

## 2.14/2.140 Problem Set 7

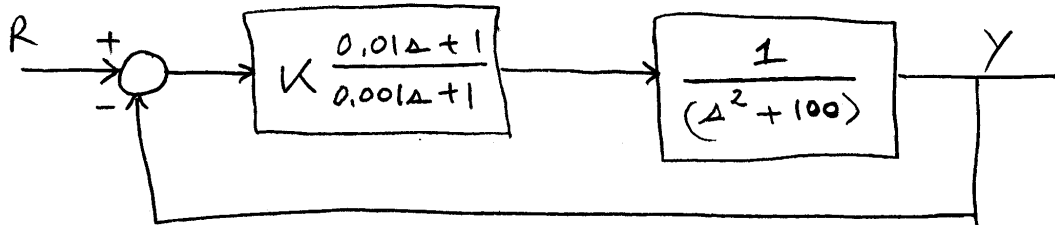
**Assigned:** Fri. April 6, 2007

**Due:** Thurs. April 12, 2007, in class

**Reading:** Nise sections 10.3–10.10; Notes Chapters 4,5,6

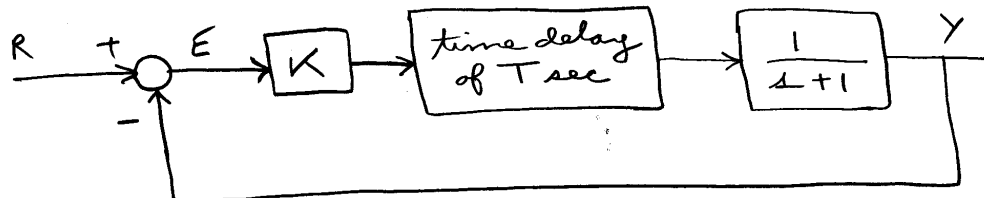
The following problems are assigned to both 2.14 and 2.140 students. In all of this problem set, you are free to use Matlab or any other computational tools to aid your solutions. There are no additional problems for 2.140 students.

**Problem 1** A unity-gain feedback loop is shown below; note that the plant has a pair of poles located on the imaginary axis.



- a) Sketch a root locus plot for  $K > 0$ .
- b) Use a Bode plot of the return ratio  $L(s)$  to choose the controller gain  $K$  such that the loop crossover frequency is 100 rad/sec. What gain  $K$  is required? What is the resulting phase margin  $\phi_m$ ?
- c) Make a Nyquist plot for this loop, which is accurately drawn in the region where  $\phi_m$  is defined. Be sure to label the corresponding points on the diagram. Note that your Nyquist analysis will require detours around the poles at  $\pm j10$ . Please clearly show the portion of the Nyquist contour corresponding to these detours.

**Problem 2** A unity-gain feedback loop is shown below. The controller in this loop is a proportional gain  $K$ , but it is implemented on a computer which requires a delay time of  $T$  seconds to produce the control output.

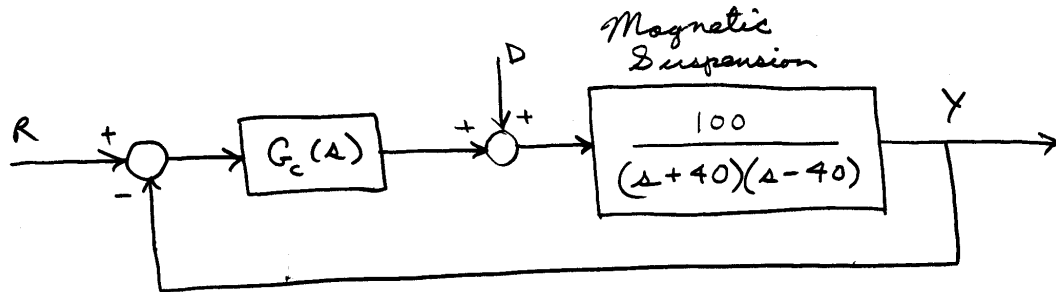


In all of this problem, assume that the time delay has a value of  $T = 10$  msec.

- a) Use a Bode plot of the return ratio  $L(s)$  to determine the maximum controller gain  $K_{max}$  such that for  $0 < K < K_{max}$  the loop is stable. For this maximal value of  $K = K_{max}$  sketch a Nyquist plot for the loop.

- b) What value of  $K < K_{max}$  will result in a stable loop with a phase margin of  $20^\circ$ ? What is the resulting crossover frequency? What is the gain margin for this loop? For this value of  $K$  sketch a Nyquist plot for the loop which shows this information.

**Problem 3** A feedback loop for a linearized magnetic suspension is shown below with a controller  $G_c(s)$ . The position of the suspended object is  $Y$ , the control reference value is  $R$ , and there is a disturbance  $D$  acting on the system.



In this problem, assume the controller takes the form

$$G_c(s) = K \frac{10\tau s + 1}{\tau s + 1}.$$

- a) Choose the values of  $K$  and  $\tau$  such that the loop has a crossover frequency of 1000 rad/sec and a phase margin which is maximized for this controller. Sketch a Nyquist plot which confirms that the loop is stable.
- b) For the value of  $K$  determined above, use Matlab to plot the response  $Y(t)$  to a unit step in reference  $R(t)$ . Also plot the response  $Y(t)$  to a unit step in disturbance  $D(t)$ .
- c) This magnetic suspension is open-loop unstable, and thus there is a minimum controller gain  $K > K_{min}$  required to stabilize the loop. Using the value of  $\tau$  determined in part a), what is the numerical value of  $K_{min}$  at this marginally-stable condition? By what ratio can  $K$  be reduced with respect to the nominal design of part a) to bring the loop to the brink of instability? Note that this ratio can be defined as the *gain-reduction* margin, as distinguished over the standard gain margin which is defined as an allowable *gain-increase* margin.