



## Introduction

In this lab, we measure the time constant of a first-order spring-damper system and then use this value to estimate the magnitude of the damping. Figure ?? shows the spring-damper system, and Figure ?? gives an idealized model which we will use here to predict its behavior.

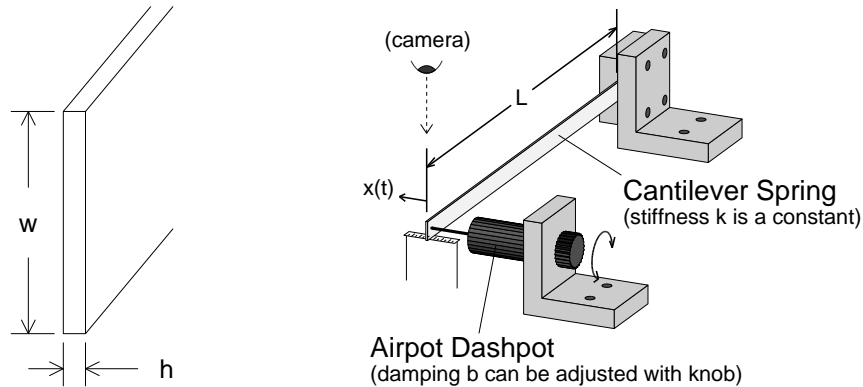


Figure 1: Picture of spring and dashpot system.  
 ( $L = 0.205$  m,  $w = 0.0127$  m,  $h = 0.00127$  m)

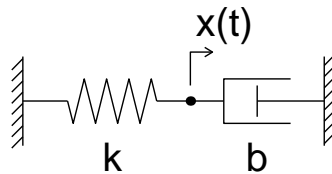


Figure 2: Idealized spring-damper system.

We will model the spring-steel cantilever as a linear spring; that is, we assume that a plot of force vs. displacement is a straight line. The constitutive relationship for a linear spring has the form

$$f_k(t) = kx(t) \quad (1)$$

where

- $f_k(t)$  = force applied to spring in  $x$  direction (N)
- $k$  = the spring stiffness (N/m)
- $x(t)$  = the displacement across the spring (m)

The stiffness  $k$  in the  $x$  direction (see Fig. 1) as seen at a position  $L$  along a uniform cantilever beam can be derived from the beam bending equations as

$$k = \frac{3EI}{L^3} \quad (2)$$

where

$$\begin{aligned} E &= \text{Young's modulus (210 GPa for spring steel)} \\ I &= wh^3/12 \\ w &= \text{width of the beam's cross section} \\ h &= \text{height (thickness) of the beam's cross section} \\ L &= \text{length of the beam} \end{aligned}$$

An Airpot<sup>1</sup> dashpot attached to the end of the cantilever adds damping to the system. The Airpot is an air piston-cylinder arrangement with a controllable "leak". The Airpot—with simplifying assumptions—can be modeled as a pure damper where the damping force is proportional to velocity, so we write its constitutive relationship as

$$f_b(t) = b\dot{x}(t) \quad (3)$$

where

$$\begin{aligned} f_b(t) &= \text{force applied to damper in the } x \text{ direction (N)} \\ b &= \text{the damping coefficient} \\ \dot{x}(t) &= \text{the velocity difference across the damper} \end{aligned}$$

From a free body diagram the spring and dashpot forces sum at the node joining them (be careful about signs as you go through this step!). Using the constitutive relationships for the spring and damper given by Equations (??) and (??), we can write a simple first-order model of the homogeneous response of our cantilever and Airpot system as

$$0 = b\dot{x}(t) + kx(t). \quad (4)$$

This is the *equation of motion* of the system. The solution to this differential equation has the form

$$x(t) = Ae^{-t/\tau} \quad (5)$$

(You are asked to derive the solution in Problem 3.)

In words, the free (or "natural") response of this model to an initial displacement decays exponentially. The exponential response of a first-order system is characterized by its *time constant*,  $\tau$ . The time constant is the time required for the magnitude of the output to "decay away" by about 63%. (See Prof. Trumper's supplementary notes on first-order system response for more details.)

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<sup>1</sup><http://www.airpot.com/>

### Problems

1. A model for the first-order spring-damper system is shown in Figure ???. The same image is labelled as “Model A” in Figure ?? below. Which of the other two models below (B or C) also model the same dynamic system? (Explain *briefly*.)

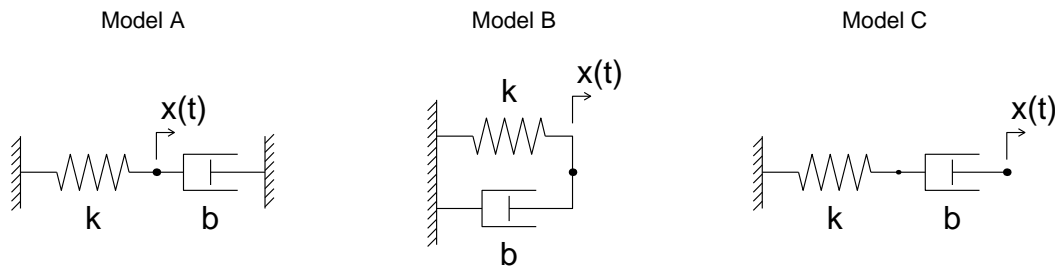


Figure 3: Idealized spring-damper systems.

2. Determine the numerical value of the spring constant  $k$  of the steel beam shown in Figure ??, at the position  $L$  where the Airpot is attached.
3. Derive the solution to Equation (??) if the system is given an initial displacement  $a_0$  and released. The solution should be of the form

$$x(t) = Ae^{-t/\tau} \quad (6)$$

where  $A$  is a constant to be determined from initial conditions, and  $\tau$  is the *time constant*.

4. Suppose that we have measured the following output data from a spring-dashpot system, and that the dimensions of the spring steel cantilever beam are as sketched in Figure ??.

time (s)	$x(t)$ (m)
0.00	1.57
0.05	1.17
0.10	0.89
0.15	0.68
0.20	0.52
0.25	0.40
0.30	0.30
0.35	0.23
0.40	0.17
0.45	0.13
0.50	0.11

- (a) Plot the data using MATLAB and include this plot in your prelab. You may use the MATLAB code on the next page to create the plot.
- (b) Use your plot to estimate the time constant. (Just get a rough estimate by “eyeballing” it.) Estimate  $A$  and  $\tau$  from the data, and compare your estimate with the Matlab plot done on the Matlab M-file exercise.
- (c) Determine the damping constant  $b$  that best fits this data, given the spring constant you calculated in Problem 2.

## MATLAB M-file exercise on plotting

You may use the code below to create your MATLAB plot of the data in Problem 4 if you wish. We are asking you to create a simple MATLAB plot in part to insure you have some (recent) experience in plotting within the MATLAB environment, since you will be asked to make similar plots during the laboratory sessions for 2.003.

When using MATLAB, you may often find it convenient to create an “m-file” to write out a sequence of MATLAB commands so you can more easily repeat the same set of commands (on multiple sets of data, for instance). An m-file can be either a function or a script. We suggest that you write a simple script to create your MATLAB plot for Problem 4, however you do **not** need to turn this script in with your prelab.

By MATLAB convention, an m-file must be given a name with a “.m” extension (hence the name). For instance, you might create an m-file saved as “lab1.m”. To run the m-file from the MATLAB command window, you would then type “lab1” (without the “.m” extension). MATLAB then executes each line in the corresponding m-file just as if you had typed them in the MATLAB command window. (Make sure your current working directory in MATLAB is the same as the one where your file is saved when you try to run your file!)

MATLAB is available on the MIT server. M-files or other work saved on the MECHENG computers will be accessible from the computers in the laboratory. As mentioned on page 1, there are several handouts on using MATLAB which are available from the “Additional Handouts” page of the course 2.003 on the MIT server.

```
% In MATLAB, any text after a '%' symbol on a line is a comment.  
% The ':' notation below creates a vector t from 0 to 0.5 with  
% step values of 0.05. t = [0 0.05 0.1 0.15 ... 0.5]
```

```
t=[0:0.05:0.5]  
y=[1.57 1.17 .89 .68 .52 .4 .3 .23 .17 .13 .11]  
plot(t,y,'b.-')  
xlabel('Time (s)')  
ylabel('Position (cm)')  
title('Position vs Time')  
grid on
```