2.14/2.140 Problem Set 8

Assigned: Thurs. April 19, 2007
Due: Thurs. April 26, 2007, in class
Reading: Nise Chapter 12
Reading for 2.140 students: ***A Vibration Isolation Platform*****, posted on course web page**

The following problems are assigned to both 2.14 and 2.140 students.

Problem 1 Nise Chapter 12, problem 4

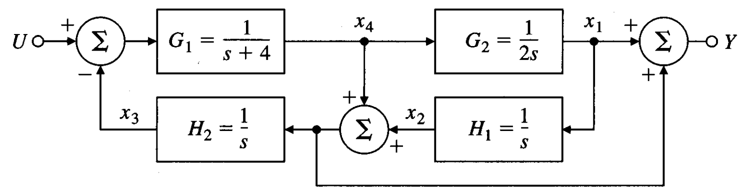
Problem 2 Nise Chapter 12, problem 6

Problem 3 Nise Chapter 12, problem 12

Problem 4 The following problem is taken from Franklin, Powell and Emami-Naeini, *Feedback Control of Dynamic Systems*, 4^{th} Edition, Pentice-Hall, 2002.

7.14. Consider the system shown in Fig. 7.81:

Figure 7.81
A block diagram for Problem 7.14



(a) Find the transfer function from U to Y.
(b) Write state equations for the system using the state variables indicated.

Problem 5 The following problem is taken from Franklin, Powell and Emami-Naeini, *Feedback Control of Dynamic Systems*, 4^{th} Edition, Pentice-Hall, 2002.

7.18. Consider the plant described by

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 7 & -4 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}u,$$

$$y = [1 \quad 3]\mathbf{x}.$$

(a) Draw a block diagram for the plant with one integrator for each state variable.
(b) Find the transfer function using matrix algebra.
(c) Find the closed-loop characteristic equation if the feedback is:

(1) $u = -[K_1 \quad K_2]\mathbf{x}$; (2) $u = -Ky$.

Problem 6 The following problem is taken from Franklin, Powell and Emami-Naeini, *Feedback Control of Dynamic Systems*, 4^{th} Edition, Pentice-Hall, 2002.

7.19. For the system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u,$$

$$y = [1 \quad 0]\mathbf{x}.$$

design a state feedback controller that satisfies the following specifications:

- Closed-loop poles have a damping coefficient $\zeta = 0.707$.
- Step-response peak time is under 3.14 sec.

Verify your design with MATLAB.

Problem 7 The following problem is taken from Franklin, Powell and Emami-Naeini, *Feedback Control of Dynamic Systems*, 4^{th} Edition, Pentice-Hall, 2002.

7.20. **(a)** Design a state feedback controller for the following system so that the closed-loop step response has an overshoot of less than 25% and a 1% settling time under 0.115 sec:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -10 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u,$$

$$y = [1 \quad 0]\mathbf{x}.$$

(b) Use the step command in MATLAB to verify that your design meets the specifications. If it does not, modify your feedback gains accordingly.

The following problems are assigned to only 2.140 students. Students in 2.14 are welcome to work these, but no extra credit will be given.

Problem G1 The following problem is taken from Franklin, Powell and Emami-Naeini, *Feedback Control of Dynamic Systems*, 4$^{\text{th}}$ Edition, Pentice-Hall, 2002.

7.36. This problem is intended to give you more insight into controllability and observability. Consider the circuit in Fig. 7.89, with an input voltage source $u(t)$ and an output current $y(t)$.

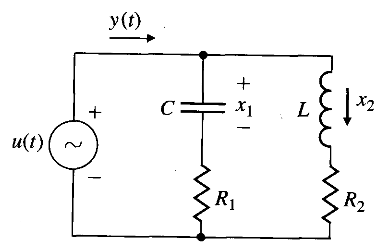
(a) Using the capacitor voltage and inductor current as state variables, write state and output equations for the system.

(b) Find the conditions relating R_1, R_2, C, and L that render the system uncontrollable. Find a similar set of conditions that result in an unobservable system.

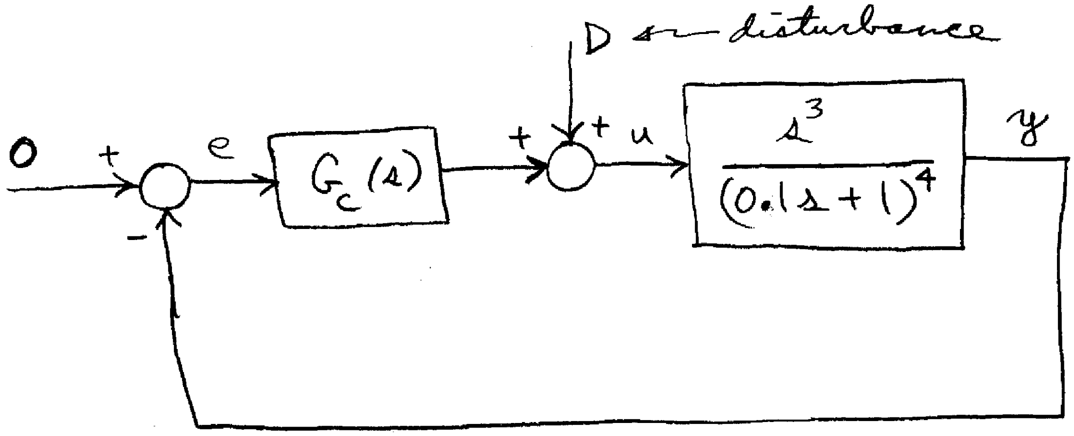
(c) Interpret the conditions found in part (b) physically in terms of the time constants of the system.

(d) Find the transfer function of the system. Show that there is a pole-zero cancellation for the conditions derived in part (b) (that is, when the system is uncontrollable or unobservable).

Figure 7.89
Electric circuit for Problem 7.36



Problem G2 This problem expands on problem 3 from Quiz 2. In order to understand the issues of controllers for AC-coupled loops, please read the paper *A Vibration Isolation Platform* which is available on the course web page. Consider the AC-coupled feedback loop shown below, which uses the plant from Quiz 2 with a more general controller $G_c(s)$.



Note that this system is a regulator, in that the reference input is zero. A disturbance D acts on the input to the plant.

Design the controller $G_c(s)$ such that the loop gain crosses *up* through unity at a frequency of 1 rad/sec, and crosses *down* through unity at a frequency of 1000 rad/sec. The loop shape must have a phase margin of 20 degrees at both of these frequencies. Hint: You will need to think about the Nyquist plot of the return ratio in order to determine the form of the controller. The assigned paper will also help you in thinking about how to design the controller $G_c(s)$. Clearly show your reasoning, and include Bode and Nyquist plots for the return ratio which verify the design meets spec. Also, calculate the closed-loop response in $y(t)$ to a unit step in the disturbance $D(t)$ acting at the input to the plant. How do the loop crossing frequencies and damping ratio (phase margins) show up in this response? How do the closed-loop poles correspond with the observed response and loop shape?