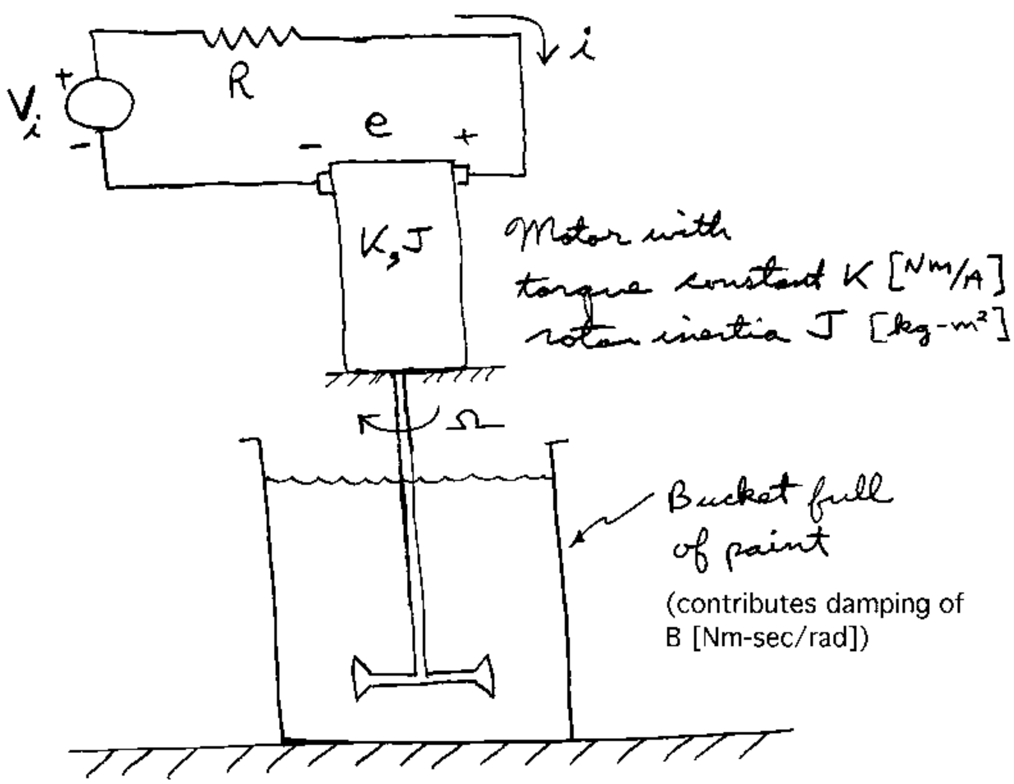
2.14/2.140 Problem Set 5

Assigned: Fri. March 16, 2007
Due: Thurs. March 22, 2007, in class
Reading: Notes Chapter 5 and 6.

The following problems are assigned to both 2.14 and 2.140 students.

Problem 1 This problem revisits the paint stirrer considered in Quiz 1. A brushed DC motor is used to stir a bucket of paint as shown in the figure below. The motor has a torque constant K [Nm/A] and the motor plus stirrer inertia is J [kg-m^2]. The motor is considered ideal in that it has no inductance and no resistance; the only resistance in the electrical circuit is R as shown. The rotor is rigidly linked to the stirrer, which has no flexibility; the combined assembly has a rotational velocity Ω rad/sec. The effect of the paint on the stirrer is modeled as a rotational damper B [Nms/rad].

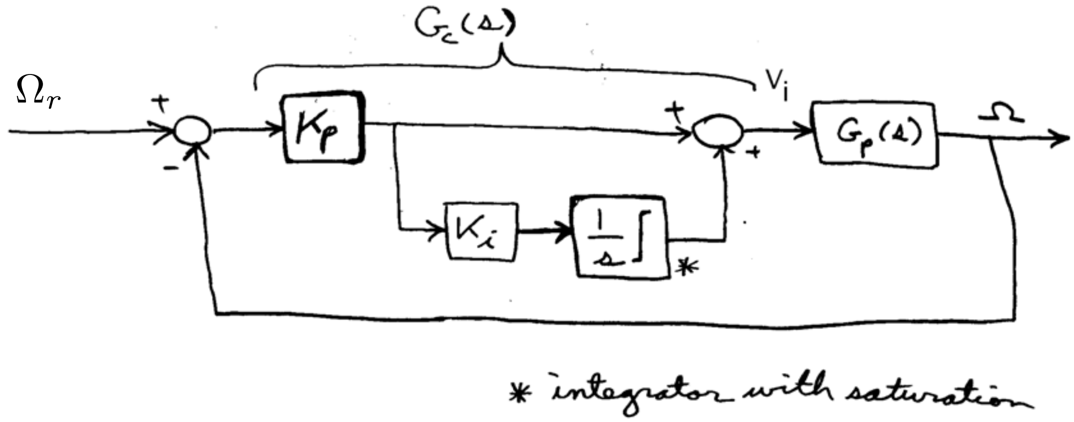


a) Solve for the transfer function $G_p(s) = \Omega(s)/V_i(s)$ in terms of the system parameters. Now set the parameter values as $K = 0.5$ Nm/A, $R = 5\ \Omega$, $B = 0.15$ Nms/rad, and $J = 0.2$ kgm^2. For these parameter values, make a hand sketch of the Bode plot for G_p.

b) Now, let the system input V_i be driven by a PI controller of the form

$$G_c(s) = K_p\left(1 + \frac{K_i}{s}\right).$$

This system has a speed reference input Ω_r and output Ω as shown in the block diagram below



Choose the controller gain values K_p and K_i to set the loop crossover frequency ω_c = 10 rad/sec, with a phase margin $\phi_m = 45°$. Show your calculations. You should be able to accomplish this design using hand-sketched Bode plots, and then using Matlab for confirmation.

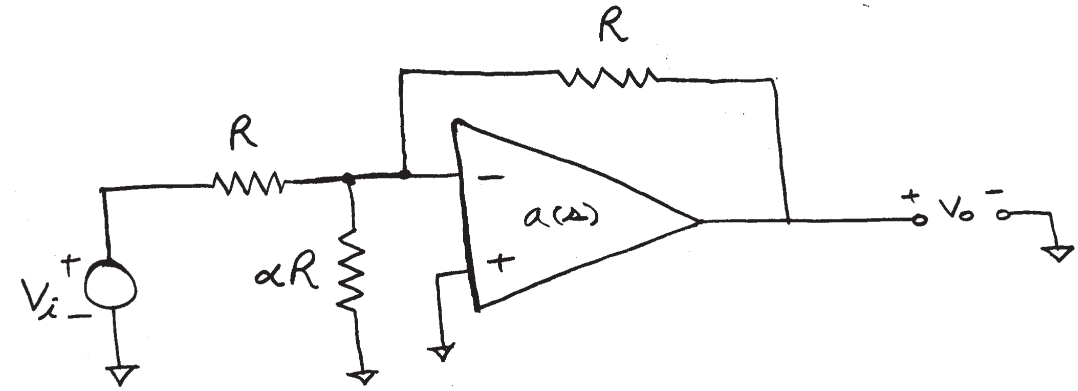
c) Create a Simulink simulation of the loop with the controller implemented as shown in the block diagram above. For now, let the integrator have an unbounded output (leave the integrator limit output box unchecked). Set your simulation to use the variable step solver with a max step size limit of 0.01 sec, and use the ode45 solver. Note that the standard simulation window has a simulation step time of 10 sec. You may want to adjust this value.

Let the input reference Ω_r take a step from zero to 10 rad/sec at $t = 0$, from initial rest conditions. Run this as a Simulink simulation and record and plot the responses $V_i(t)$ and $\Omega(t)$. What is the maxiumum value of the V_i during this transient? What is the steady-state value of V_i after the transient has settled?

d) Now set the integrator limits to ± 1.5 times the steady-state value of V_i from part c), and check the integrator limit box. (This insures that the integrator term is able to supply the required steady-state control effort for a speed of 10 rad/sec, but not much more.) Again let the input reference Ω_r take a step from zero to 10 rad/sec at $t = 0$, from initial rest conditions. Run this as a Simulink simulation and record and plot the responses $V_i(t)$ and $\Omega(t)$. How do these differ from the unlimited case in part c)?

e) Be sure to keep a copy of your calculations and this model, as we will continue studying it in the next problem set.

Problem 2 This problem revisits the op amp circuit from Quiz 1, which is shown below



Here the resistors take values of R, R, and αR, respectively. The resistors R set the ideal gain of the circuit, which is -1. The resistor αR can be used to detune the bandwidth of the opamp loop when the amplifier dynamics are too "hot" for the circuit configuration. In particular, like all real systems, op amps have additional high frequency dynamics which can give bad stability in some circuits. To understand this, let

$$a(s) = \frac{5 \times 10^7}{s(10^{-7}s + 1)^2}.$$

The additional pair of poles represent high-frequency dynamics in the internal opamp circuit.

a) First, we remove the resistor αR by letting $\alpha \to \infty$ (infinite resistance is an open circuit). For this configuration, calculate the loop crossover frequency and phase margin. Use Matlab to plot the unit step response of the circuit. You should find that this system is unstable. Compare the ring frequency of the unstable step response signal with the natural frequency of the closed loop dominant pole pair and with the loop crossover frequency. You should see that $\omega_n \approx \omega_c$. (When making ring frequency measurements on the step response, be sure to convert from Hz to rad/sec.)

b) Now pick a value of α which results in a loop phase margin $\phi_m = 45°$. For this value of α, what is the loop crossover frequency ω_c? Use Matlab to plot the unit step response of this detuned circuit. Compare the natural frequency of the closed loop dominant pole pair and with the loop crossover frequency.

Problem 3 In Prelab 2, you have/are designing P and PI controllers for motor velocity. This problem considers those designs from a loop-shaping perspective. Please use your prelab calculation results for this question. In all sections of this problem, you can ignore the antiwindup saturation block and gain K_a, since we are only considering linear issues here.

a) In part 2 of the prelab, you designed a proportional velocity controller with gain K_p. For your designed value of K_p, write an expression for the transfer function of the return ratio $L(s)$. Plot the Bode plot of $L(s)$. What are the crossover frequency and phase margin for this loop? What is the closed-loop pole location? How does this compare with the crossover frequency? What is the closed-loop bandwidth? How does this compare with the crossover frequency?

b) In part 3 of the prelab, you designed a PI velocity controller with gains K_p and K_i. For your designed values of K_p and K_i, write an expression for the transfer function of the return ratio $L(s)$. Plot the Bode plot of $L(s)$. What are the crossover frequency and phase margin for this loop? What are the closed-loop pole locations and natural frequency and damping ratio? How do these compare with the crossover frequency and phase margin on the basis of the approximations introduced in class? What is the closed-loop bandwidth? How does this compare with the crossover frequency?

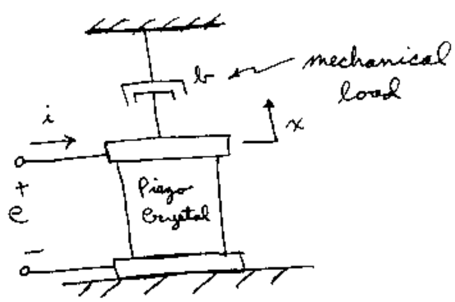
c) Now, let's look at a higher bandwidth design with lower phase margin. In this section, modify the PI velocity controller so that you achieve a crossover frequency of 200 rad/sec, with a phase margin of 15 degrees. What values of K_p and K_i result? Be sure to explain how you arrived at these parameter values. Plot the closed-loop poles that result. What are the values of ζ and ω_n associated with these root locations? How do these root locations compare with the those predicted by the approximations introduced in class on the basis of phase margin and crossover frequency?

The following problem is assigned to only 2.140 students. Students in 2.14 are welcome to work these, but no extra credit will be given.

Problem G1

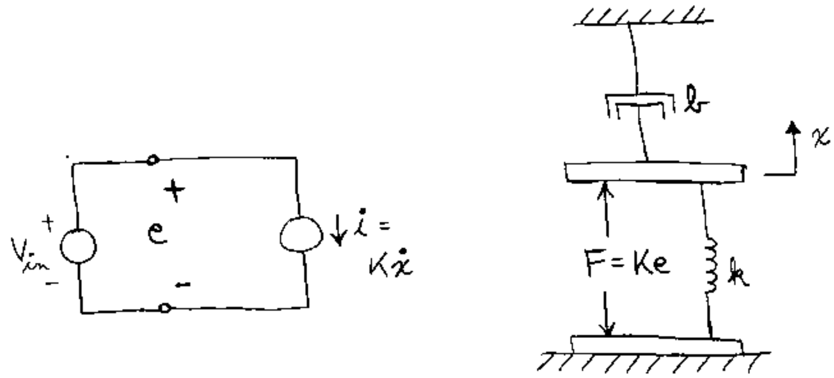
This problem was used as the ME quals systems written exam in 2005. It considers modeling and control issues associated with positioning systems driven with a *piezoelectric* actuator. A piezoelectric positioner is driven with an electrical input in order to produce a mechanical output and vice versa.

The figure below shows a model of a system incorporating a piezoelectric device.



Here the electrical terminals of the device are defined as having an input voltage $e(t)$ and an input current $i(t)$. The piezo crystal is sandwiched between end plates, with the bottom plate connected to mechanical ground. The motion of the upper plate in the vertical direction is defined as $x(t)$. A mechanical load consisting of a damper with value b is connected between the upper plate and mechanical ground. We model the piezoelectric device and plates as massless. Motion is considered to be constrained to the x direction.

The electrical/mechanical coupling of the device is modeled as shown below



Here, the piezoelectric actuator is modeled internally as a dependent force source $F(t)$ in parallel with a spring k. The force depends linearly upon the input voltage as $F(t) = Ke(t)$, where K is a scale factor with units of N/V. The spring k models the internal stiffness of the piezoelectric actuator. The force F is applied to the massless upper plate which connects the damper and spring.

The electrical portion of the model is shown on the left in the figure. Here, a dependent current source has a value $i(t) = K\dot{x}(t)$. The system is driven with a voltage source $V_{in}(t)$.

a) Calculate the transfer function $X(s)/V_{in}(s)$. Clearly show the steps in your development.

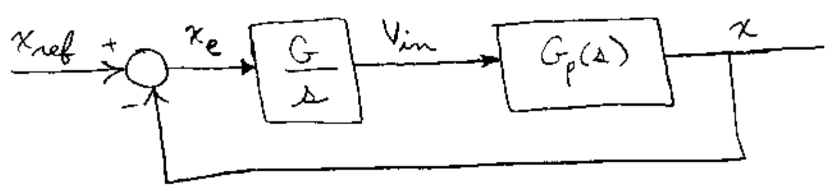
b) Assume initial rest conditions. Let the input voltage be a unit step: $V_{in}(t) = u(t)$. Calculate a closed-form solution for the resulting displacement $x(t)$ and make a graph of $x(t)$ versus time.

c) Develop a closed-form expression for the input electrical power $P(t) = e(t)i(t)$ associated with the transient you solved for in part b) above, and make a graph of $P(t)$ versus time.

We now learn that the top plate has finite mass, and so the model developed earlier needs to be augmented. Measurements indicate that the input/output transfer function is now given by

$$G_p(s) \equiv \frac{X(s)}{V_{in}(s)} = \frac{1}{10^{-2}s^2 + 10s + 10^6}.$$

This experimentally-adjusted model is to be used to design the feedback loop shown below.



The controller for this loop is an integral controller $G_c(s) \equiv G/s$, where G is an adjustable gain associated with the integrator.

d) Sketch a root locus plot for this control loop as the gain $G > 0$ is varied. For what range of gains G is the loop stable?

e) The loop transfer function (sometimes called the return ratio) for this loop is given by $L(s) = G_c(s)G_p(s)$. Make a careful hand sketch of the Bode plot for $L(s)$, showing the effect of G as a parameter, and using the numerical representation of $G_p(s)$ given above.

f) What value of G will result in a loop crossover frequency $\omega_c = 100$ rad/sec? (Recall that the crossover frequency is the frequency for which the loop transfer function magnitude crosses through unity, that is, we require $|L(j100)| = 1$.)

What are the phase margin and gain margin for the loop with this crossover frequency? Indicate these parameters on a Bode plot for the loop.