

### 18.701 Problem Set 3

1. Chapter 2, Problem 6.12.
2. The dihedral group  $G = D_4$  is generated by two elements  $x, y$  with the relations  $x^4 = 1, y^2 = 1, yx = x^3y$ . Every element is a product  $x^i y^j$ , with  $0 \leq i \leq 3$  and  $0 \leq j \leq 1$ .
  - (a) Partition  $G$  into conjugacy classes explicitly.
  - (b) Find all subgroups of  $G$ , and determine which are normal subgroups.
3. Let  $G$  be the symmetric group  $S_4$ , and let  $H$  be the subset consisting of the four elements

$$\{(1), (12)(34), (13)(24), (14)(23)\}.$$

Prove that  $H$  is a normal subgroup of  $G$ , and identify the quotient group  $G/H$ .

4. Chapter 2, Misc. Problem 7(a).
5. Chapter 2, Misc. Problem 9(a).
6. Chapter 3, Problem 2.14.
7. Chapter 3, Problem 2.17.