

Practice Quiz 2

1. Factor $3 + i$ in the ring $\mathbb{Z}[i]$ of Gauss integers.
2. Let R' be the ring obtained from the real numbers \mathbb{R} by adjoining an element ϵ with the relation $\epsilon^2 = 0$.
 - (a) How many ideals does R' contain?
 - (b) Let $\phi : \mathbb{R}[x] \rightarrow R'$ be the map defined by the substitution $x = 1 + \epsilon$. Determine the kernel and the image of ϕ .
3. Let I be the principal ideal of $R = \mathbb{Q}[x]$ generated by the element $x^3 + x + 1$. Show that $\overline{R} = R/I$ is a field.
4. Let $R = \mathbb{Z}[\delta]$, where $\delta = \sqrt{-10}$.
 - (a) Decide whether or not 3 remains prime in R .
 - (b) The prime 7 splits. Find generators for one of the prime factors of the principal ideal (7) in R .
 - (c) Find a lattice basis for the product AB of the ideals $A = (2, \delta)$ and $B = (5, \delta)$.
5. Describe as a direct sum of cyclic groups the abelian group which is presented by the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 0 \\ 5 & 1 \end{pmatrix}.$$

6. Find a basis for the free \mathbb{Z} -module of integer solutions of the equation $6x - 5y + 3z = 0$.