December 4, 2003

18.701 Solutions to Practice Quiz 3

1. Let V be the vector space of real 2×2 matrices. Is the bilinear form $\langle A, B \rangle = \text{trace}\, AB$ on V symmetric? Is it positive definite?

2. Let W be the subspace of $V = \mathbb{R}^3$ spanned by the vectors $(1,1,0)^t$ and $(0,1,1)^t$. Write a formula that computes the orthogonal projection of a vector $v = (x,y,z)^t \in V$ to W.

3. In these questions, A denotes a 3×3 real skew-symmetric matrix($A^t = -A$).
(i) Prove that the determinant of A is zero.
(ii) What can be said about the complex eigenvalues of A?
(iii) Prove that the linear operator on $\mathbb{C}^3$ defined by A is a normal operator.
(iv) What does the spectral theorem say about this operator?
(v) Show that $P = e^A$ is an element of SO_3. (Don't forget to show that $\det P = 1$.)
(vi) Describe the axis of rotation of $P = e^A$ in terms of the matrix A.

4. Let G be the subgroup of $GL_2(\mathbb{R})$ consisting of the matrices

$$P = \begin{pmatrix} x & y \\ 0 & x^{-1} \end{pmatrix},$$

where $x > 0$ and y is an arbitrary real number.
(i) Decompose the set of matrices $P \in G$ with $x = 1$ into conjugacy classes.
(ii) Determine the matrices A such that e^{At} is a one parameter group in G.