MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.710 Optics **Fall '04**

Problem Set #5 Posted Wednesday, Oct. 13, 2004 — Due Wednesday, Oct. 20, 2004

1. Your friend Alice uses corrective eyeglasses of power -10D.

1.a) When Alice is not using her eyeglasses, is her retina located in front of or behind her eye's focal plane?

1.b) When you look at Alice's eye behind her eyeglasses, how much smaller or bigger does the pupil of her eye appear compared to its actual size?

1.c) If Alice wants to look at the moon through a Keplerian astronomical telescope, without her eyeglasses on, how does she have to adjust the eyepiece in order to see a clear image?

2. Two positive lenses L_1 and L_2 are separated by 5cm. Their diameters are 6 and 4cm, respectively, and their focal lengths are $f_1 = 9$cm and $f_2 = 3$cm. If a diaphragm with a hole 1cm in diameter is located between them, 2cm away from L_2, find

2.a) the aperture stop,

2.b) the locations and sizes of the pupils for an axial object point located 12cm to the left of L_2.

3. Plane waves and phasor representations Throughout this problem, by "complete expression" of a wave we mean the space-time representation, e.g. $f(x, y, z, t) = A\cos(kx - \omega t)$ is a plane wave of wave-vector magnitude k and frequency ω propagating in the direction of the $\hat{\boldsymbol{x}}$ coordinate axis. By "phasor expression" we mean the complex representation of the wave, e.g. Ae^{ikx} for the same wave.

3.a) Write the complete and phasor expressions for a plane wave $f_1(x, y, z, t)$ propagating at an angle 30° relative to the $\hat{\boldsymbol{z}}$ axis on the xz-plane (*i.e.*, the plane $y = 0$). The wavelength is $\lambda = 1\mu$m, and the wave speed is $c = 3 \times 10^8 \text{m} \cdot \text{sec}^{-1}$.

3.b) Write the complete and phasor expressions for a plane wave $f_2(x, y, z, t)$ of the same wavelength and wave speed as f_1 but propagating at angle 60° relative to the $\hat{\boldsymbol{z}}$ axis on the yz-plane.

3.c) Use the complete expression to plot $f_1(x, y, z = 0, t = 0)$ and $f_2(x, y, z = 0, t = 0)$ using MATLAB. (*Note:* you will probably need to use `surf` or an equivalent command.)

3.d) The plane $z = 0$ is illuminated by the superposition of the two waves f_1 and f_2. Plot the waveform received at points A, B, C, D, E with Cartesian coordinates, respectively,

$$(0,0,0), \left(\frac{1}{4}, -\frac{1}{4\sqrt{3}}, 0\right), \left(\frac{1}{2}, -\frac{1}{2\sqrt{3}}, 0\right), \left(\frac{3}{4}, -\frac{3}{4\sqrt{3}}, 0\right), \left(1, -\frac{1}{2\sqrt{3}}, 0\right)$$

(all units in microns) as function of time. What do you observe?

4. Consider the dielectric displacement field $\boldsymbol{D}$ corresponding to a plane wave of wave–vector $\boldsymbol{k}$. Use Maxwell's equations to show that $\boldsymbol{D} \cdot \boldsymbol{k} = 0$, *i.e.* that the two vectors are perpendicular to each other.