November 1, 2003

18.701 Practice Quiz 2

You are required to present your reasoning for each problem.

1. (15 points) Determine the complex eigenvalues and eigenvectors of the rotation matrix

$$\begin{pmatrix} c & -s \\ s & c \end{pmatrix},$$

where $c = cos\theta$ and $s = sin\theta$.

2. (20 points) Let G be the group of symmetries of an equilateral triangular prism P, including the orientation reversing symmetries.
(a) Determine the order of the group G.
(b) Determine the stabilizer of one of the rectangular faces of P.

picture of prism goes here

3. (15 points) Let L be the lattice spanned by the lattice basis $(2, 0)$ and $(1, 1)$. Suppose the letter T is placed at each lattice point p, right side up if the coordinates of p are even, and upside down if they are odd. Determine the point group of this pattern.

4. (20 points) Let s denote the reflection of the plane about the shifted vertical axis $x = 1$.
(a) Find an isometry g of the plane such that $s = grg^{-1}$, where r is the standard reflection about the x-axis.
(b) Write s explicitly in the form $t_a\rho_\theta r$.

5. (30 points) The class equation of a group G is

$$20 = 1 + 4 + 5 + 5 + 5.$$

(a) Let x be an element in the conjugacy class of order 4. What is the order of x?
(b) Show that G contains an element of order 2.
(c) Does G have a subgroup of order 4? If so, is it a normal subgroup?
(d) Does G have a subgroup of order 5? If so, is it a normal subgroup?