $3 + 5 = 8$ is true, but $AX = B$ is neither true nor false: It is the equation we are supposed to solve. Writing $AX = B$ is a natural way to begin, but simply restating the equation doesn't make it true. Keeping this in mind, write the logic of each step carefully in words, to see what the sequence shows, and what it does not show.

3. The problem here is to determine the positions in the plane that can be reached in a certain game. The starting position is the point $(1, 1)$, and a permissible "move" replaces a point (a, b) by one of the points $(a + b, b)$ or $(a, a + b)$. So the position after the first move will be either $(2, 1)$ or $(1, 2)$. The position after two moves can be $(3, 1)$, $(2, 3)$, $(3, 2)$, or $(1, 3)$. Determine all points that can be reached in a finite number of moves.

4. Prove that the two matrices

$$E = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \text{ and } E' = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

generate the group $SL_2(\mathbb{Z})$ of all *integer* matrices with determinant 1. Remember that the subgroup they generate consists of all elements that can be expressed as products using the four elements E, E', E^{-1}, E'^{-1} .

5. Chapter 6, Problem 6.15.

Diagnostic Problem

Chapter 2, Problem 2.11.

This is a problem on logical reasoning. I will read your solution, but it will not count towards your grade. Please work the problem entirely by yourself. Try to write as perfect a solution as you can, and put it, with your name, on a separate sheet of paper. Hand it in with the first problem set.