February 12 , 2003

18.702 Problem Set 2

due Wednesday, February 19

1. Show how to construct a unitary matrix from a character table, and use this to prove that the columns of a character table are orthogonal.

2. Chapter 9, exercise 5.13 a,b.

3. Chapter 5, exercise 5.15 a.

4. Let ρ be a d-dimensional representation of a finite group G, and let σ be a one-dimensional representation of G. Define $\rho'_g = \sigma_g \rho_g$, where the right side stands for the product of a scalar and a matrix. Show that ρ' is a representation, and describe its character.

5. The symmetric group S_n operates on $\mathbb{C}^n$ by permuting the coordinates. Decompose this representation into irreducible representations.

Comment: This problem is closely related to one assigned in 18.701, to find the rank of the matrix whose rows are the permutations of a given vector. I recommend against using the orthogonality relations.

6. Determine the character table of a nonabelian group of order 21.