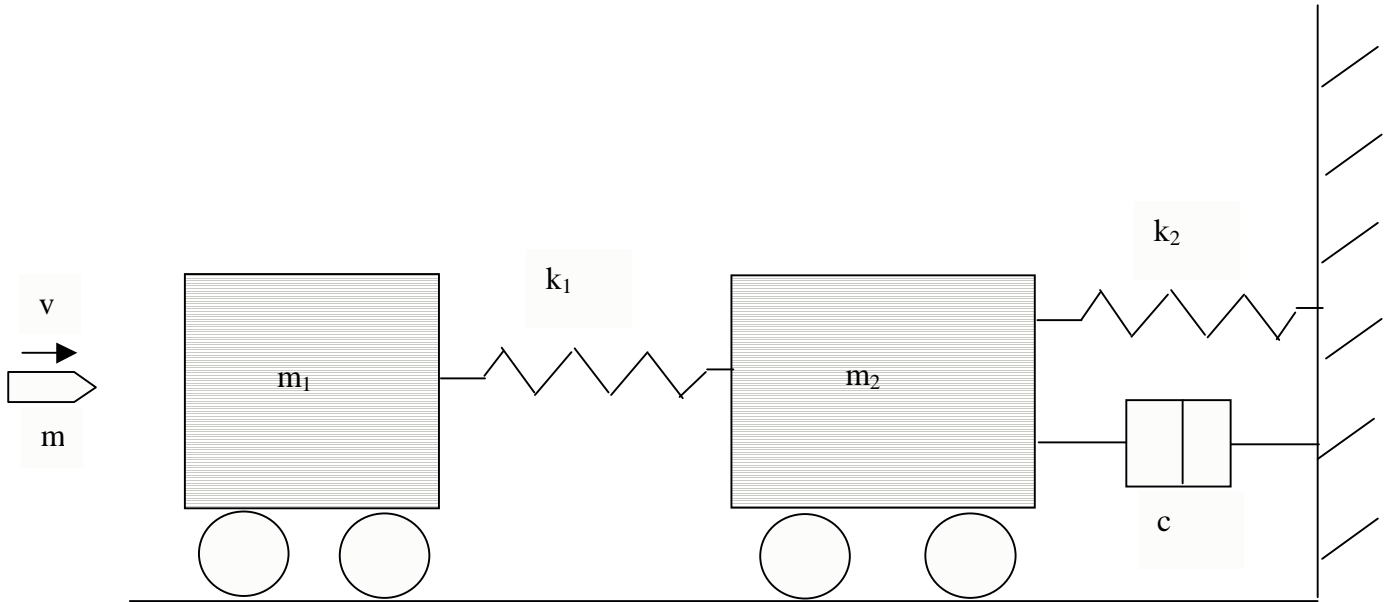


Figure for Problem 1:



Problem 1:

(25 Points)

At time $t=0$ a projectile of mass m and velocity v collides with mass m_1 and gets stuck to it. Just before the collision masses m_1 and m_2 are in static equilibrium and the springs k_1 and k_2 are unstretched.

- a) Using the direct method find the ordinary differential equations of motion *after* the collision.

(10 Points)

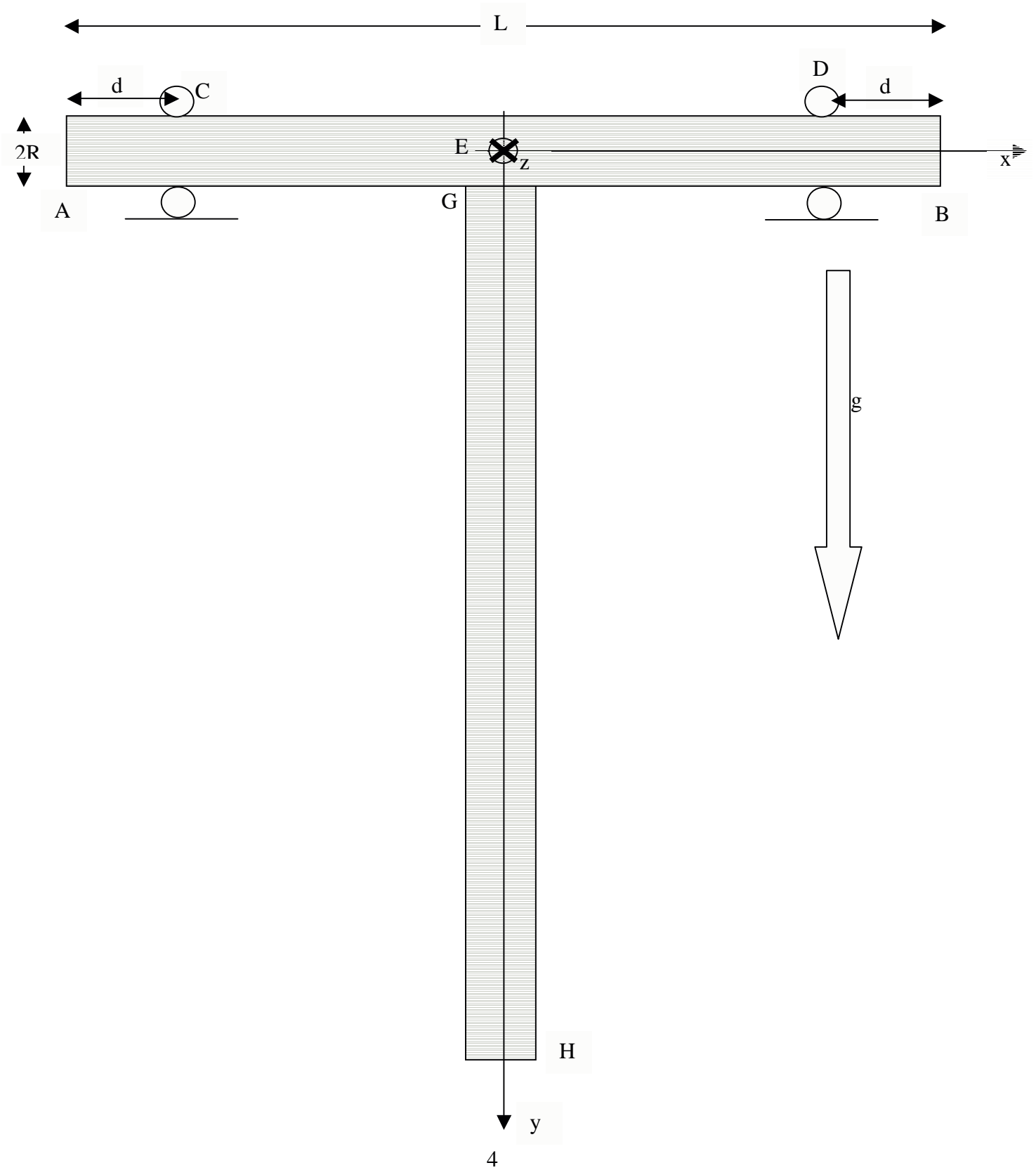
- b) Using the indirect method find the ordinary differential equations of motion *after* the collision.

(10 Points)

- c) What are the initial conditions for solving the resulting system of ordinary differential equations?

(5 Points)

Figure for Problem 2:



Problem 2:**(25 Points)**

A circular cylinder AB of radius R , length L and mass M is able to rotate about its axis of rotational symmetry, Ex which is horizontal, being supported by two frictionless bearings C and D , at a distance d from A and B .

Another identical circular cylinder GH is rigidly attached to the mid point of AB so that the axes of the two cylinders are orthogonal.

At time $t < 0$ the system is at rest as in the figure in a gravity field g . Then at $t = 0$ the system is suddenly given an initial angular velocity ω_0 , about the axis of cylinder AB , i.e. about the axis Ex .

a) Find the inertia tensor of the system of the two cylinders in the $Exyz$ system.

(10 Points)

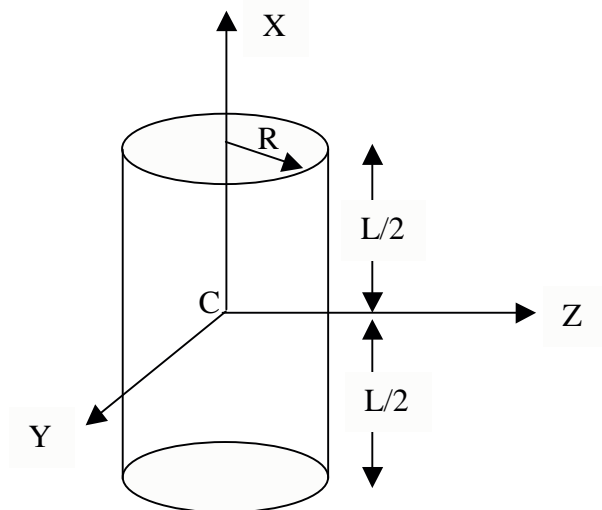
b) Determine the ordinary differential equation of motion of the system for $t > 0$.

(10 Points)

c) What are the initial conditions for solving this ordinary differential equation?

(5 Points)

Hint: For a homogeneous cylinder with centroid C , mass M , radius R and length L :

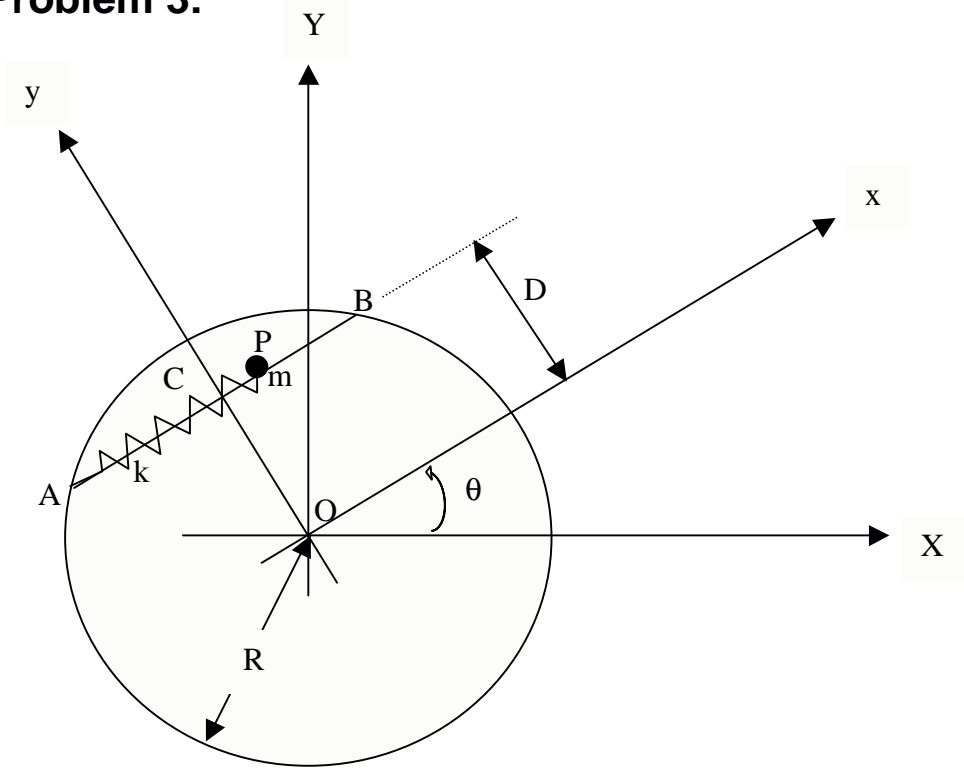


$$I_{xx} = \frac{MR^2}{2}$$

$$I_{yy} = I_{zz} = \frac{M}{12} [3R^2 + L^2]$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

Figure for Problem 3:



Problem 3:**(50 Points)**

A circular disk of radius R , with mass moment of inertia $I_o = MR^2/2$ about the center O is able to rotate about a fixed vertical axis OZ without friction. A bead of mass m can move along a massless track AB at a distance D from O without friction. The bead is restrained to point A on the disk via a massless spring of spring constant k . The unstretched length of the spring is AC , where C is the mid point of AB .

Consider a rotating coordinate system Oxy attached to the disk, and the angle between the inertial axis OX and Ox is $\theta(t)$. Further let $CP=x$. Use θ and x as generalized coordinates to position the disk and the bead.

- a) Using the indirect method derive the following two nonlinear ordinary differential equations of motion of the system:

$$\ddot{\bar{x}} - \ddot{\theta} + [\omega_o^2 - \dot{\theta}^2]\bar{x} = 0 \quad \text{and} \quad \ddot{\theta} - \lambda[\ddot{\bar{x}} - \ddot{\theta}(1 + \bar{x}^2) - 2\dot{\theta}\bar{x}\dot{\bar{x}}] = 0$$

$$\text{where } \bar{x} = x/D, \quad \omega_o^2 = \frac{k}{m} \quad \text{and} \quad \lambda = 2\frac{m}{M}\left(\frac{D}{R}\right)^2$$

Hint: For questions (b) to (e), use of the above two nonlinear ordinary differential equations of motion from question (a) without solution of question (a) is permitted and will carry full credit.

(15 Points)

- b) Now assume the disk is forced to rotate at a fixed angular velocity $\dot{\theta} = \omega = \text{constant}$ via an external torque about the axis OZ . Derive the ordinary differential equation of motion of the bead in terms of \bar{x} .

(10 Points)

- c) Under the assumptions of question (b), derive an expression for the external torque on the disk, necessary to keep $\dot{\theta} = \omega$ a given constant.

Hint: Determine the force from the bead to the track AB normal to the track AB and the force from the spring on the disk at point A .

(15 Points)

- d) Using the differential equation derived in question (b), find a condition under which the static equilibrium $\bar{x} = 0$ of the bead is *unstable*.

(10 Points)

- e) Assume that the condition derived in question (d) holds and that at time $t=0$ the bead has, $\dot{x}(0) = v_o > 0$, $x(0) = 0$. Find how much time will elapse until the bead reaches point B , distinguishing the two cases:

$$\omega = \pm \sqrt{\frac{k}{m}} \quad \text{or} \quad |\omega| > \sqrt{\frac{k}{m}}$$

(Extra: 10 Points)