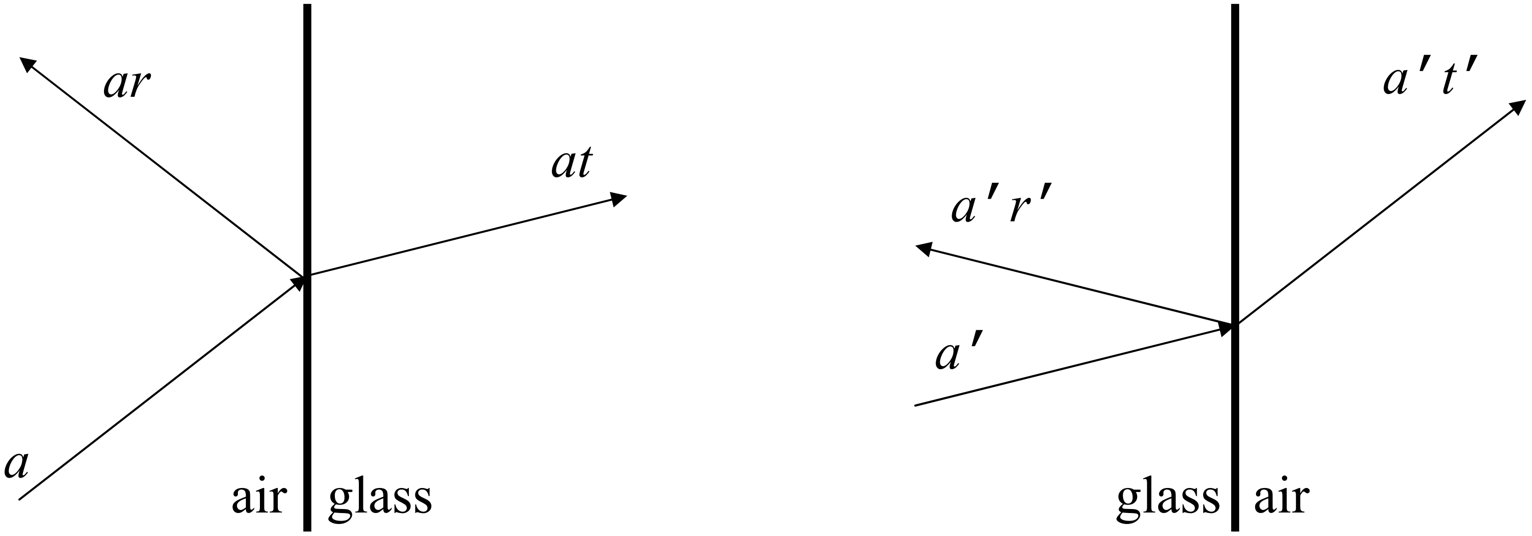
Today’s summary

- Multiple beam interferometers: Fabry-Perot resonators
  - Stokes relationships
  - Transmission and reflection coefficients for a dielectric slab
  - Optical resonance
- Principles of lasers
- Coherence: spatial / temporal

Fabry-Perot interferometers

Relation between r, r' and t, t'



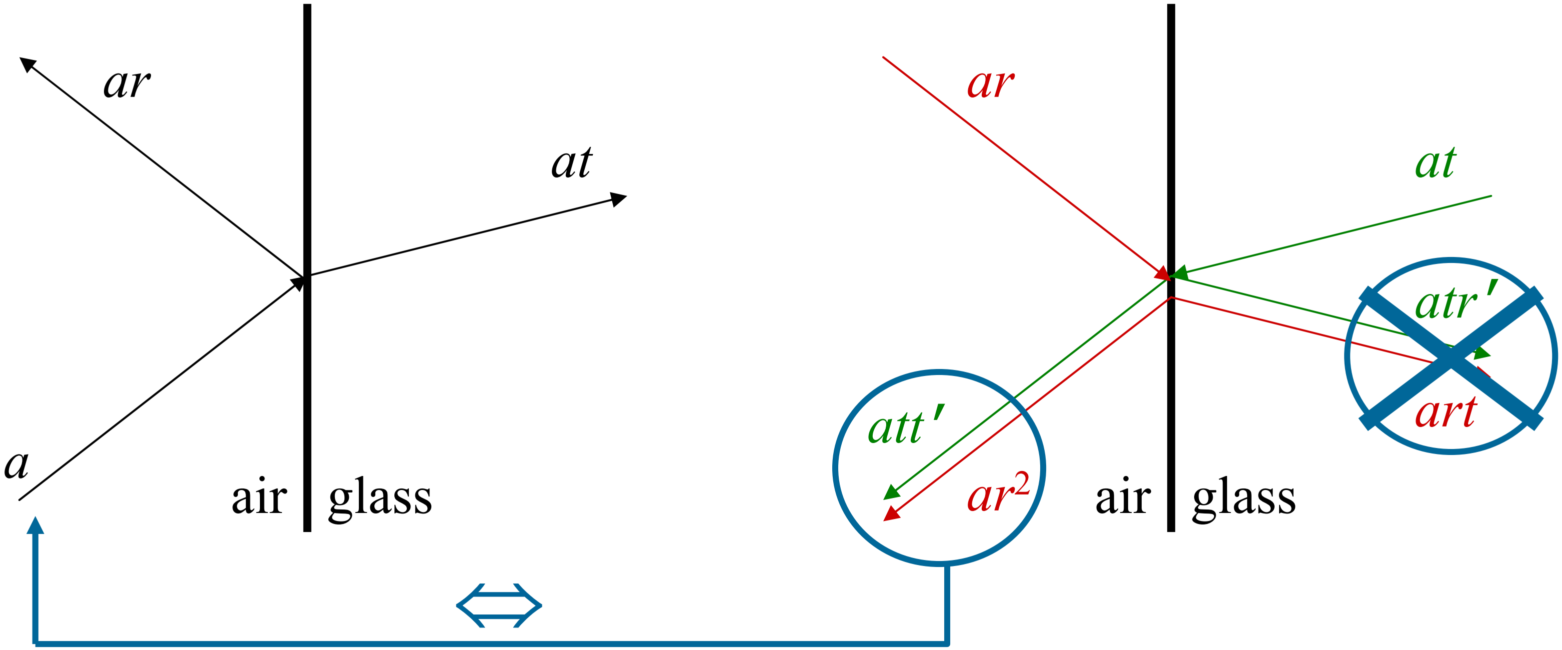
$$r' = -r$$

$$r^2 + tt' = 1$$

Stokes relationships

<u>Proof</u>: algebraic from the Fresnel coefficients or using the property of *preservation of the field properties upon time reversal*

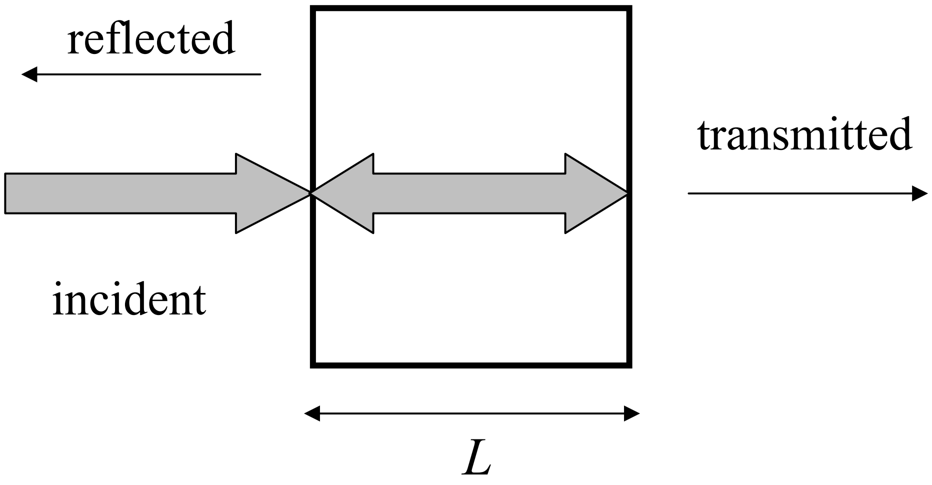
Proof using time reversal



$$a\left(r+r'\right)t=0\Rightarrow r=-r'$$

$$a\left(r^{2}+tt'\right)=a\Rightarrow r^{2}+tt'=1$$

Fabry-Perot Interferometer



Resonance condition: reflected wave = 0

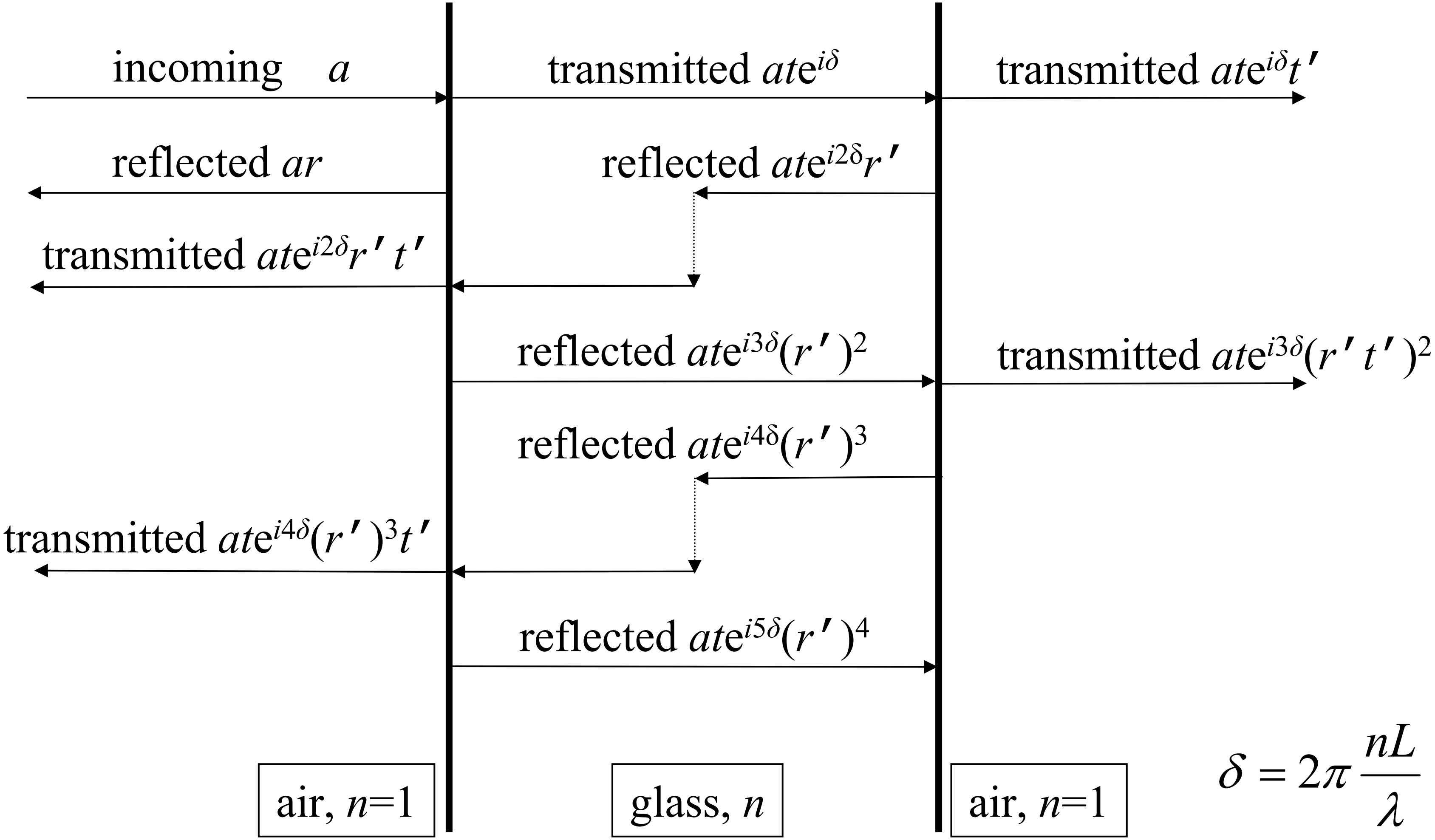
⇔ all reflected waves interfere destructively

$$L = \frac{m\lambda}{2n}$$

← wavelength in free space

← refractive index

Calculation of the reflected wave



incoming a

transmitted $ate^{i\delta}$

transmitted $ate^{i\delta}t'$

reflected ar

reflected $ate^{i2\delta}r'$

transmitted $ate^{i2\delta}r't'$

reflected $ate^{i3\delta}(r')^2$

transmitted $ate^{i3\delta}(r't')^2$

reflected $ate^{i4\delta}(r')^3$

transmitted $ate^{i4\delta}(r')^3t'$

reflected $ate^{i5\delta}(r')^4$

air, n=1

glass, n

air, n=1

$$\delta = 2\pi \frac{nL}{\lambda}$$

Calculation of the reflected wave

$$a_{\text{reflected}} = a\left\{r + tt'r'\,\mathrm{e}^{i2\delta}\left(1 + r'^2\,\mathrm{e}^{i2\delta} + r'^4\,\mathrm{e}^{i4\delta} + \cdots\right)\right\}$$

$$= a\left\{r + tt'r'\,\mathrm{e}^{i2\delta}\,\frac{1}{1 - r'^2\,\mathrm{e}^{i2\delta}}\right\}$$

Use Stokes relationships

$$r' = -r$$

$$r^2 + tt' = 1$$

$$a_{\text{reflected}} = a\,\frac{r\left(1 - \mathrm{e}^{i2\delta}\right)}{1 - r^2\,\mathrm{e}^{i2\delta}}$$

Transmission & reflection coefficients

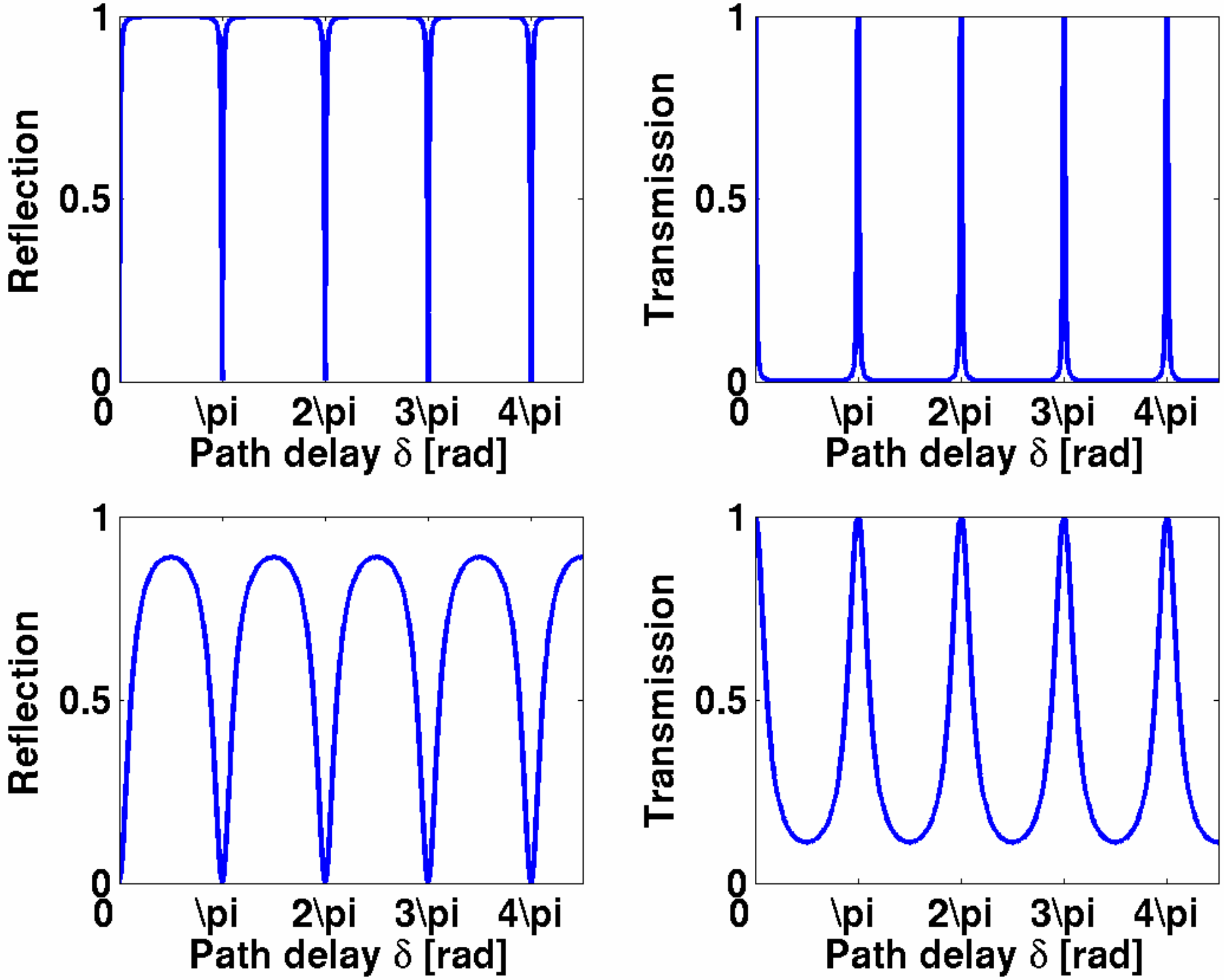
$$a_{\text{reflected}} = a\,\frac{\left(1-\mathrm{e}^{i2\delta}\right)r}{1-r^2\,\mathrm{e}^{i2\delta}} \qquad a_{\text{transmitted}} = a\,\frac{tt'}{1-r^2\,\mathrm{e}^{i2\delta}}$$

$$R \equiv |r|^2$$

$$\begin{pmatrix}\text{reflection}\\ \text{coefficient}\end{pmatrix} = \left|\frac{a_{\text{reflected}}}{a}\right|^2 = \frac{4R\sin^2\delta}{(1-R)^2+4R\sin^2\delta}$$

$$\begin{pmatrix}\text{transmission}\\ \text{coefficient}\end{pmatrix} = \left|\frac{a_{\text{transmitted}}}{a}\right|^2 = \frac{(1-R)^2}{(1-R)^2+4R\sin^2\delta}$$

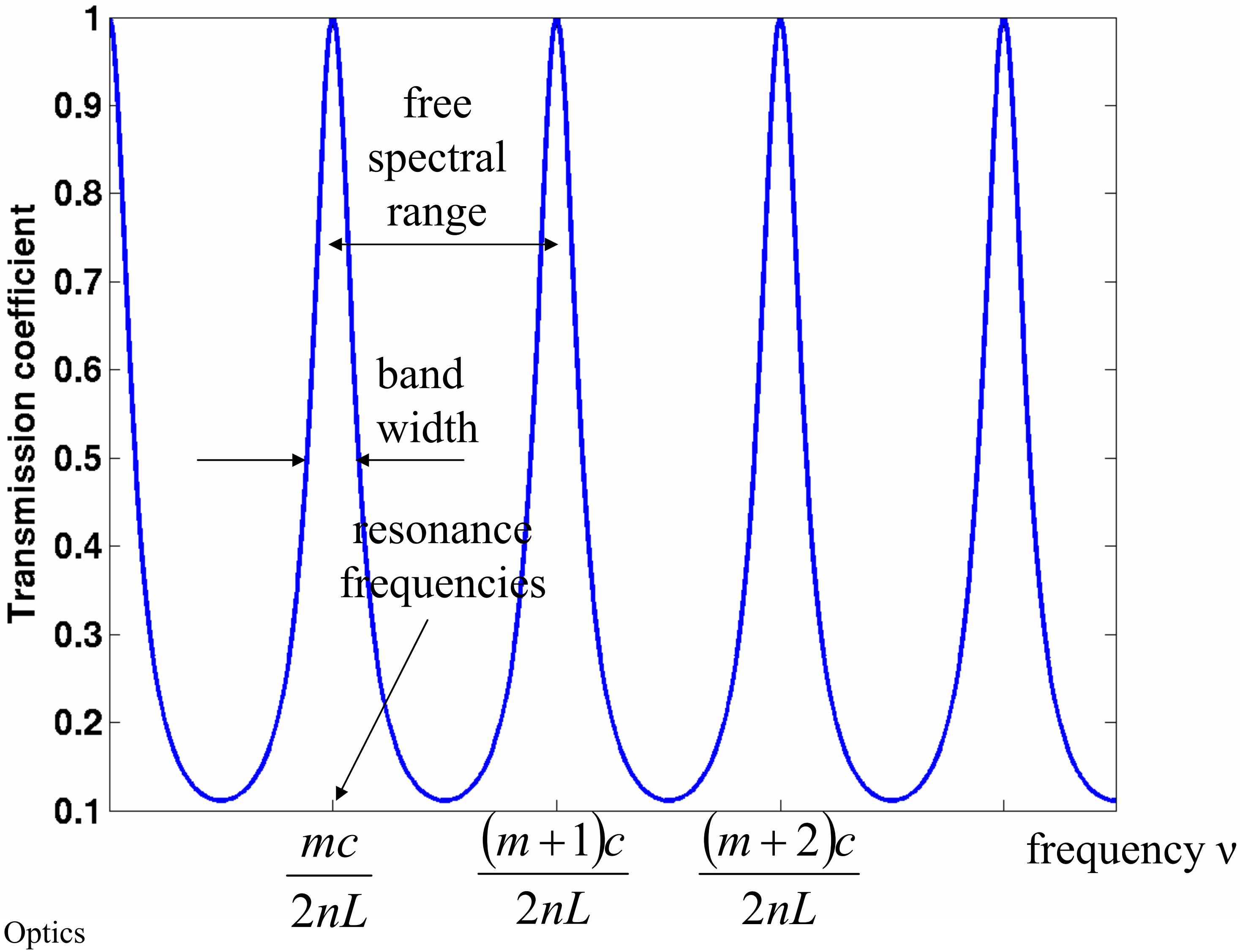
Transmission & reflection *vs* path



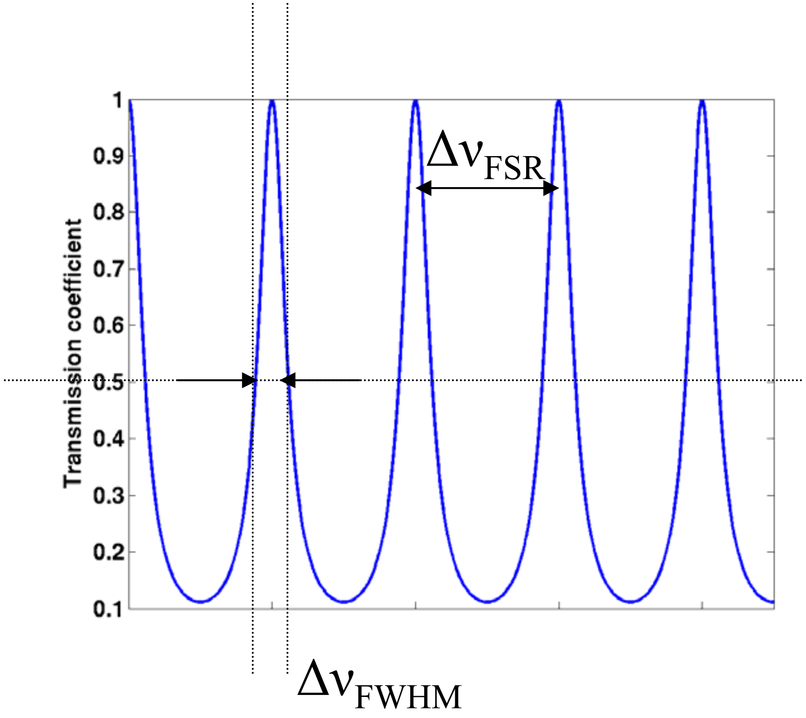
R=0.95

R=0.5

Fabry-Perot terminology



Fabry-Perot terminology



FWHM Bandwidth is inversely proportional to the *finesse* ***F*** (or *quality factor*) of the cavity

$$F \equiv \frac{\pi\sqrt{R}}{1-R}$$

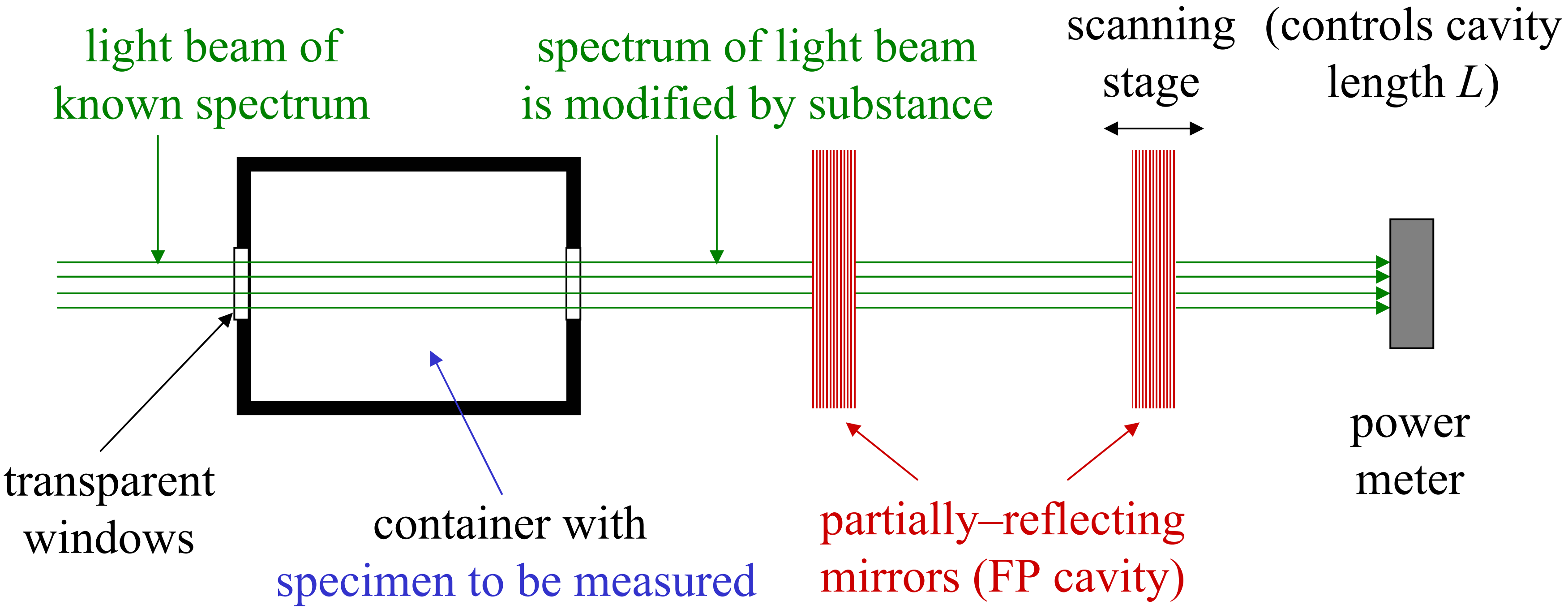
$$\Delta\nu_{\mathrm{FWHM}} = \frac{c}{2nLF}$$

$$\Delta\nu_{\mathrm{FWHM}} = \frac{\Delta\nu_{\mathrm{FSR}}}{F} \qquad (\text{bandwidth}) = \frac{(\text{free spectral range})}{(\text{finesse})}$$

Spectroscopy using Fabry-Perot cavity

Goal: to measure the specimen's absorption as function of frequency ω

Experimental measurement principle:



Spectroscopy using Fabry-Perot cavity

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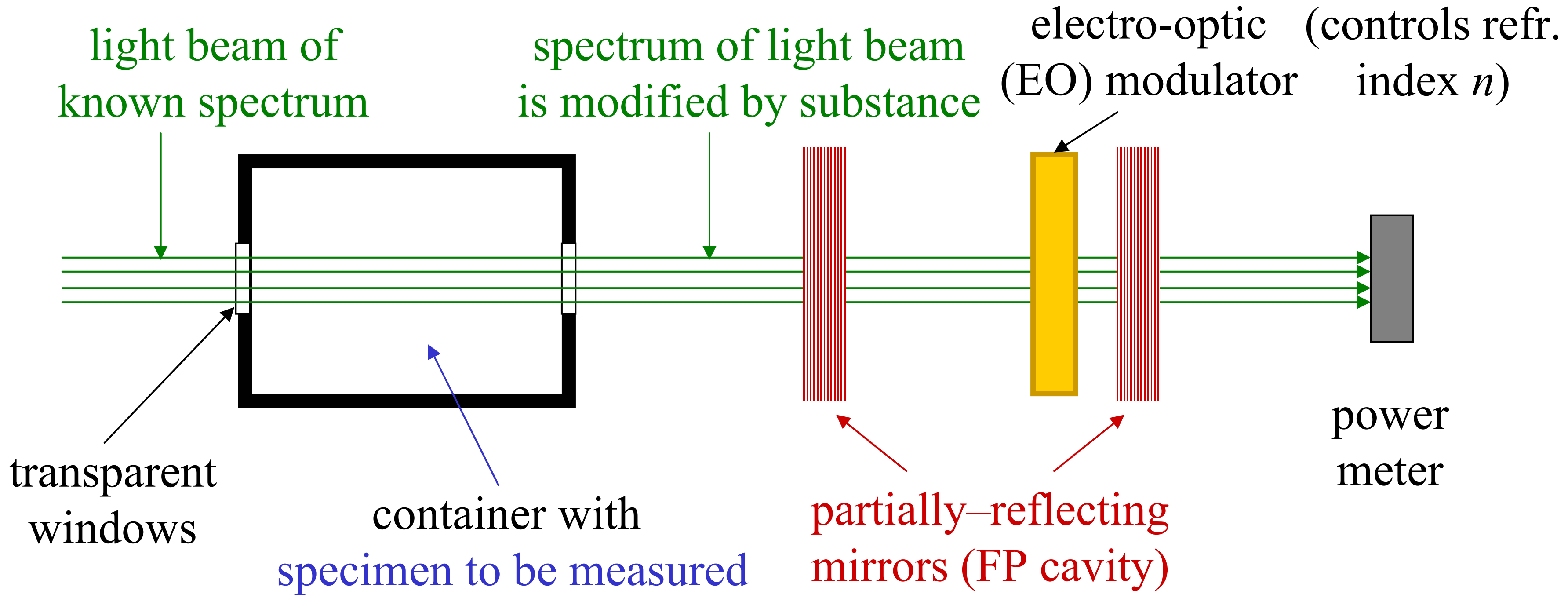
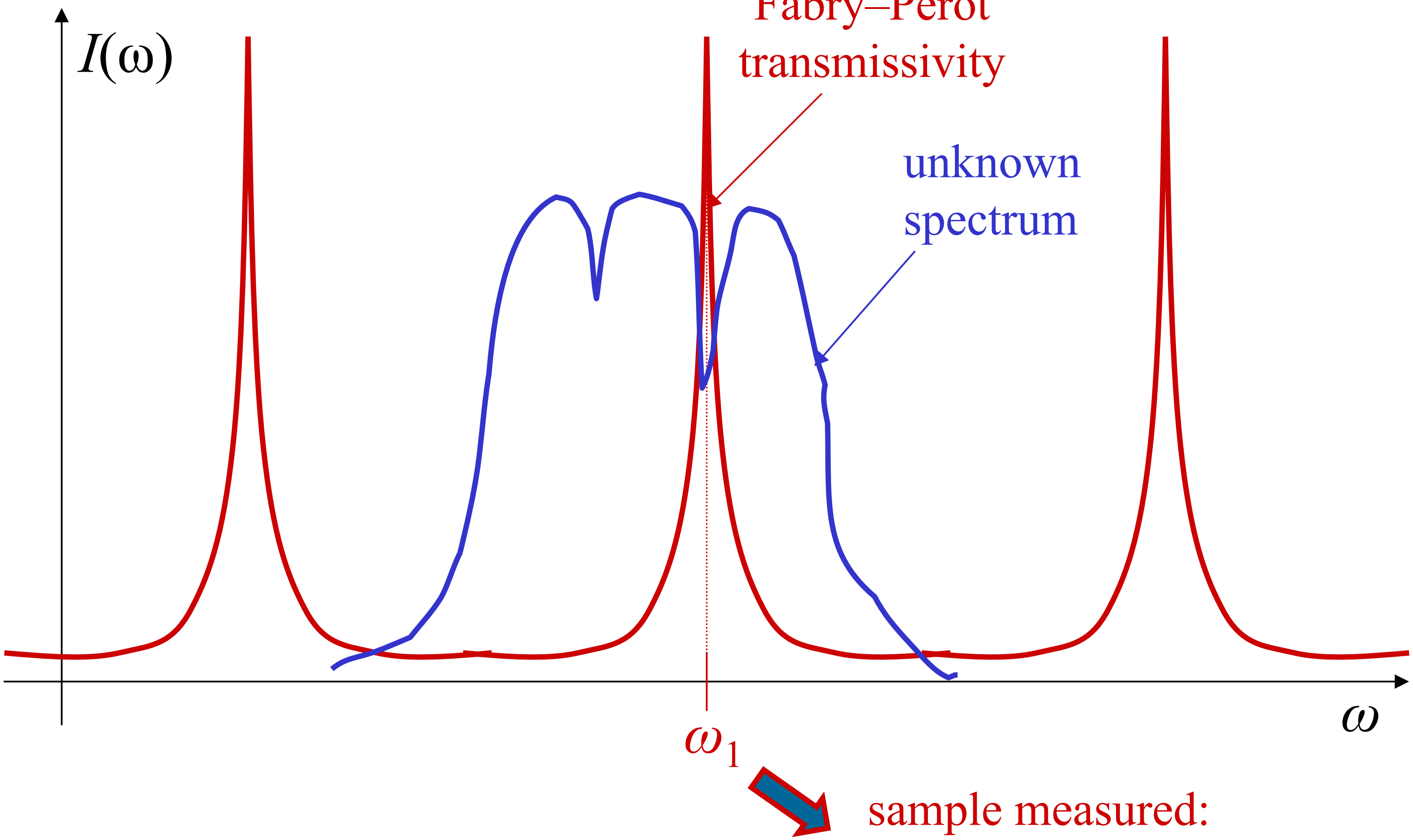
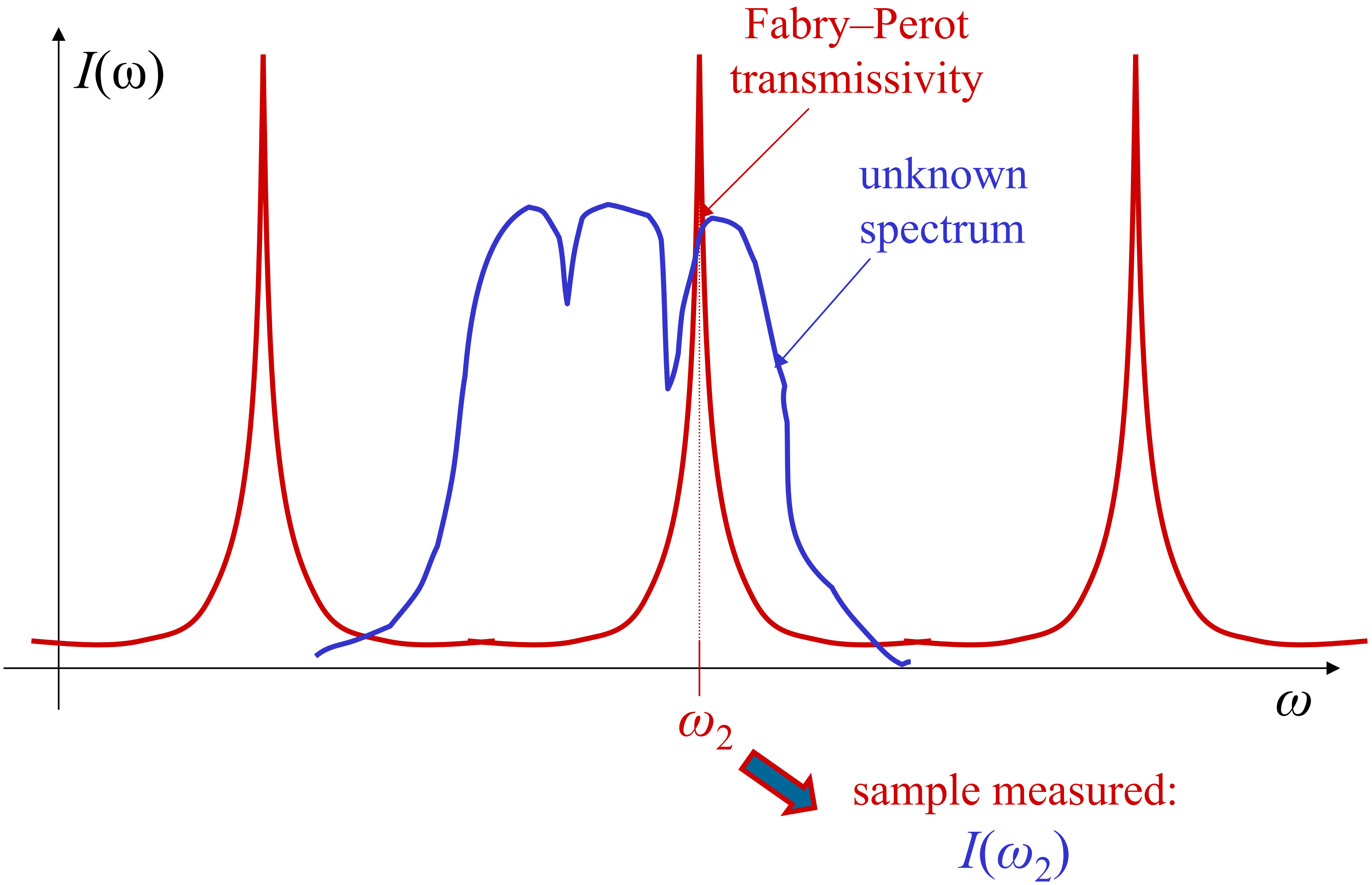


Spectroscopy using Fabry-Perot cavity

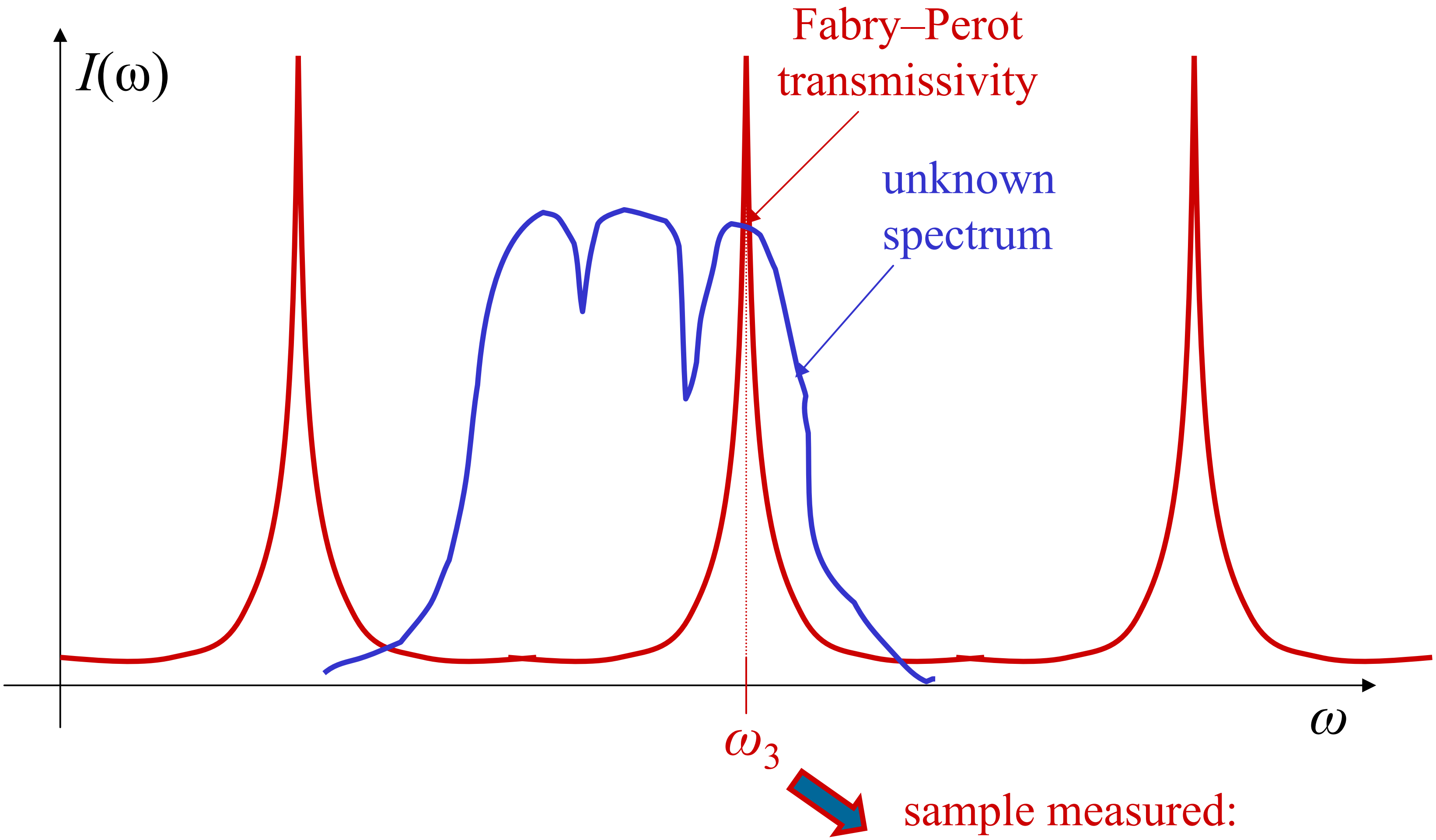


sample measured:
$I(\omega_1)$

Spectroscopy using Fabry-Perot cavity

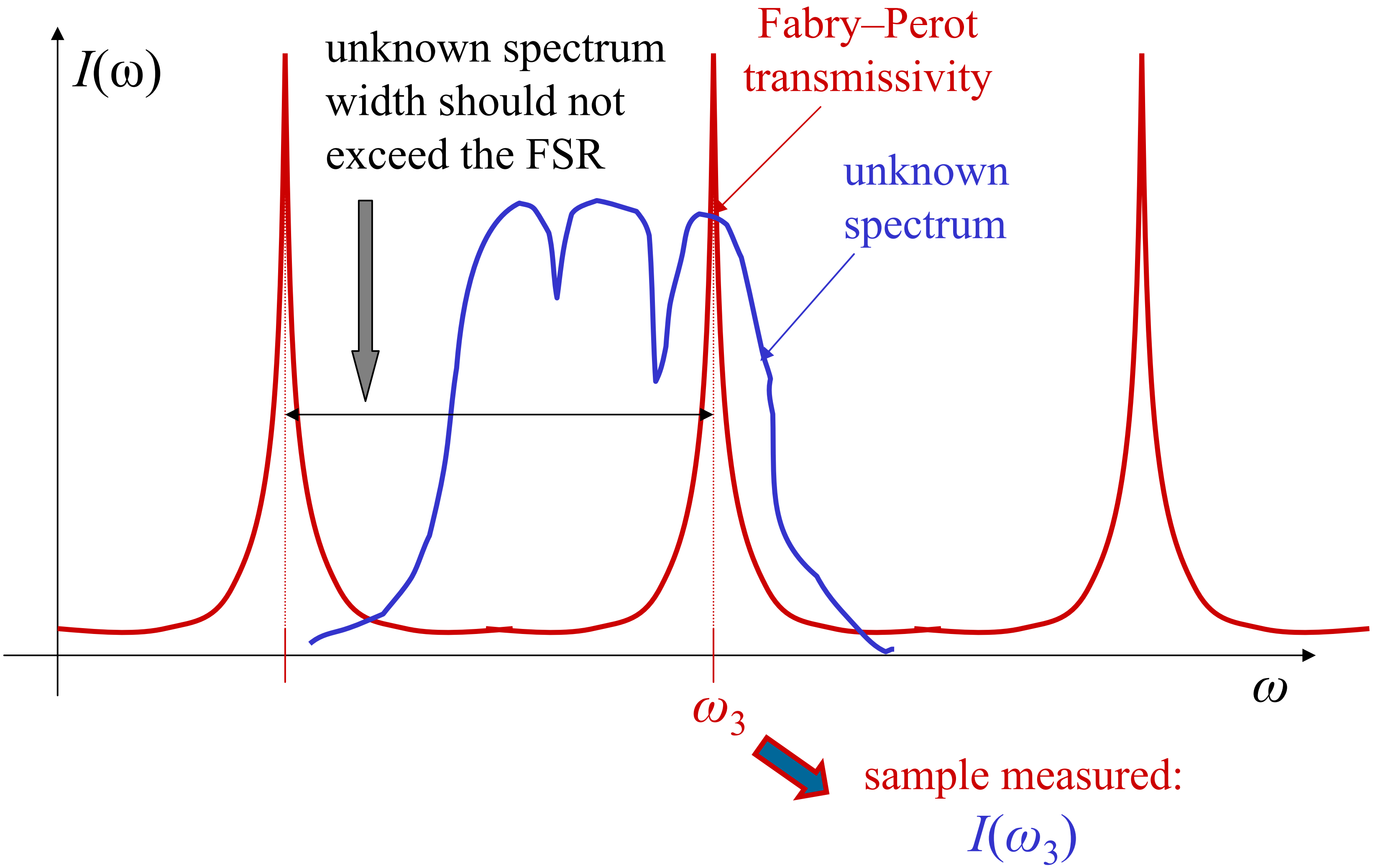


Spectroscopy using Fabry-Perot cavity

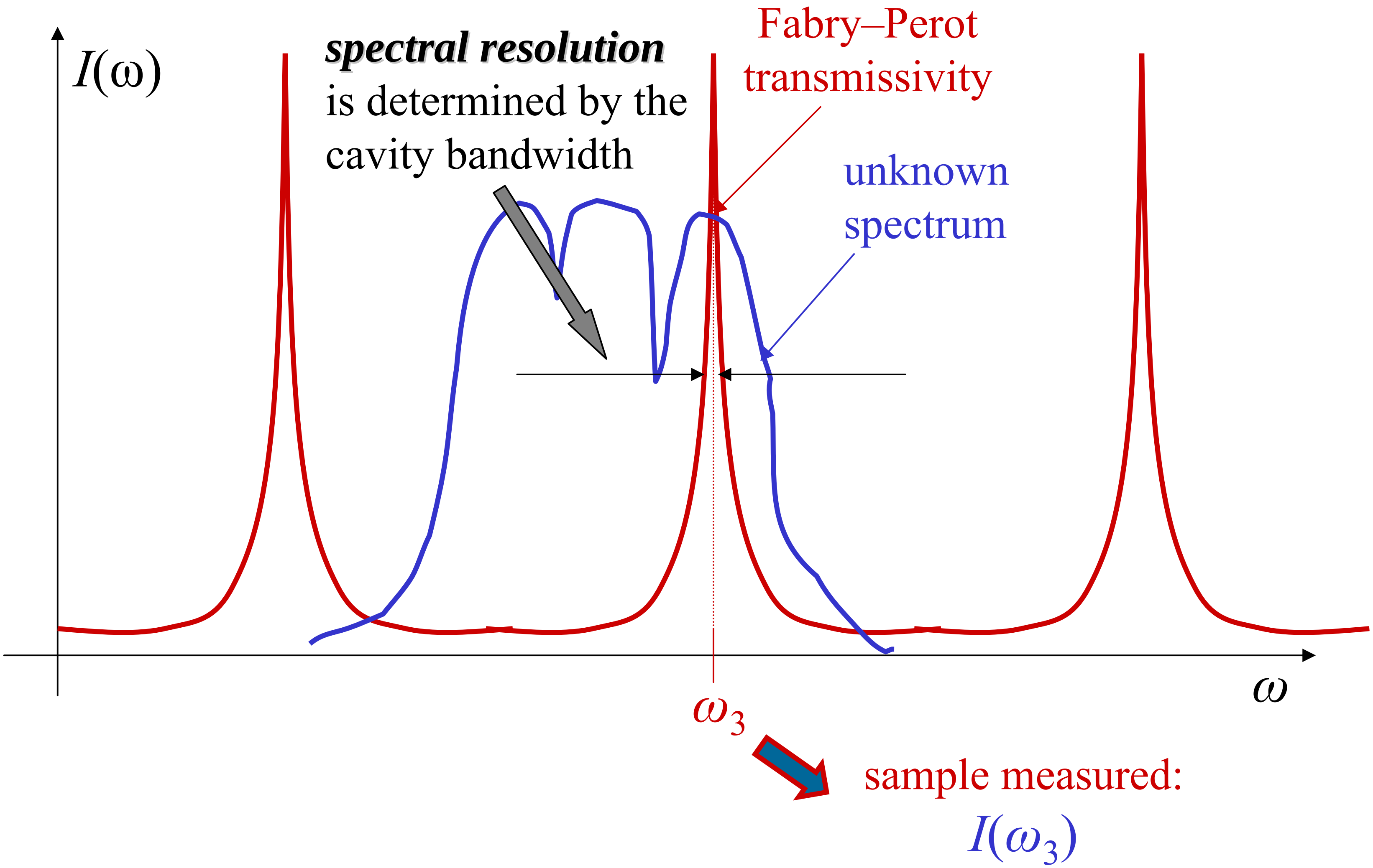


sample measured:
$I(\omega_3)$

Spectroscopy using Fabry-Perot cavity



Spectroscopy using Fabry-Perot cavity



spectral resolution is determined by the cavity bandwidth

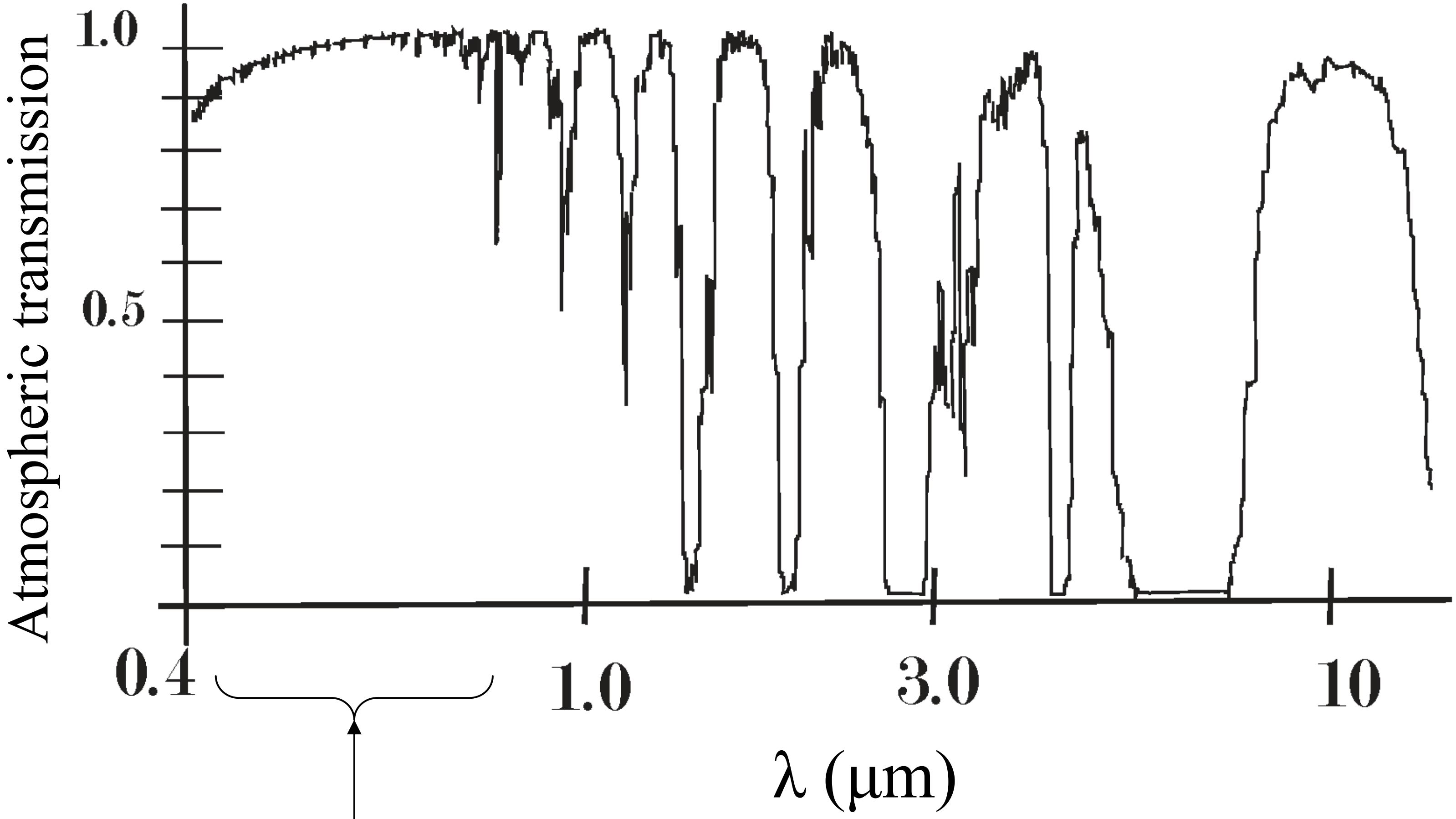
Fabry–Perot transmissivity

unknown spectrum

sample measured: $I(\omega_3)$

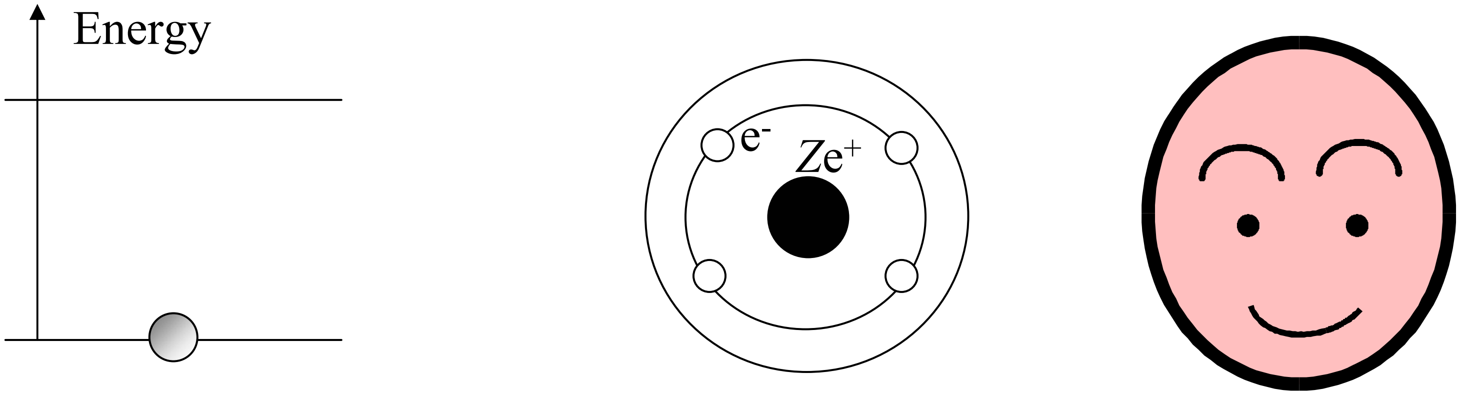
Lasers

Absorption spectra

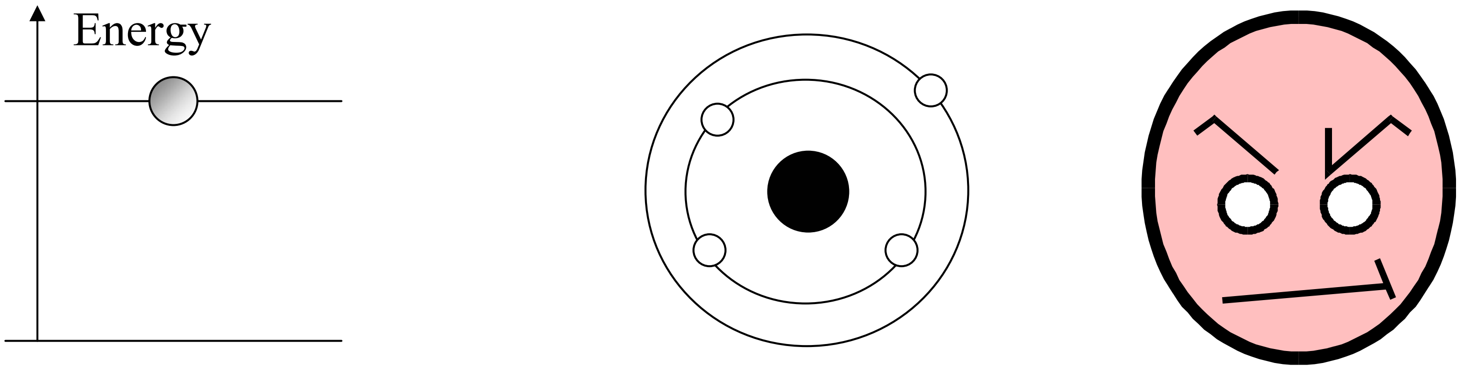


human vision

Semi-classical view of atom excitations

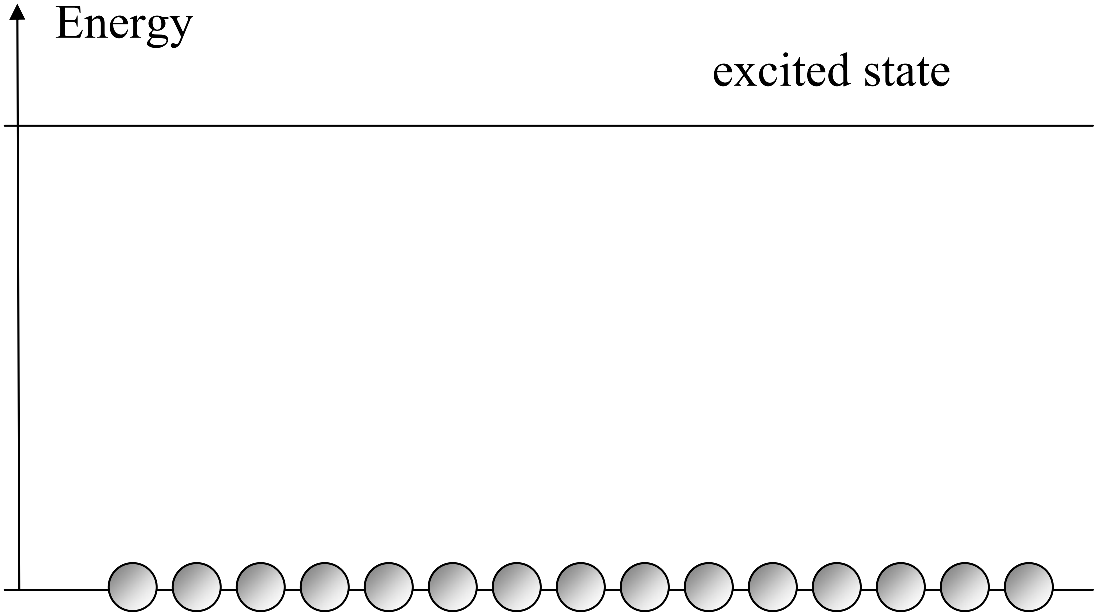


Atom in ground state



Atom in excited state

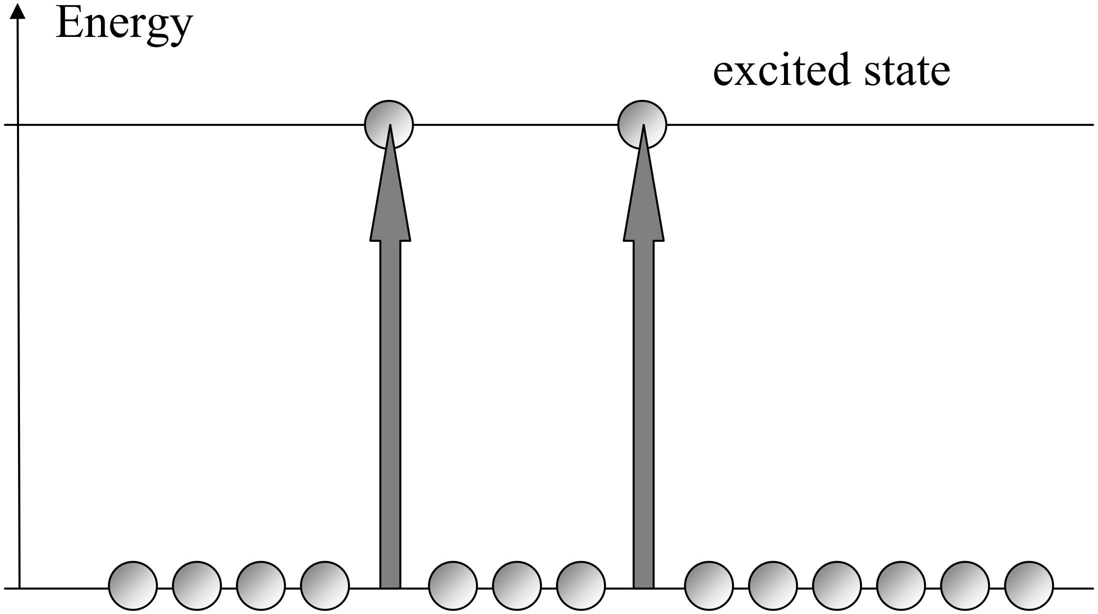
Light generation



equilibrium: most atoms in ground state

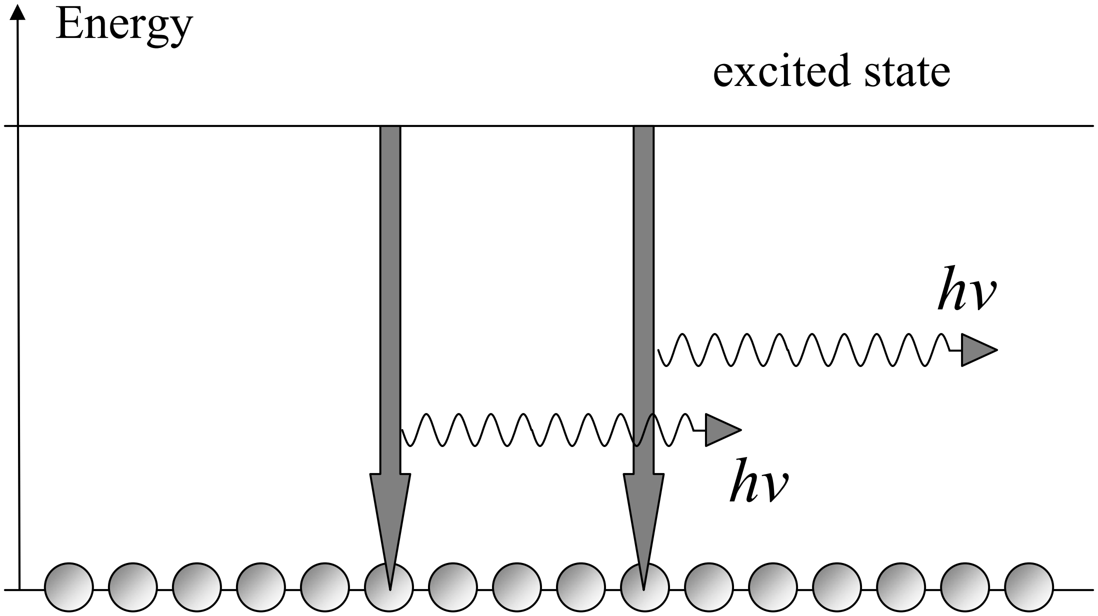
ground state

Light generation



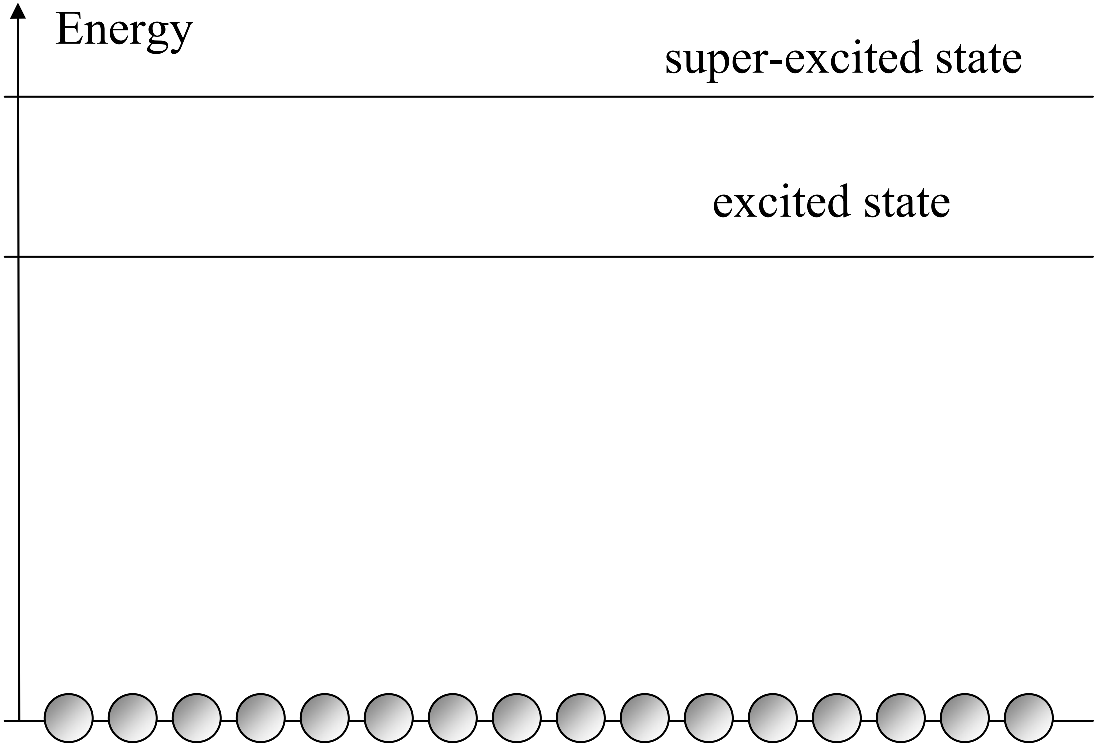
A **pump** mechanism (e.g. thermal excitation or gas discharge) ejects some atoms to the excited state

Light generation



The excited atoms **radiatively decay**, emitting one photon each

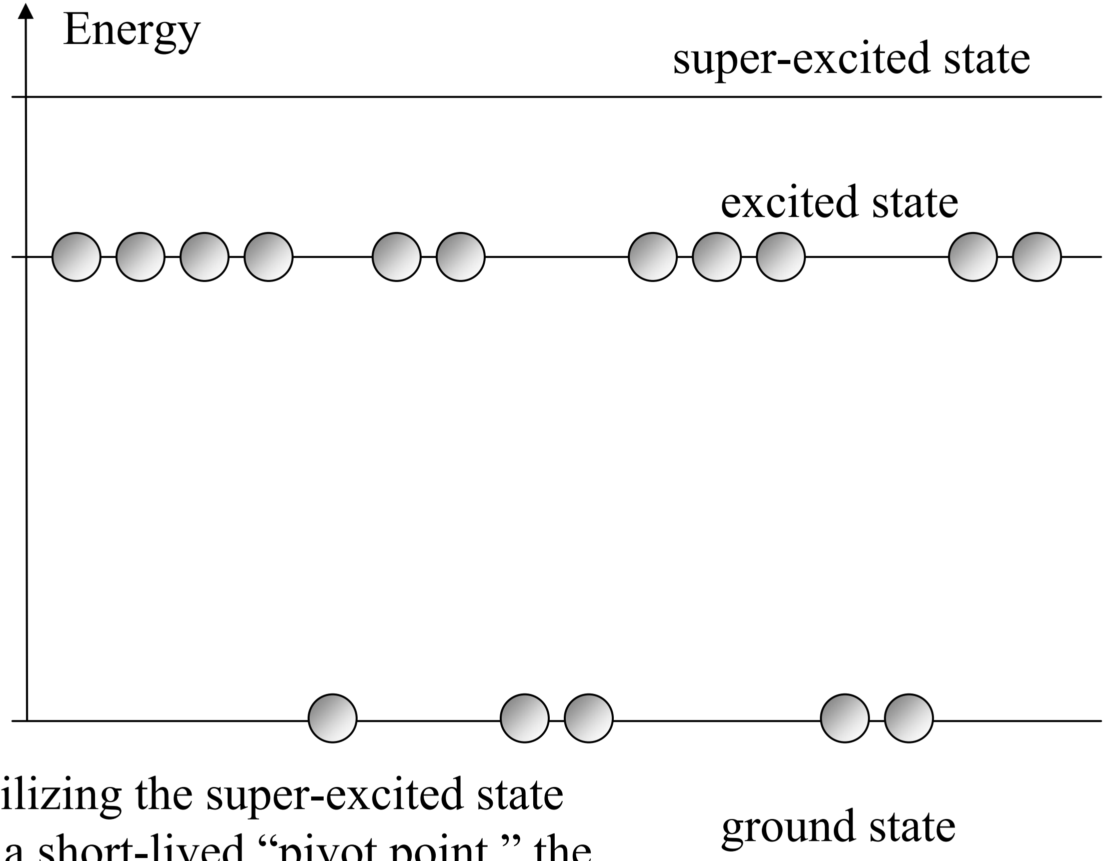
Light amplification: 3-level system



ground state

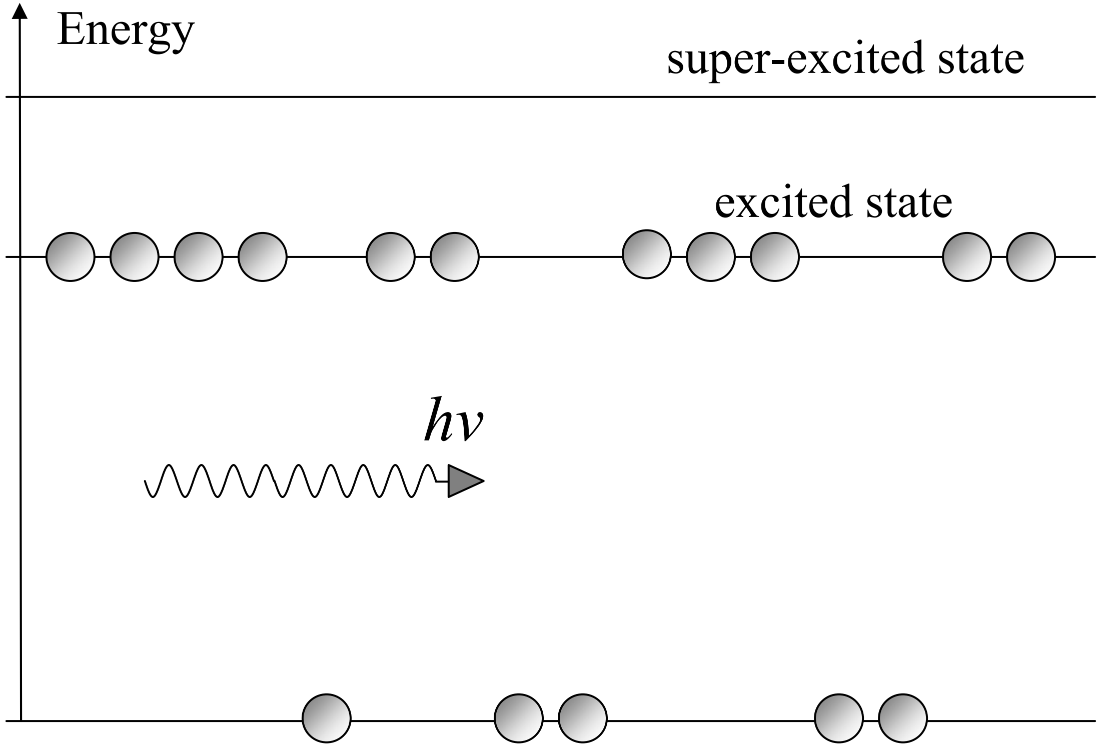
equilibrium: most atoms
in ground state; note the existence
of a third, “super-excited” state

Light amplification: 3-level system



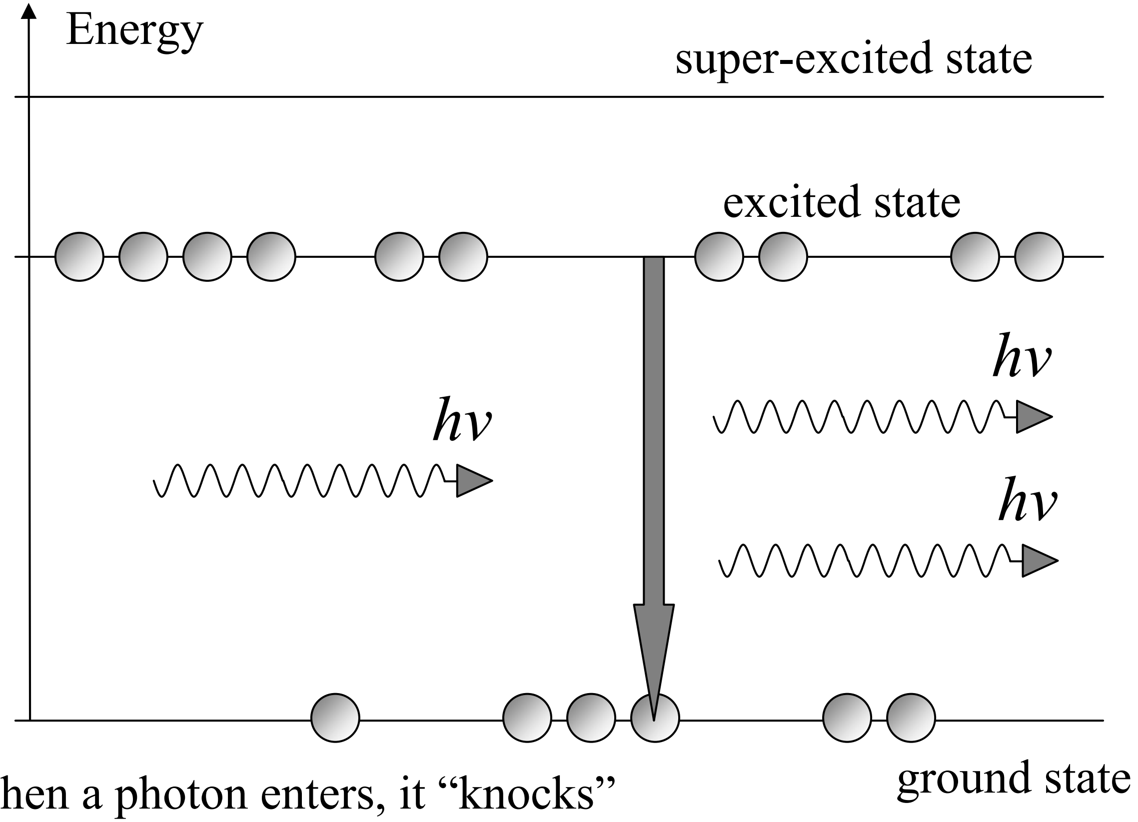
Utilizing the super-excited state
as a short-lived “pivot point,” the
pump creates a **population inversion**

Light amplification: 3-level system



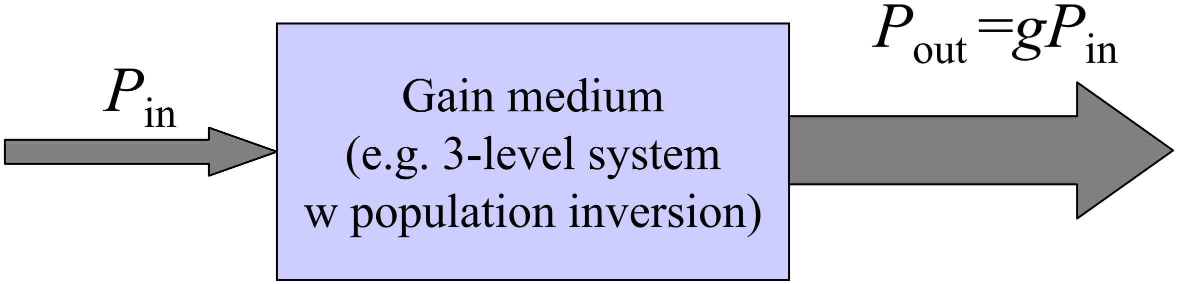
When a photon enters, ...

Light amplification: 3-level system



When a photon enters, it “knocks”
an electron from the inverted population
down to the ground state, thus creating
a new photon. This amplification process
is called **stimulated emission**

Light amplifier

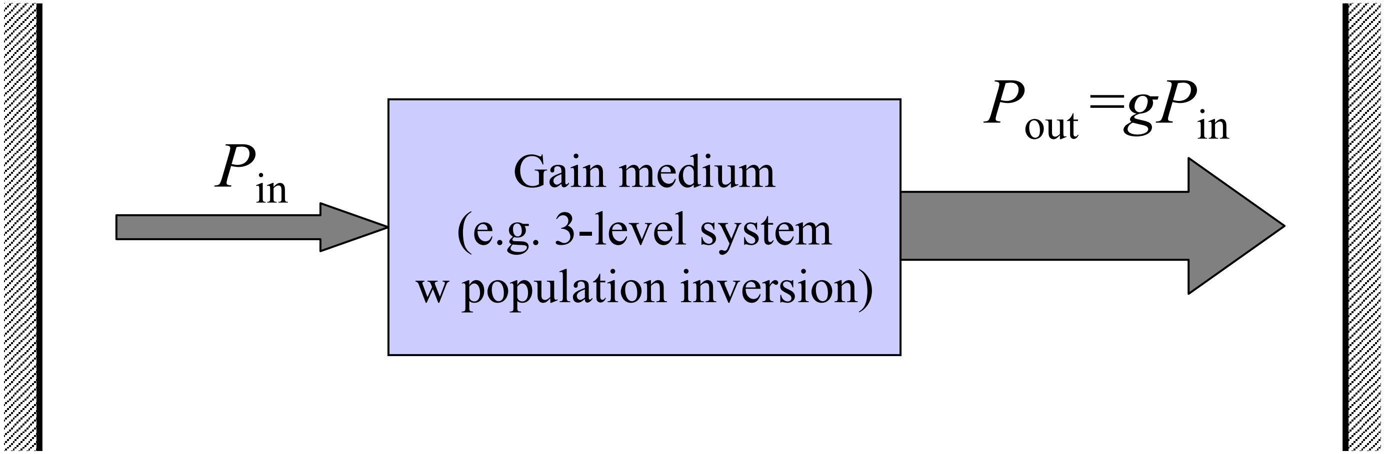


P_{in}

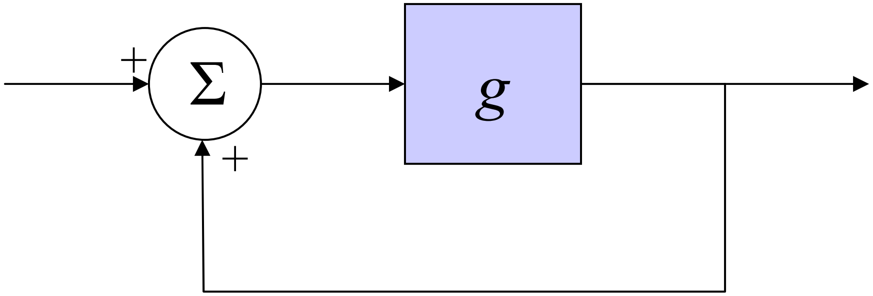
Gain medium
(e.g. 3-level system
w population inversion)

$P_{\text{out}} = gP_{\text{in}}$

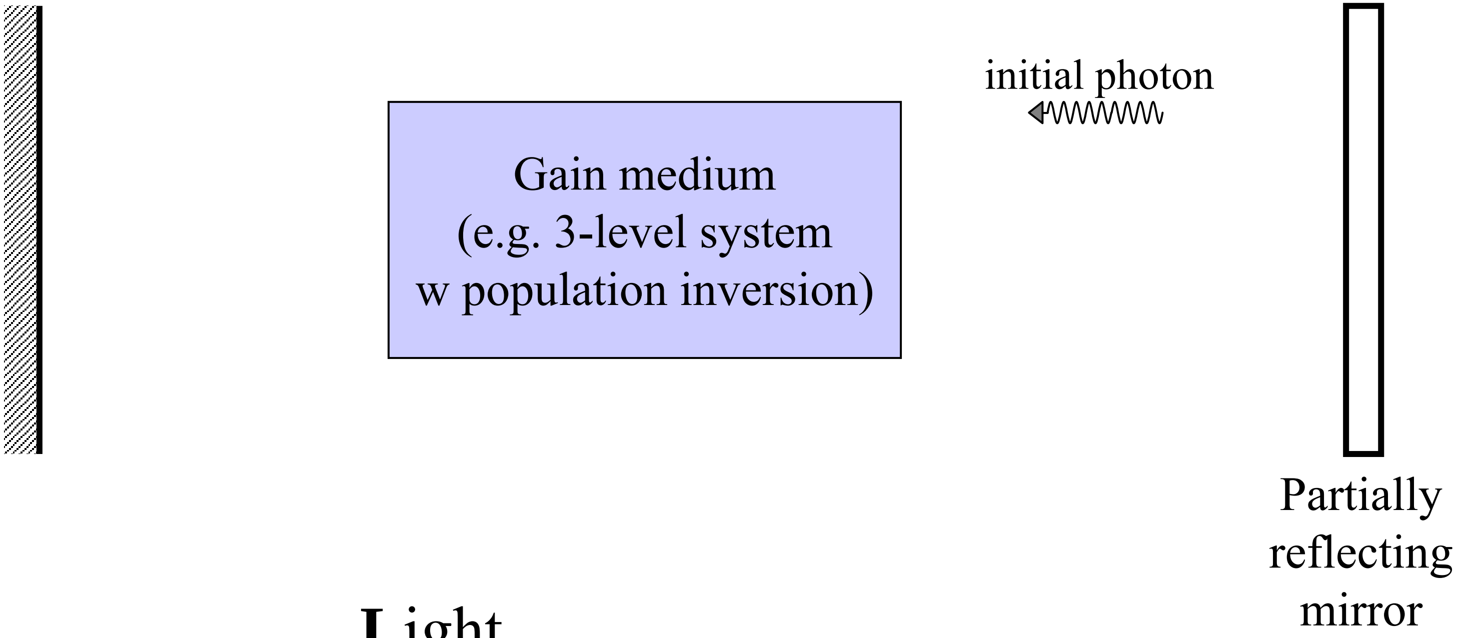
Light amplifier w positive feedback



When the gain exceeds the roundtrip losses, the system goes into **oscillation**

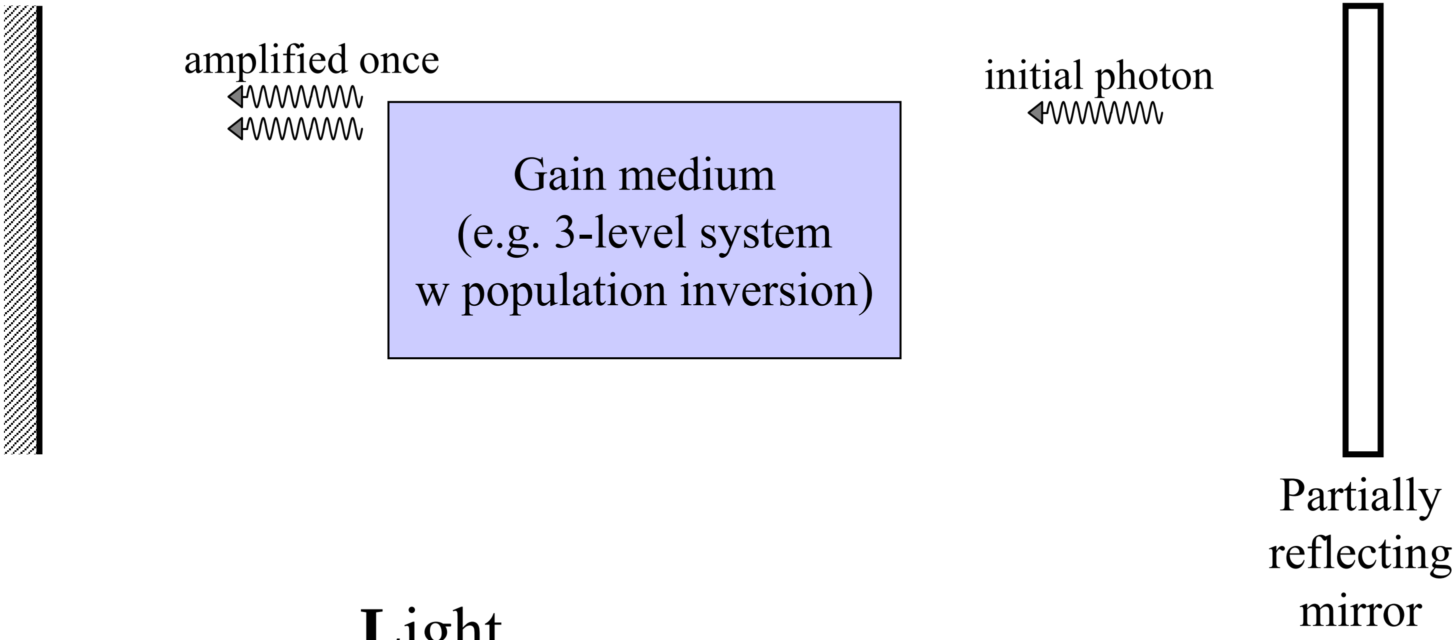


Laser



Light
Amplification through
Stimulated
Emission of
Radiation

Laser



amplified once

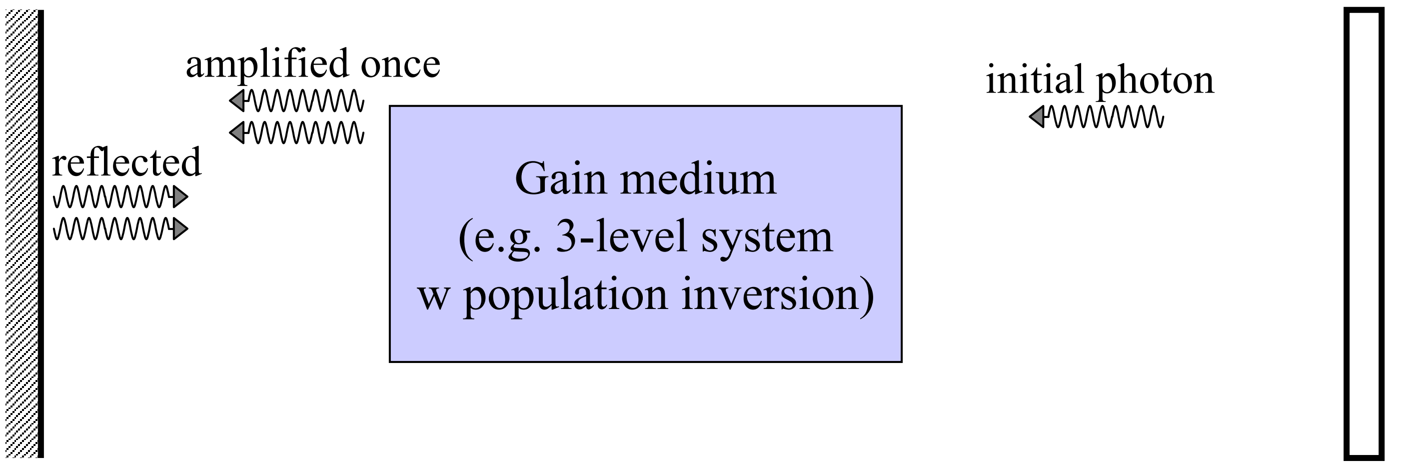
initial photon

Gain medium
(e.g. 3-level system
w population inversion)

Partially
reflecting
mirror

Light
Amplification through
Stimulated
Emission of
Radiation

Laser



amplified once

reflected

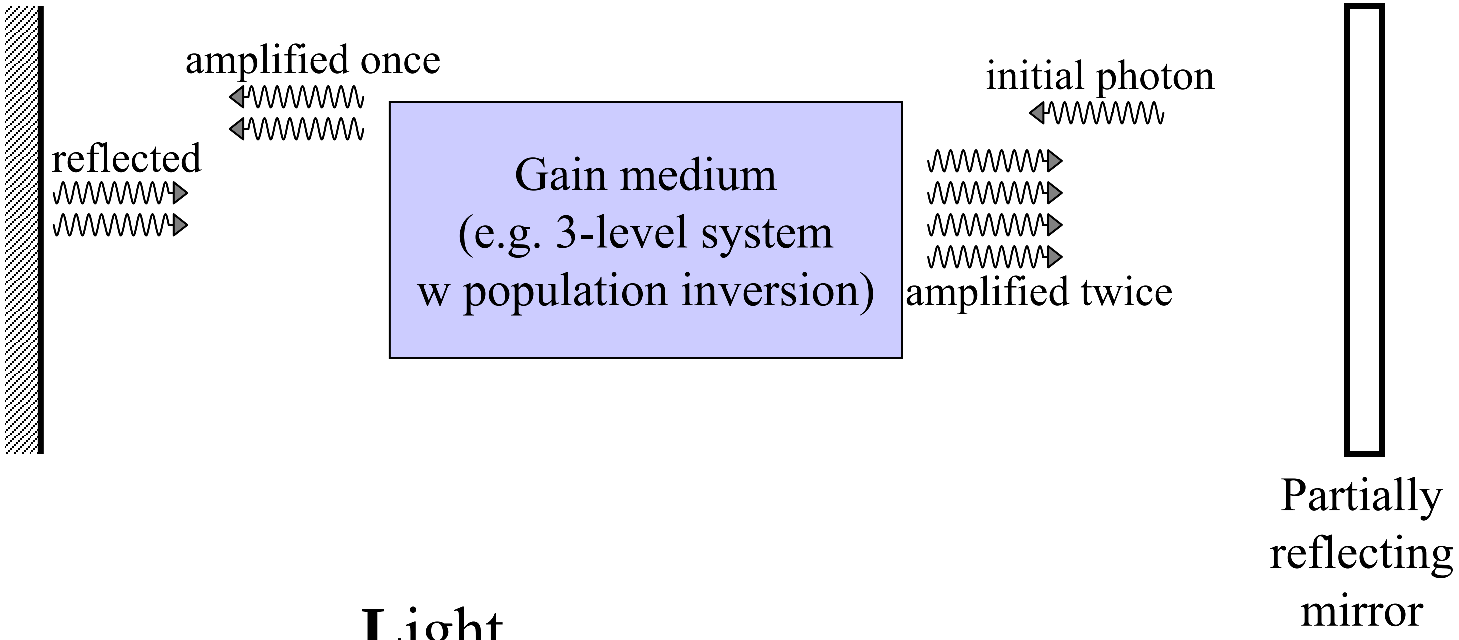
Gain medium
(e.g. 3-level system
w population inversion)

initial photon

Partially
reflecting
mirror

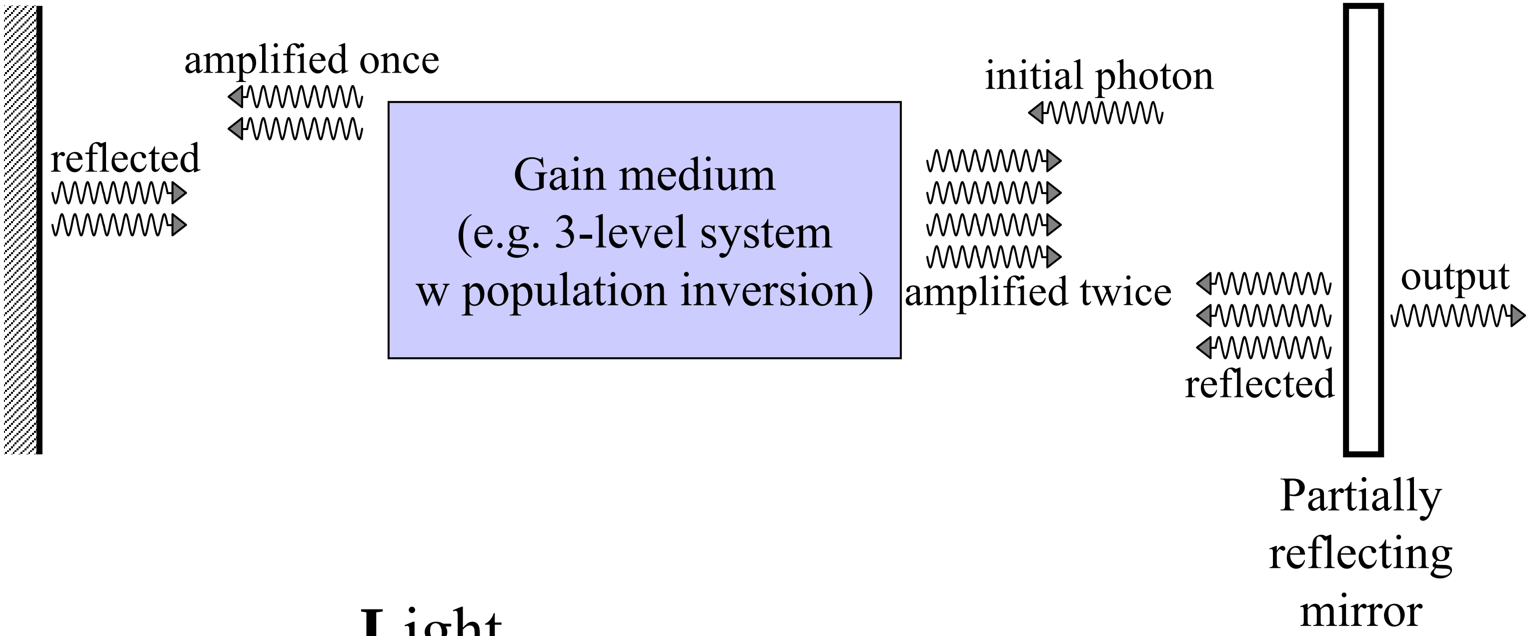
Light
Amplification through
Stimulated
Emission of
Radiation

Laser



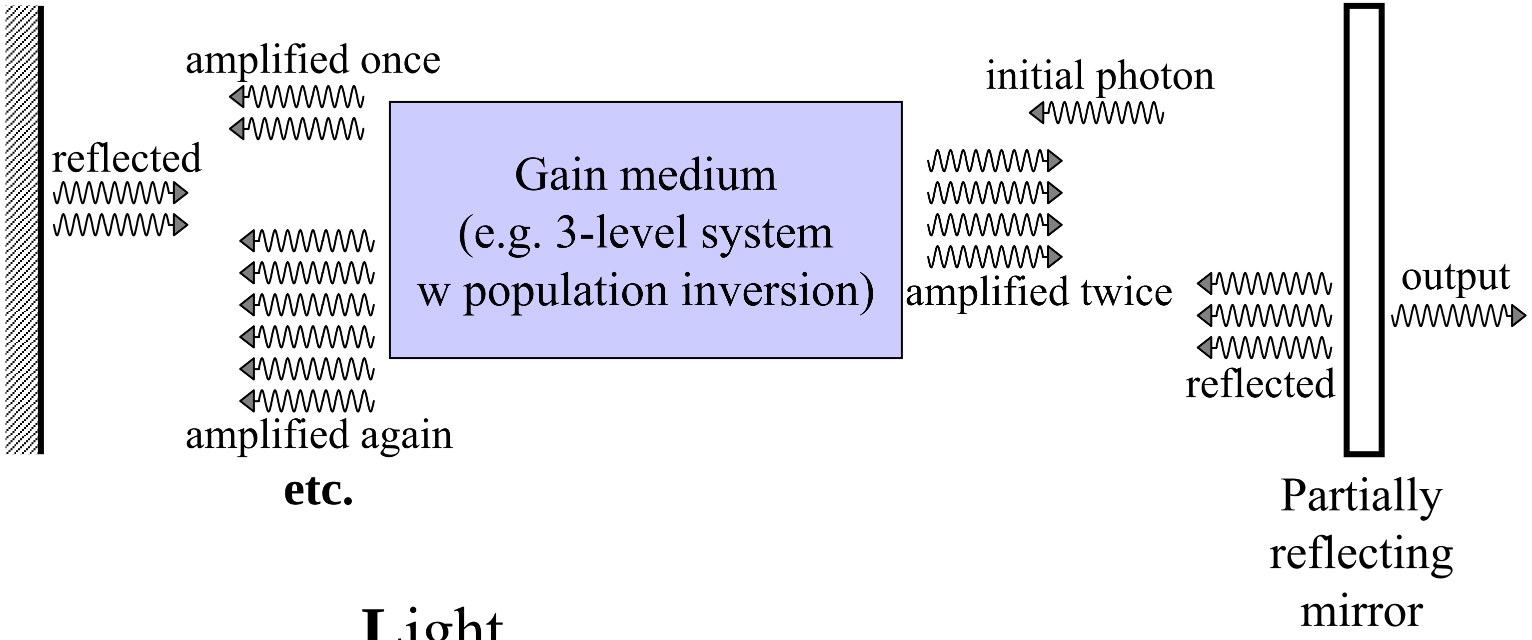
Light
Amplification through
Stimulated
Emission of
Radiation

Laser



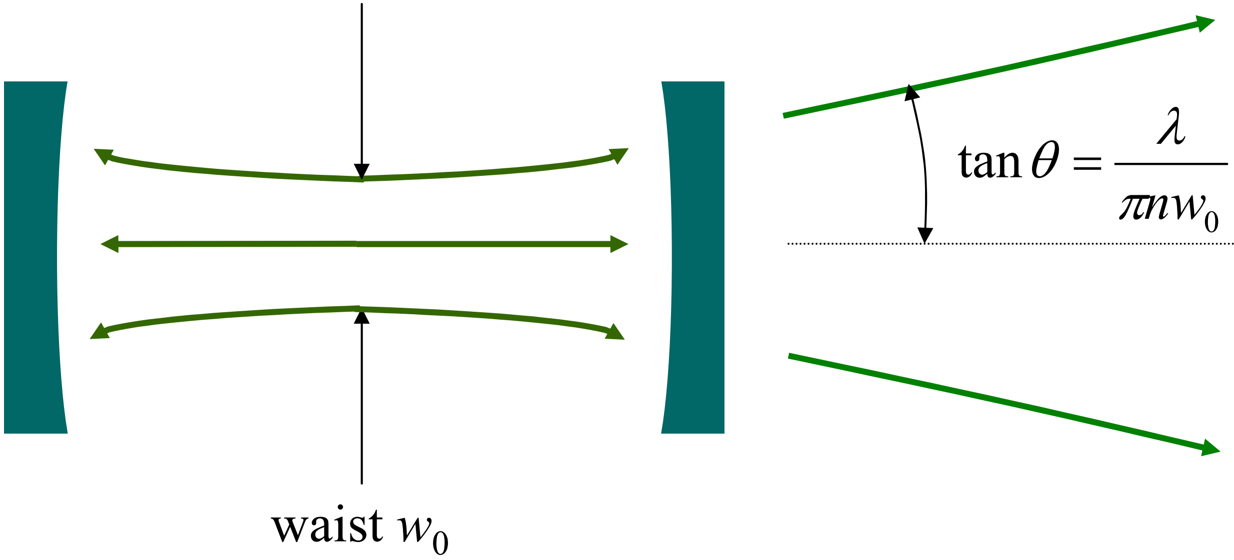
Light
Amplification through
Stimulated
Emission of
Radiation

Laser



Light
Amplification through
Stimulated
Emission of
Radiation

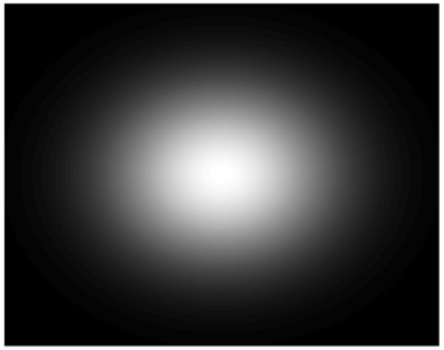
Confocal laser cavities



waist w_0

$$\tan\theta = \frac{\lambda}{\pi n w_0}$$

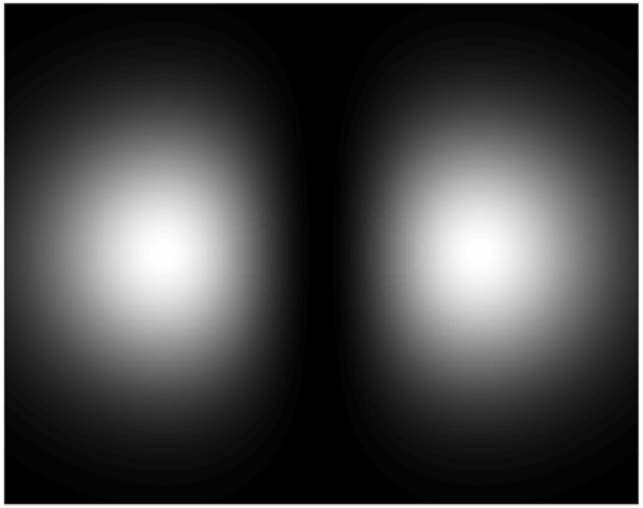
diffraction angle



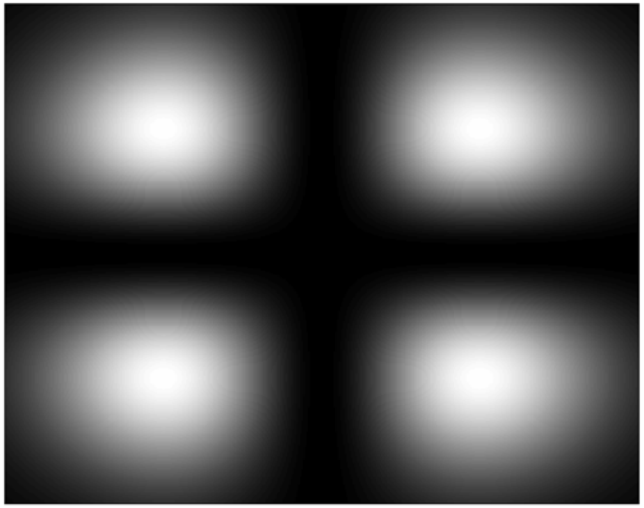
Beam profile:
2D Gaussian function

"TE_{00} mode"

Other "transverse modes"



TE10



TE11

(usually undesirable)

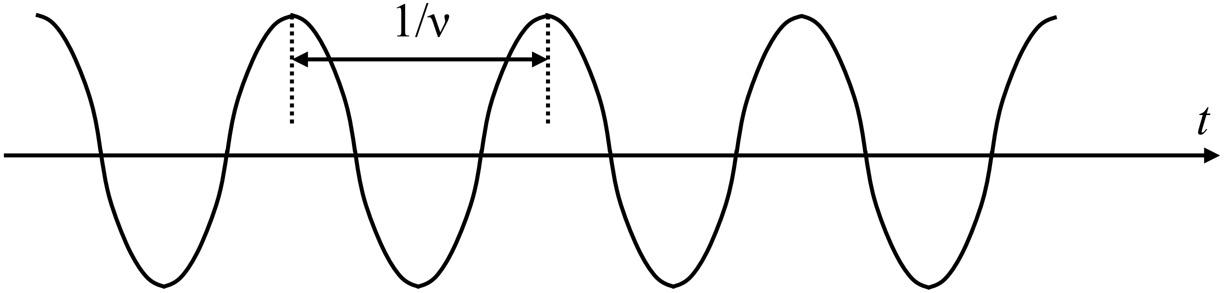
Types of lasers

- Continuous wave (cw)
- Pulsed
  - Q-switched
  - mode-locked

- Gas (Ar-ion, HeNe, CO_2)
- Solid state (Ruby, Nd:YAG, Ti:Sa)
- Diode (semiconductor)
- Vertical cavity surface-emitting lasers –VCSEL– (also sc)
- Excimer (usually ultra-violet)

CW (continuous wave lasers)

Laser oscillation well approximated by a sinusoid



Typical sources:

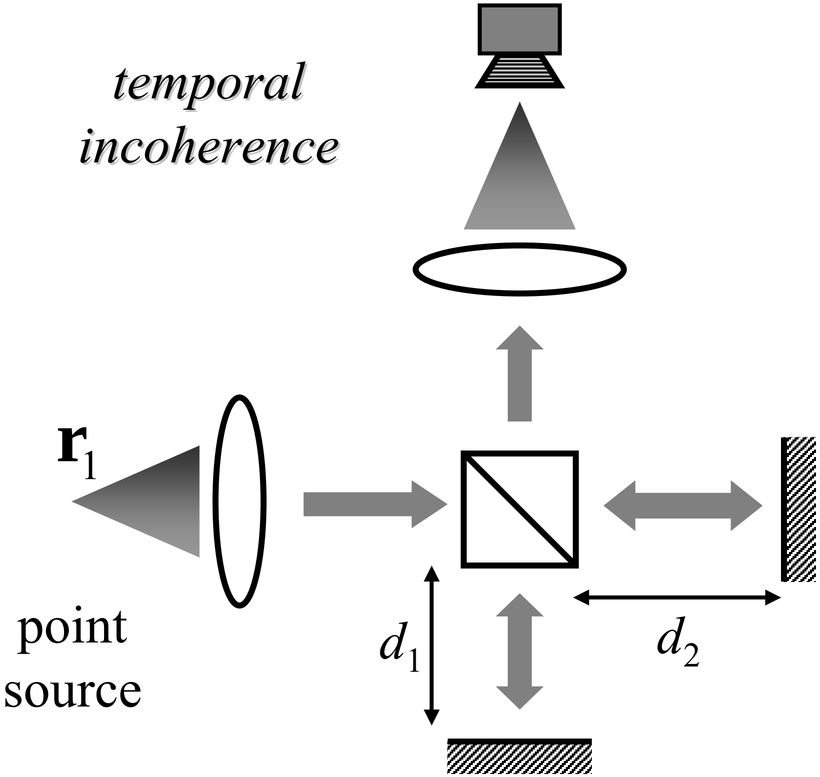
- Argon-ion: 488nm (blue) or 514nm (green); power ~1-20W
- Helium-Neon (HeNe): 633nm (red), also in green and yellow; ~1-100mW
- doubled Nd:YaG: 532nm (green); ~1-10W

Quality of sinusoid maintained over a time duration known as
"coherence time" t_c

Typical coherence times ~20nsec (HeNe), ~10μsec (doubled Nd:YAG)

Two types of incoherence

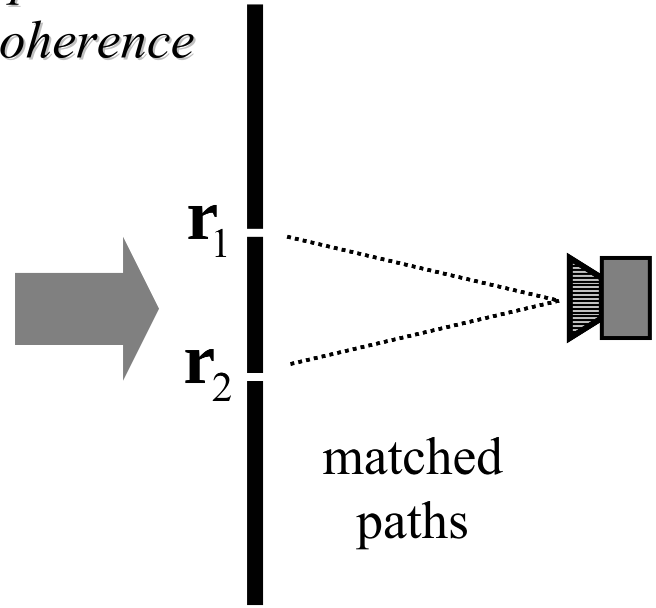
temporal incoherence



Michelson interferometer

poly-chromatic light
(=multi-color, broadband)

spatial incoherence

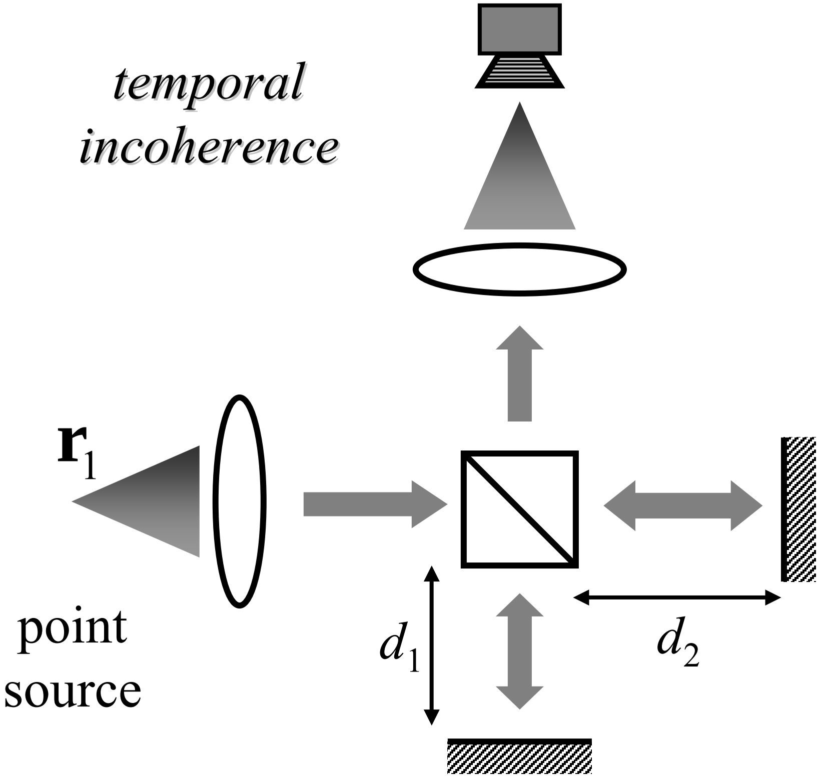


Young interferometer

mono-chromatic light
(= single color, narrowband)

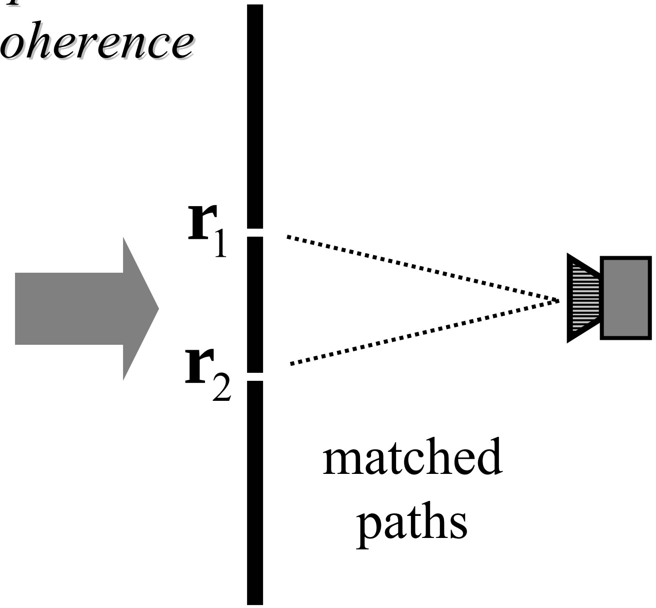
Two types of incoherence

temporal incoherence



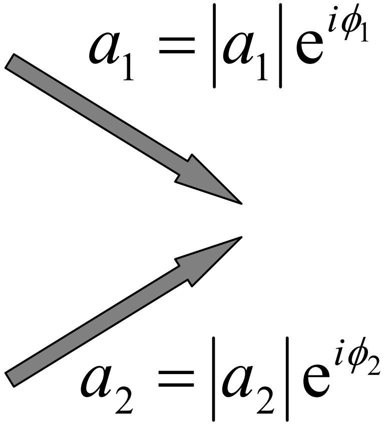
waves from unequal paths do not interfere

spatial incoherence



waves with equal paths but from different points on the wavefront do not interfere

Coherent vs incoherent beams



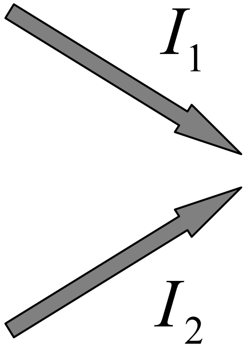
$$a_1 = |a_1| \mathrm{e}^{i\phi_1}$$

$$a_2 = |a_2| \mathrm{e}^{i\phi_2}$$

Mutually coherent: superposition field *amplitude* is described by *sum of complex amplitudes*

$$a = a_1 + a_2 = |a_1| \mathrm{e}^{i\phi_1} + |a_2| \mathrm{e}^{i\phi_2}$$

$$I = |a|^2 = |a_1 + a_2|^2$$



$$I_1$$

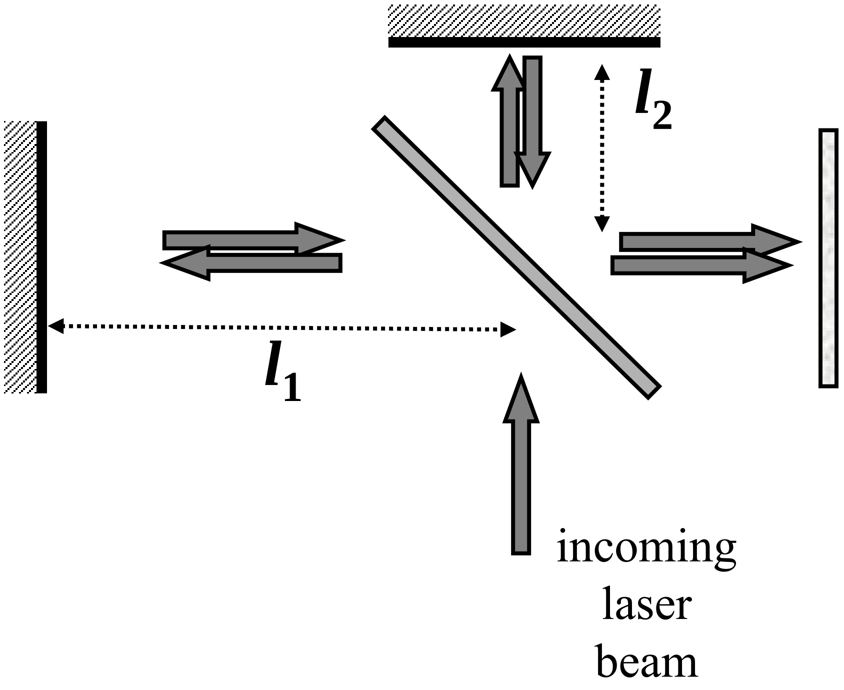
$$I_2$$

Mutually incoherent: superposition field *intensity* is described by *sum of intensities*

$$I = I_1 + I_2$$

(the phases of the individual beams vary randomly with respect to each other; hence, we would need statistical formulation to describe them properly — *statistical optics*)

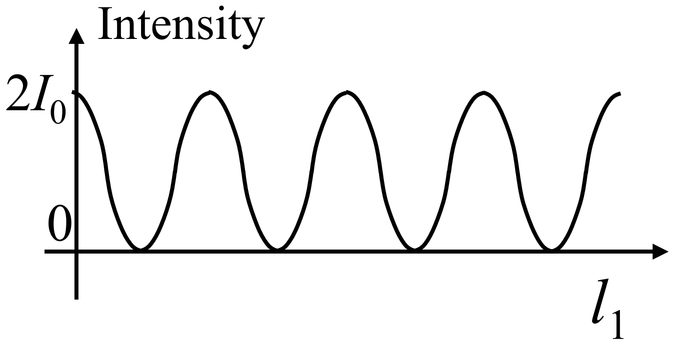
Coherence time and coherence length



Michelson interferometer

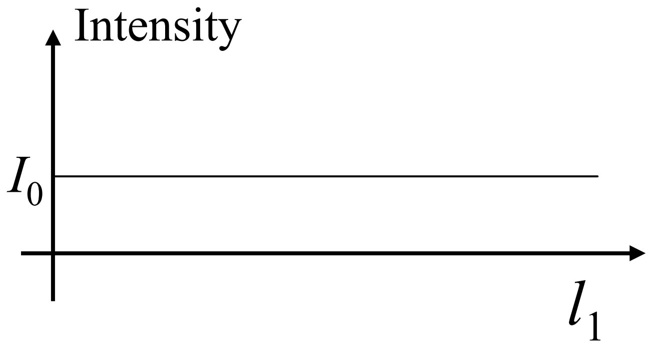
- l_1-l_2 much shorter than "coherence length" ct_c

  sharp interference fringes

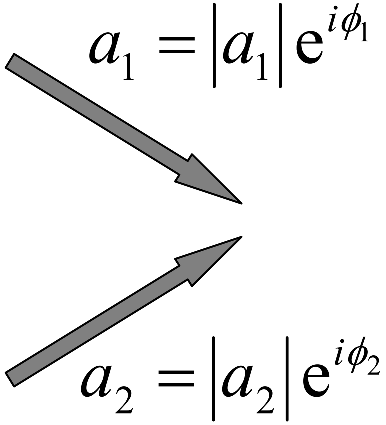


- l_1-l_2 much longer than "coherence length" ct_c

  no interference



Coherent vs incoherent beams



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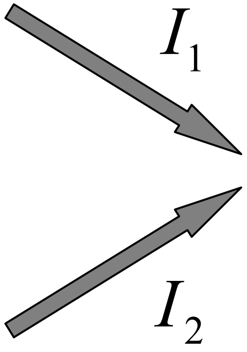
$$a_2 = |a_2| \mathrm{e}^{i\phi_2}$$

Coherent: superposition field *amplitude* is described by *sum of complex amplitudes*

$$a = a_1 + a_2 = |a_1| \mathrm{e}^{i\phi_1} + |a_2| \mathrm{e}^{i\phi_2}$$

$$I = |a|^2 = |a_1 + a_2|^2$$



$$I_1$$

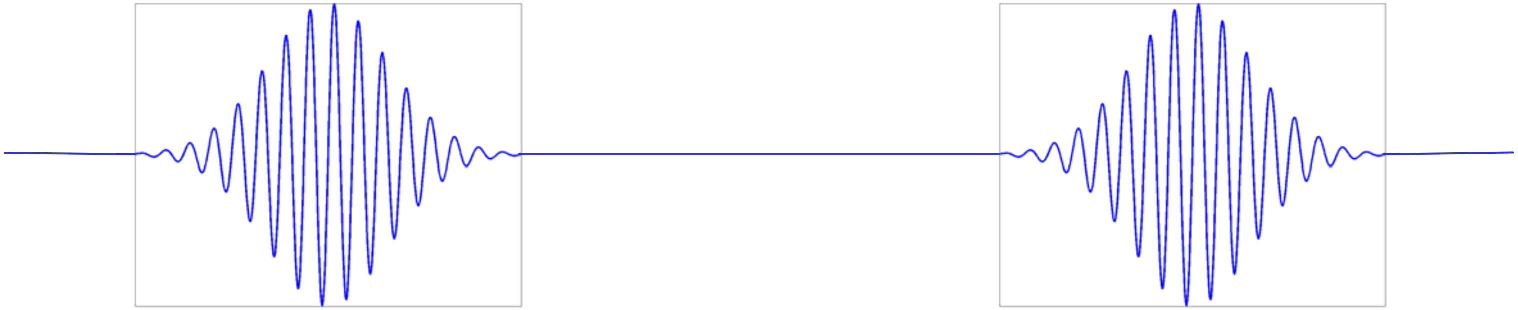
$$I_2$$

Incoherent: superposition field *intensity* is described by *sum of intensities*

$$I = I_1 + I_2$$

(the phases of the individual beams vary randomly with respect to each other; hence, we would need statistical formulation to describe them properly — *statistical optics*)

Mode-locked lasers



Typical sources: Ti:Sa lasers (major vendors: Coherent, Spectra Phys.)

Typical mean wavelengths: 700nm – 1.4µm (near IR)

can be doubled to visible wavelengths
or split to visible + mid IR wavelengths using OPOs or OPAs
(OPO=optical parametric oscillator;
OPA=optical parametric amplifier)

Typical pulse durations: ~psec to few fsec

(just a few optical cycles)

Typical pulse repetition rates (“rep rates”): 80-100MHz

Typical average power: 1-2W; peak power ~MW-GW

Overview of light sources

non-Laser

Thermal: polychromatic, spatially incoherent (e.g. light bulb)

Gas discharge: monochromatic, spatially incoherent (e.g. Na lamp)

Light emitting diodes (LEDs): monochromatic, spatially incoherent

Laser

Continuous wave (or cw): strictly monochromatic, spatially coherent (e.g. HeNe, Ar$^+$, laser diodes)

Pulsed: quasi-monochromatic, spatially coherent (e.g. Q-switched, mode-locked)

Q-switched: ~nsec

mode-locked: ~psec to few fsec

pulse duration

mono/poly-chromatic = single/multi color