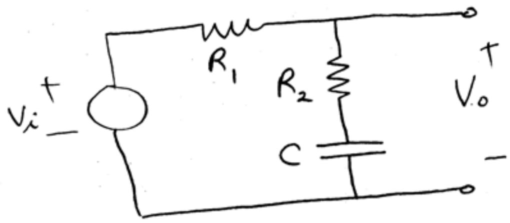
2.14/2.140 Problem Set 2

Assigned: Thurs. Feb. 15, 2007
Due: Thurs. Feb. 22, 2007, in class
Reading: Nise 2.1–2.4; Notes from course web page: Circuits; Op-amps; Frequency response
Reading for 2.140 students: Subtleties of the Laplace Transform, from course web page.

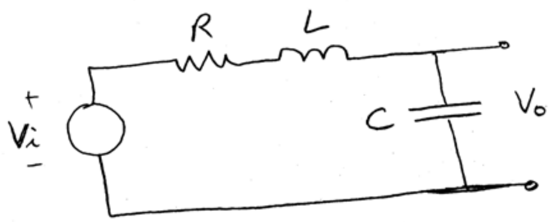
The following problems are assigned to both 2.14 and 2.140 students.

Problem 1 This problem reconsiders the circuit from last week, shown below



a) Make a carefully-dimensioned sketch of the Bode plot for the transfer function $V_o(s)/V_i(s)$ which you calculated last week, for the same values $R_1 = 3k\Omega$, $R_2 = 12k\Omega$, and $C = 10\ \mu$F. Do not use Matlab to plot this response; you need to be able to work out such simple cases by hand!

Problem 2 This problem reconsiders the circuit shown below



a) Make a carefully-dimensioned sketch of the Bode plot for the transfer function $V_o(s)/V_i(s)$ which you calculated last week, for the same values $L = 0.01$ H and the remaining parameters of the circuit such that $\omega_n = 10^5$ and $\zeta = 0.05$. Do not use Matlab to plot this response; you need to be able to work out such simple cases by hand!

Problem 3 Archive Problem 9.4

Problem 4 Archive Problem 9.6

Problem 5 Archive Problem 9.10

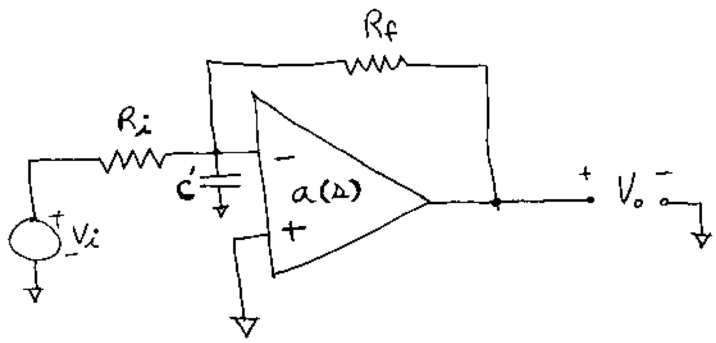
Problem 6 Archive Problem 11.5

Problem 7 Archive Problem 17.4

The following problems are assigned to only 2.140 students. Students in 2.14 are welcome to work these, but no extra credit will be given.

Problem G1 Consider the second-order mechanical system presented as an example in the notes on Subtleties of the Laplace Transform, available on the course web page. This example is solved via Laplace techniques for 3 different initial conditions. Repeat this analysis using an impulse-matching argument; be sure to show your reasoning.

Problem G2 The circuit shown below is an inverting op amp connection, with a capacitor C' connected between the inverting input and ground. Adding a capacitor in this location can have a significant effect on the stability of the op amp connection. This problem explores such an effect.

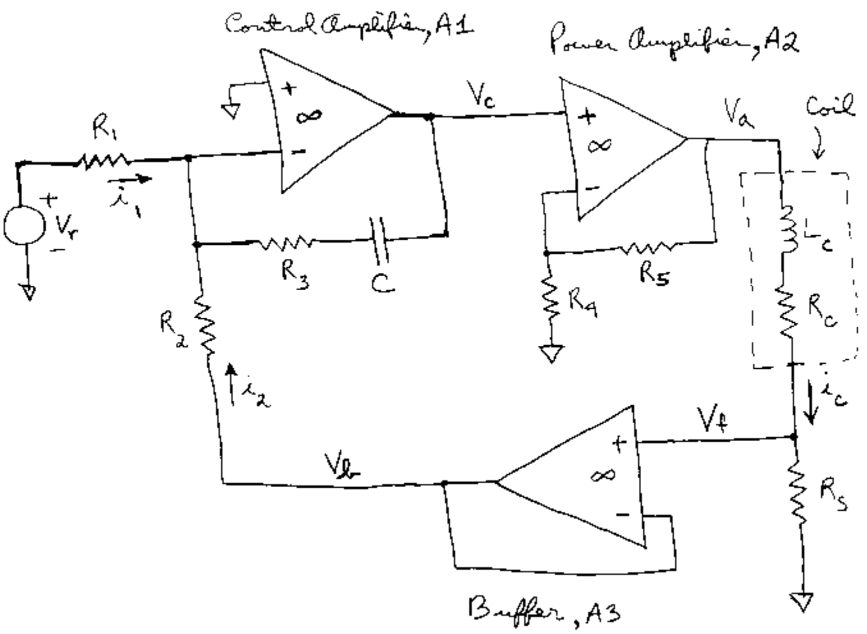


a) For this connection, draw a block diagram representing the circuit.

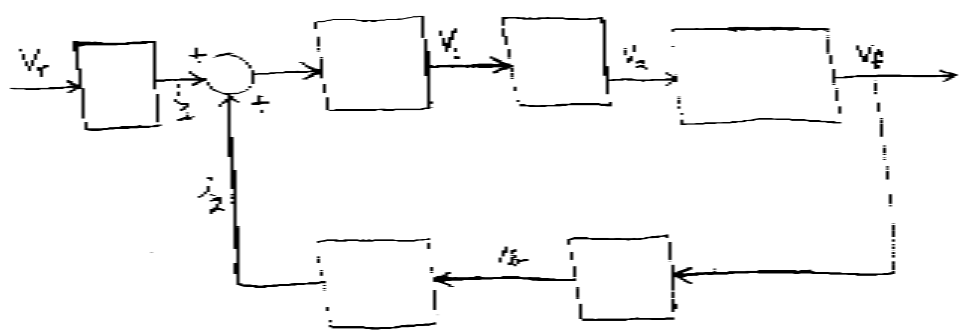
b) Let the components take the values $R_i = R_f = 10\ \text{k}\Omega$, $C = 10\ \mu\text{F}$, and $a(s) = 2\pi 10^6/s$. Calculate the transfer function $V_o(s)/V_i(s)$. What are the closed-loop damping ratio and natural frequency? Plot the system poles on the s-plane.

c) Make a carefully dimensioned hand-drawn Bode plot for this transfer function.

Problem G3 In the lab you will see power amplifiers used to control the *voltage* on a load such as a voice coil or motor. In practice, it is also common to use power amplifiers in a feedback loop to control the *current* through a load. This problem investigates the design of such current-controlled amplifier configurations. In the circuit shown below, the load is represented by a coil with inductance L_c and resistance R_c. The load current i_c is supplied by the power amplifier $A2$. The current is measured by passing it through the sense resistor R_s; the voltage V_f is then proportional to the load current. Thus if V_f is kept at a desired level, the load current can be regulated. The buffer amplifier $A3$ is used to avoid loading the main coil circuit; that is, current i_2 is supplied by amplifier $A3$. The control amplifier $A1$ is used to compare the reference voltage V_r with the feedback voltage V_b, and then to add dynamics to the loop via R_3 and C. That is, amplifier $A1$ is the compensation for the current feedback loop. Note that all the op amps are modeled as having infinite gain. All voltages labeled on the circuit are with respect to circuit ground.



a) The current feedback loop may be represented in the form of the block diagram shown below.



Develop expressions for each of the transfer functions in the blocks of this block diagram, and fill them in on your own version of the block diagram. Carefully note the system variables already shown on the block diagram, and fill in the transfer functions appropriately.

b) Suppose the input V_r has the constant value $V_r = V_0$. What is the steady-state load current i_C for this constant reference input? Explain your results.

c) Now let the parameters take the values $R_1 = R_2 = R_4 = 10$ kΩ, $R_5 = 50$ kΩ, $R_s = 1$ Ω, $L_c = 100$ mH, and $R_c = 10$ Ω. Chose the values of R_3 and C such that the closed-loop system poles have a natural frequency of $\omega_n = 1000$ rad/sec and a damping ratio of $\zeta = 0.4$.

d) For these parameter values, let the input be a negative unit step $V_r = -u_s(t)$. Make a graph of the response in current $i_C(t)$.