

September 28, 2003

18.701 Quiz 1

(This is last year's quiz. We spent about one lecture more on vector spaces this year, so there may be slightly more emphasis there than on this year's quiz.)

You are required to present your reasoning for each problem.

1. (10 points) What integers are possible as orders of an element of a group order 20?
2. (15 points) Let $\phi : G \rightarrow G'$ be a group homomorphism. Suppose that you know $|G| = 18$, $|G'| = 15$, and that ϕ is not the trivial homomorphism. (The trivial homomorphism maps all elements of G to 1.) What can be said about the order of the image of ϕ ?
3. (20 points) Let G be the general linear group $GL_2(\mathbb{R})$, and let H be the subset of matrices $A \in G$ such that $EAE^{-1} = A$, where E is the matrix

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Determine whether H is a subgroup of G and whether H is a normal subgroup of G .

4. (15 points) Let G be a group. Define a relation on G , by $a \equiv b$ if $b = gag^{-1}$ for some element $g \in G$ (i.e., b is a conjugate of a). Show that this is an equivalence relation.
5. (20 points) Prove that the three functions x^2 , $\cos x$, and e^x are linearly independent over the field of real numbers.
6. (20 points) Determine the number of subspaces of the vector space F^2 , when F is the field of integers modulo 3.