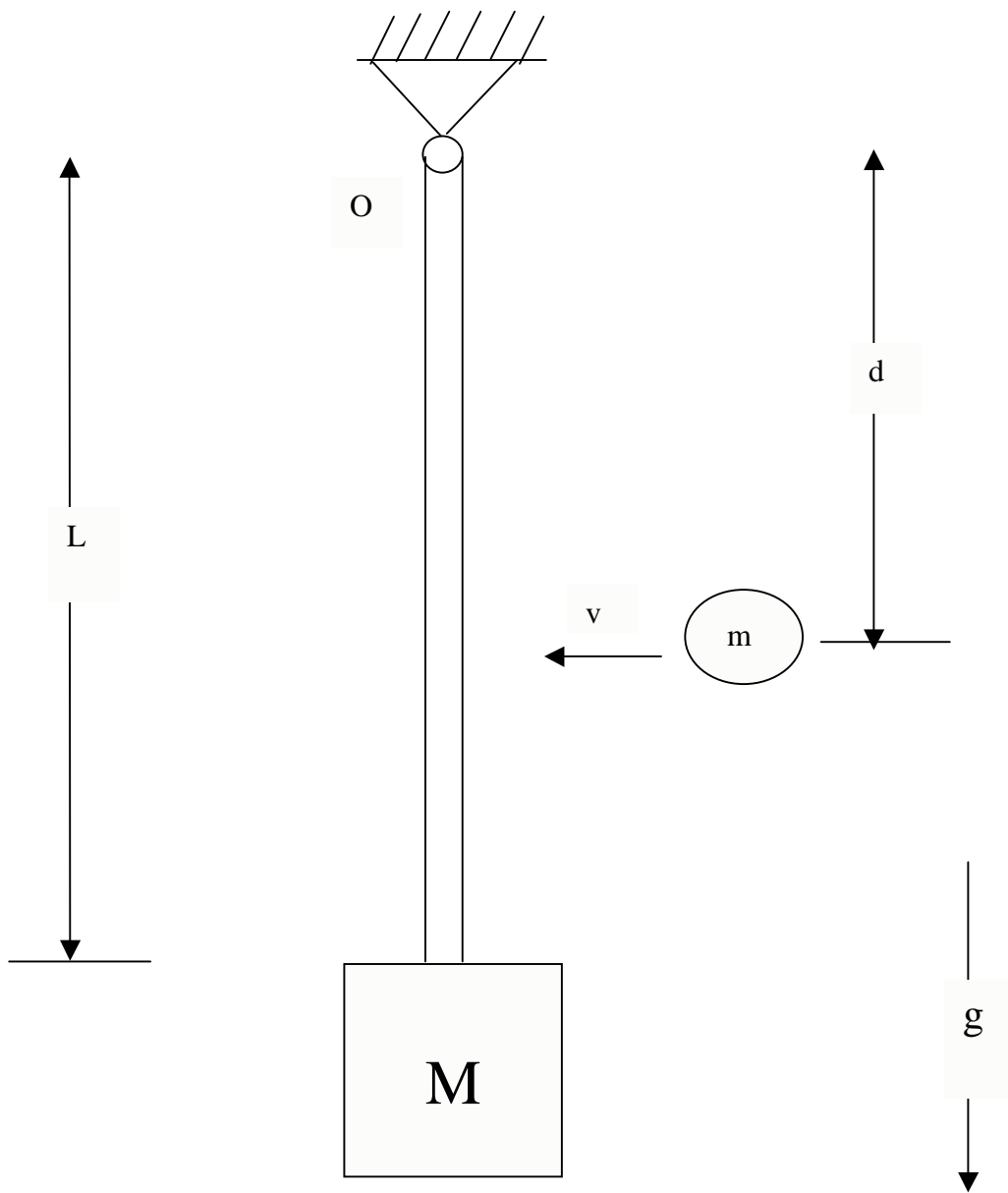


Figure for Problem 1:



Problem 1:**(50 Points)**

A pendulum of mass M with a massless rigid rod of length L hanging from a fixed frictionless pin O is initially at rest in a field with acceleration due to gravity g . The mass M can be considered as a point mass (particle with negligible dimensions). A particle of mass m and horizontal velocity v flying at a vertical distance d from the pin collides with the pendulum rod and sticks to it.

- a) a i. Explain why kinetic energy and linear momentum are not in general conserved during the collision.
- a ii. Determine the angular velocity and kinetic energy of the system immediately after the collision, and the kinetic energy loss during the collision, as a fraction of the initial kinetic energy of the mass m .
- a iii. Determine the impulse from the pin O on the rod during the time of the collision.

Hint: Impulse is the integral of the force $\vec{F}(t)$ over the time of collision $\vec{J} = \int_{0^-}^{0^+} \vec{F}(t) dt$, assuming the time interval of collision is very small.

(2 + 10 + 5 = 17 Points)

- b) Determine the second order (nonlinear) ordinary differential equation of motion of the system after the collision. The motion can be described in terms of the angle $\theta(t)$ the rod makes with the vertical line through O .

(10 Points)

- c) c i. Determine the maximum angle θ_{max} the pendulum will make with the vertical after the collision.
- c ii. Determine under what condition for the initial speed v will the maximum angle of the pendulum after collision be equal to π radians.

(5 + 3 = 8 Points)

- d) Determine an expression for the force from the rod on the pin after the collision as a function of $\theta(t)$.

Hint: Eliminate any dependence on $\dot{\theta}(t)$ and leave only the dependence on $\theta(t)$ using an appropriate conservation law valid after the collision.

(10 Points)

- e) Determine an *integral expression* for the time that will elapse between the collision and the maximum angle θ_{max} of question (c i).

Hint: Using an appropriate conservation law valid after the collision (just as in *question d*), express $\frac{d\theta}{dt} = f(\theta)$, where $f(\theta)$ is a function of θ .

(5 Points)

Figure for Problem 2:

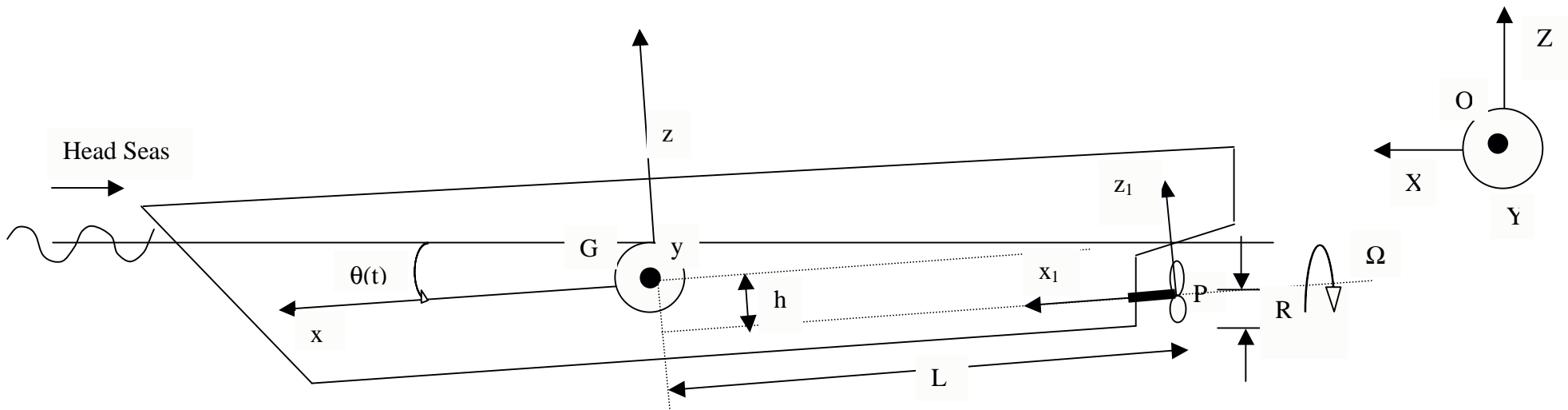
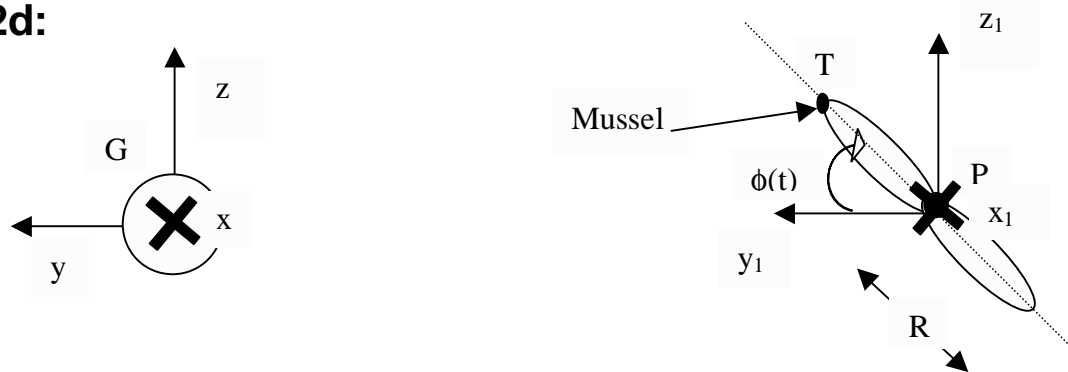


Figure for Problem 2c, 2d:



Problem 2:**(50 Points)**

A ship is traveling at a constant velocity V_{mean} with respect to the earth, along the axis OX , where $OXYZ$ is an inertial system (with unit vectors $\vec{I}, \vec{J}, \vec{K}$) attached to the earth. A moving system $Gxyz$ (with unit vectors $\vec{i}, \vec{j}, \vec{k}$) is attached to the ship at its center of gravity G , so that the axis Gx makes an angle $\theta(t)$ to the horizontal axis OX . Due to head seas (a storm with waves whose crests are parallel to OY axis and propagating along OX axis) the ship oscillates in the vertical plane heaving and surging, and rotates about an axis orthogonal to its center plane (ie. pitching). The velocity of G with respect to the inertial frame is

$$\vec{V}_G = V_{mean} \vec{I} + u(t) \vec{i} + w(t) \vec{k}$$

where $u(t)$ and $w(t)$ are the surge and heave velocities of the ship. The angular velocity of the ship with respect to $OXYZ$ is $\vec{\omega} = \omega_2(t) \vec{J}$, where $\omega_2(t) = \dot{\theta}(t)$, also known as pitch angular velocity.

At the same time the ship's propeller of radius R is rotating with its shaft at a constant angular velocity Ω with respect to the ship, ie.

$$\vec{\omega}_{propeller, ship} = \Omega \vec{i}$$

a) Determine the inertial velocity \vec{v}_p and acceleration \vec{a}_p of the center of the propeller, P using the ship fixed system $Gxyz$ for your calculations. For simplicity of algebra express $\vec{v}_p = V_{mean} \vec{I} + \alpha \vec{i} + \beta \vec{k}$ and $\vec{a}_p = \gamma \vec{i} + \delta \vec{k}$, and determine α, β, γ and δ . **(15 Points)**

b) Determine the angular velocity of the propeller with respect to $OXYZ$ and its time rate of change with respect to $OXYZ$. **(5 Points)**

c) The line PT makes an angle $\phi(t)$ with the Y direction (same as direction y_1) and assume, for simplicity that the line PT is on a plane Py_1z_1 parallel to the plane Gyz . Note that $\dot{\phi}(t) = \Omega$. Find the inertial velocity \vec{v}_T and the acceleration \vec{a}_T of the tip T for an arbitrary $\phi(t)$ and in particular when $\phi = \frac{3\pi}{2}$, ie. when the tip

T is at the lowest point.

Hint: Use a system $Px_1y_1z_1$ fixed to the ship with origin at the propeller center P and with the axes parallel to $Gxyz$, respectively. **(20 Points)**

d) Imagine that a mussel of mass m happens to be attached to the tip T , of the propeller. For the condition of question (c) and $\phi = \frac{3\pi}{2}$ find the contact force from the blade on the mussel assuming the

(static and dynamic) fluid force on the mussel is known and equal to $\vec{F}_f(t)$. Note that the acceleration of gravity is g . Assume that the propeller does not emerge from the ocean surface during the oscillations of the ship. **(10 Points)**