2.094 Finite Element Analysis of Solids and Fluids
Spring 2008

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FINITE ELEMENT ANALYSIS OF SOLIDS AND FLUIDS

SPRING 2008

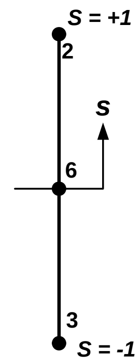
Quiz #1 - Solution

Instructor: Prof. K. J. Bathe Date: 04/03/2008

Problem 1 (10 points)

The interpolation functions are

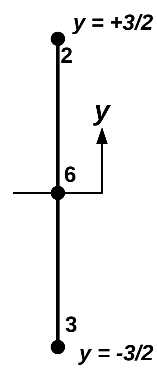
$$h_2 = \frac{1}{2}s(1+s) \quad ; h_3 = -\frac{1}{2}s(1-s) \quad ; h_6 = (1-s^2)$$



Then,

$$\underline{R}_S = \int_{-1}^{1} \begin{bmatrix} h_2 \\ h_3 \\ h_6 \end{bmatrix} (p)(4)\left(\frac{3}{2}\right) ds$$

Or

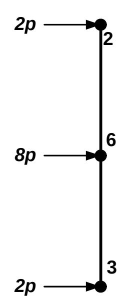


$$h_2 = \frac{1}{3}y\left(1+\frac{2}{3}y\right) \quad ; h_3 = -\frac{1}{3}y\left(1-\frac{2}{3}y\right) \quad ; h_6 = \left(1-\frac{4}{9}y^2\right)$$

$$\underline{R}_S = \int_{-3/2}^{3/2} \begin{bmatrix} h_2 \\ h_3 \\ h_6 \end{bmatrix} (p)(4)\, dy$$

Actually

$$\underline{R}_S = \begin{bmatrix} 2p \\ 2p \\ 8p \end{bmatrix}$$



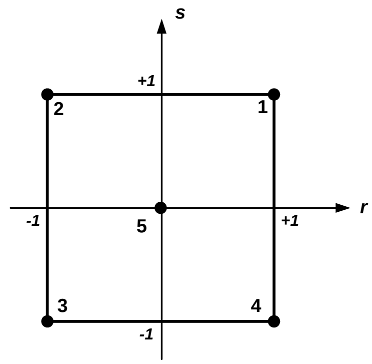
Problem 2 (10 points)

(a)

$$h_5 = (1-r^2)(1-s^2)$$

$$h_1 = \frac{1}{4}(1+r)(1+s) - \frac{1}{4}h_5 \quad ; h_2 = \frac{1}{4}(1-r)(1+s) - \frac{1}{4}h_5$$

$$h_3 = \frac{1}{4}(1-r)(1-s) - \frac{1}{4}h_5 \quad ; h_4 = \frac{1}{4}(1+r)(1-s) - \frac{1}{4}h_5$$

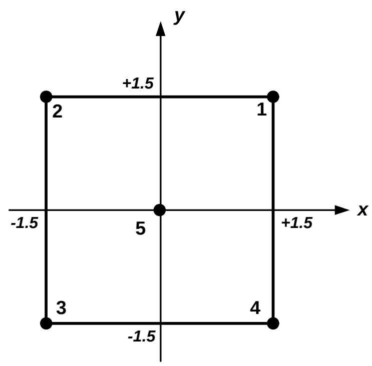


Or

$$h_5 = \left(1-\frac{4}{9}x^2\right)\left(1-\frac{4}{9}y^2\right)$$

$$h_1 = \frac{1}{4}\left(1+\frac{2}{3}x\right)\left(1+\frac{2}{3}y\right) - \frac{1}{4}h_5 \quad ; h_2 = \frac{1}{4}\left(1-\frac{2}{3}x\right)\left(1+\frac{2}{3}y\right) - \frac{1}{4}h_5$$

$$h_3 = \frac{1}{4}\left(1-\frac{2}{3}x\right)\left(1-\frac{2}{3}y\right) - \frac{1}{4}h_5 \quad ; h_4 = \frac{1}{4}\left(1+\frac{2}{3}x\right)\left(1-\frac{2}{3}y\right) - \frac{1}{4}h_5$$



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and

$$h_p = 1$$

(b)

$$\underline{J} = \begin{bmatrix} \frac{3}{2} & 0 \\ 0 & \frac{3}{2} \end{bmatrix} \text{ and } \underline{J}^{-1} = \begin{bmatrix} \frac{2}{3} & 0 \\ 0 & \frac{2}{3} \end{bmatrix}$$

$$\underline{B}_V\Big|_{u_1} = \begin{bmatrix} h_{1,x} \end{bmatrix} \qquad \text{and} \qquad \underline{B}_D\Big|_{u_1} = \begin{bmatrix} \frac{2}{3}h_{1,x} \\ -\frac{1}{3}h_{1,x} \\ h_{1,y} \\ -\frac{1}{3}h_{1,x} \end{bmatrix}$$

where

$$h_{1,x} = J_{11}^{-1}h_{1,r} + J_{12}^{-1}h_{1,s} = \frac{2}{3}h_{1,r} = \frac{1}{6}\left(1+2r+s-2rs^2\right)$$

$$h_{1,y} = J_{21}^{-1}h_{1,r} + J_{22}^{-1}h_{1,s} = \frac{2}{3}h_{1,s} = \frac{1}{6}\left(1+r+2s-2r^2s\right)$$

Or

$$\underline{B}_V\Big|_{u_1} = \begin{bmatrix} h_{1,x} \end{bmatrix} \qquad \text{and} \qquad \underline{B}_D\Big|_{u_1} = \begin{bmatrix} \frac{2}{3}h_{1,x} \\ -\frac{1}{3}h_{1,x} \\ h_{1,y} \\ -\frac{1}{3}h_{1,x} \end{bmatrix}$$

where

$$h_{1,x} = \frac{1}{6}\left(1+\frac{2}{3}y\right)+\frac{2}{9}x\left(1-\frac{4}{9}y^2\right)$$

$$h_{1,y} = \frac{1}{6}\left(1+\frac{2}{3}x\right)+\frac{2}{9}y\left(1-\frac{4}{9}x^2\right)$$