Holography

- Preamble: modulation and demodulation
- The principle of wavefront reconstruction
- The Leith-Upatnieks hologram
- The Gabor hologram
- Image locations and magnification
- Holography of three-dimension scenes
- Transmission and reflection holograms
- Rainbow hologram

Modulation & Demodulation

- Principle borrowed from radio telecommunications
- Idea is to take **baseband signal** (e.g. speech, music, with maximum frequencies up to ~20kHz) and **modulate** it onto a **carrier signal** which is a simple tone at the frequency where the radio station emits, e.g. 104.3 MHz (that's Boston's WBCN station)
- One of the benefits of modulation is that radio stations can be **multiplexed** by using a different emission frequency each
- After selecting the desired station, the receiver follows a process of **demodulation** which recovers the baseband signal and sends it to the speakers.

Types of modulation

- Amplitude modulation (AM)
- Frequency modulation (FM)
- Phase modulation (PM)
- Digital methods (Amplitude Shift Keying – ASK, Frequency Shift Keying – FSK, Phase Shift Keying – PSK, etc.)

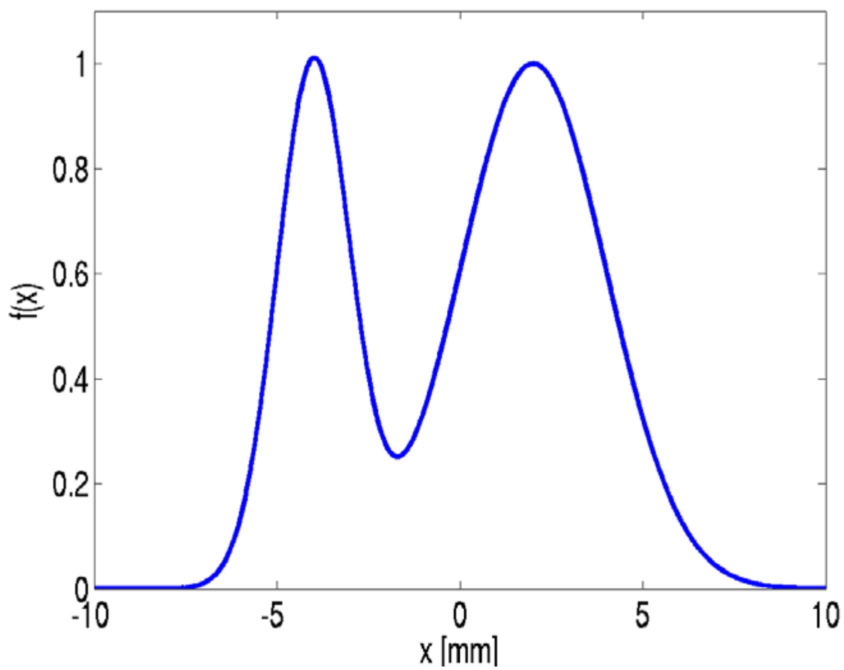
used in radio at low frequencies only ("AM band" = 535kHz to 1.7MHz) ; as we will see, it is an almost-exact analog of holography

dominant in commercial radio ("FM band" = 88MHz to 108MHz) ; there is an analog in optics, called "spectral holography," but it is beyond the scope of the class

Amplitude modulation

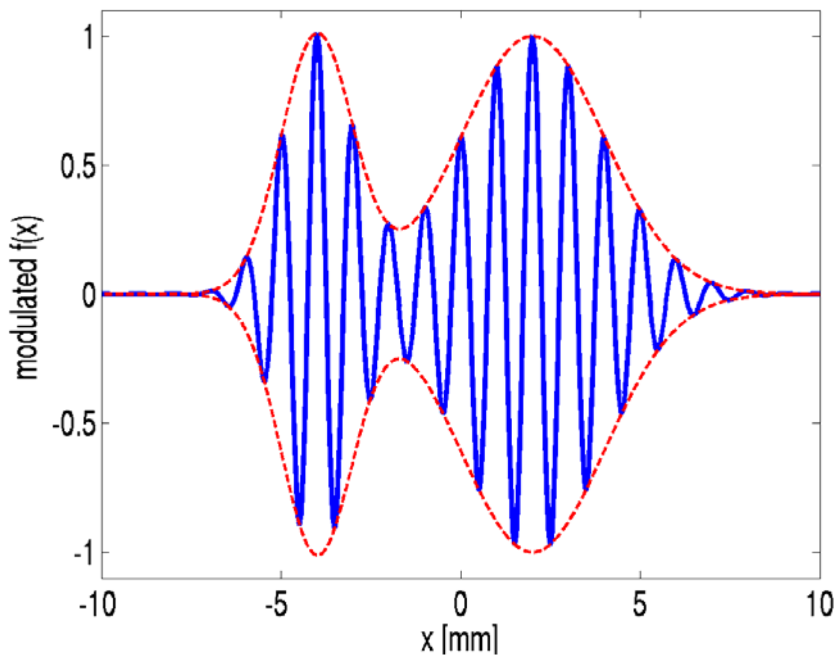
$f(x)$

(baseband)



modulated $f(x) =$

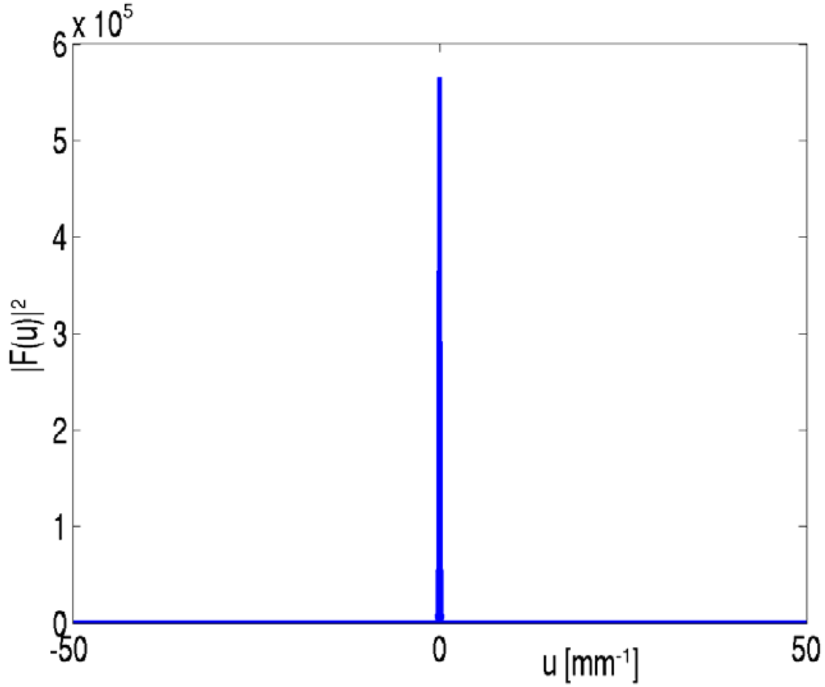
$= f(x) \times \cos(2\pi u_c x)$



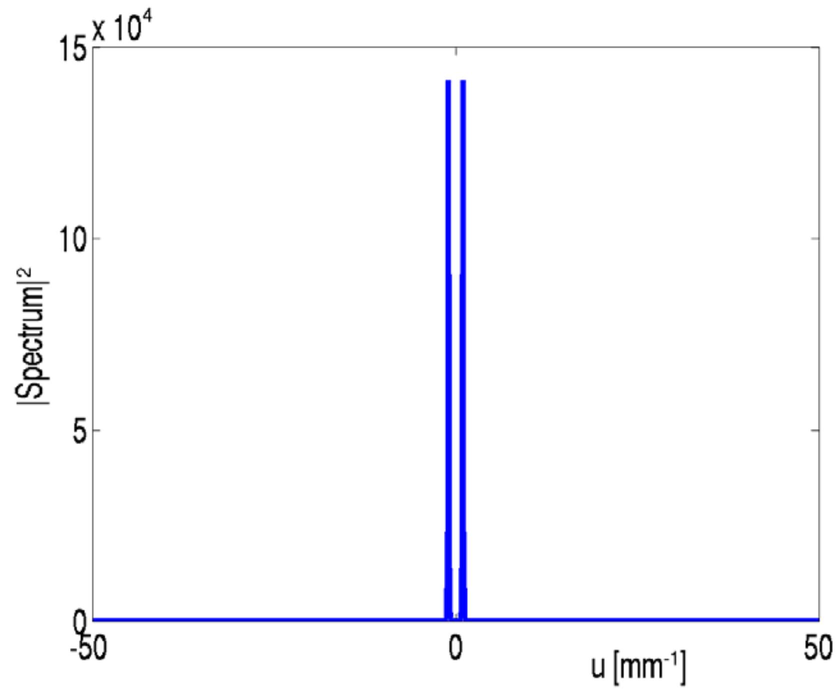
u_c: carrier frequency

AM in the frequency domain

spectrum of $f(x)$



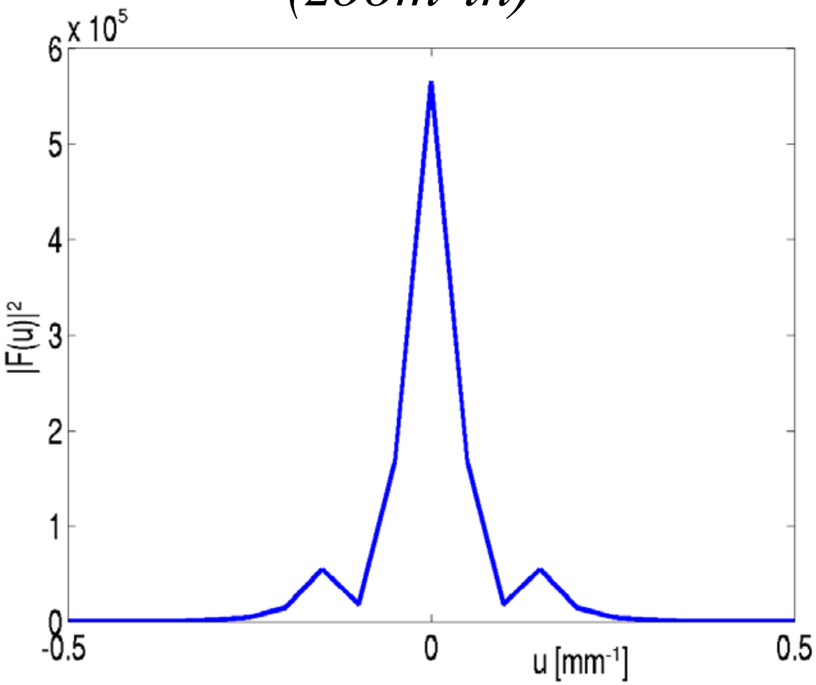
spectrum of modulated $f(x)$



AM in the frequency domain

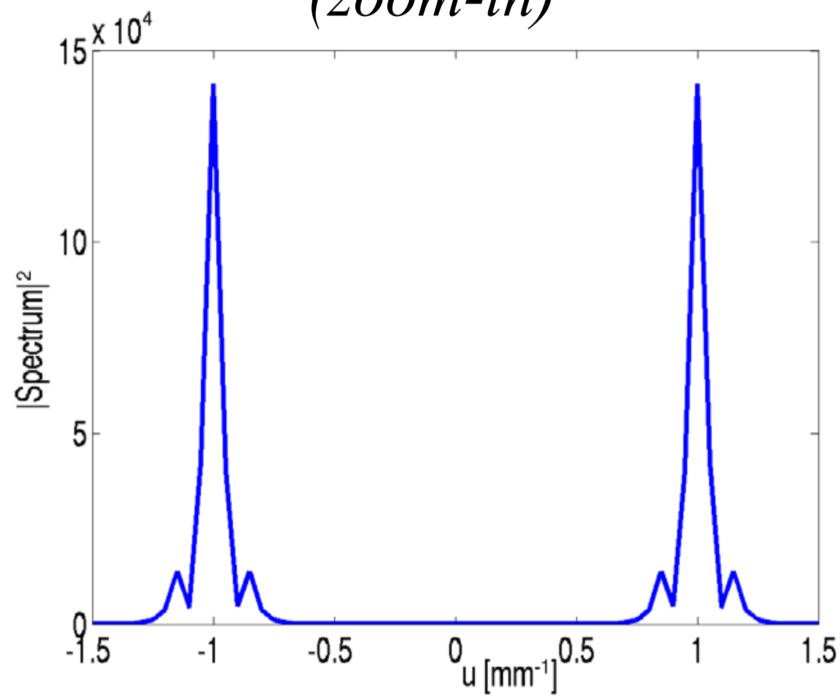
spectrum of $f(x)$

(zoom-in)



spectrum of modulated $f(x)$

(zoom-in)



AM in the frequency domain

$$f(x) \rightarrow F(u)$$

$$f(x)\cos(2\pi u_c x) = f(x) \times \frac{1}{2}\left[e^{i2\pi u_c x} + e^{-i2\pi u_c x}\right]$$

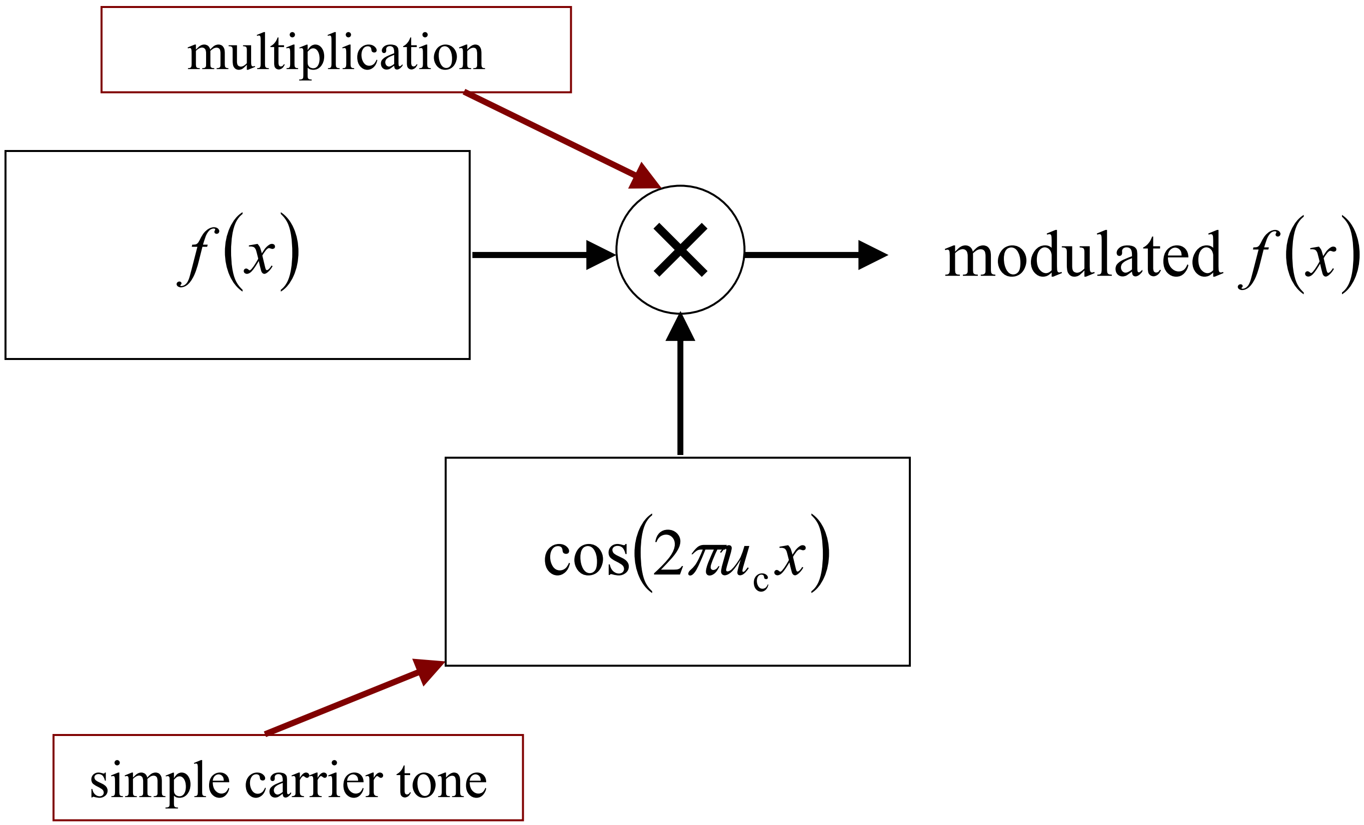
modulation in the space domain

$$\rightarrow F(u) * \frac{1}{2}\left[\delta(u - u_c) + \delta(u - u_c)\right] =$$

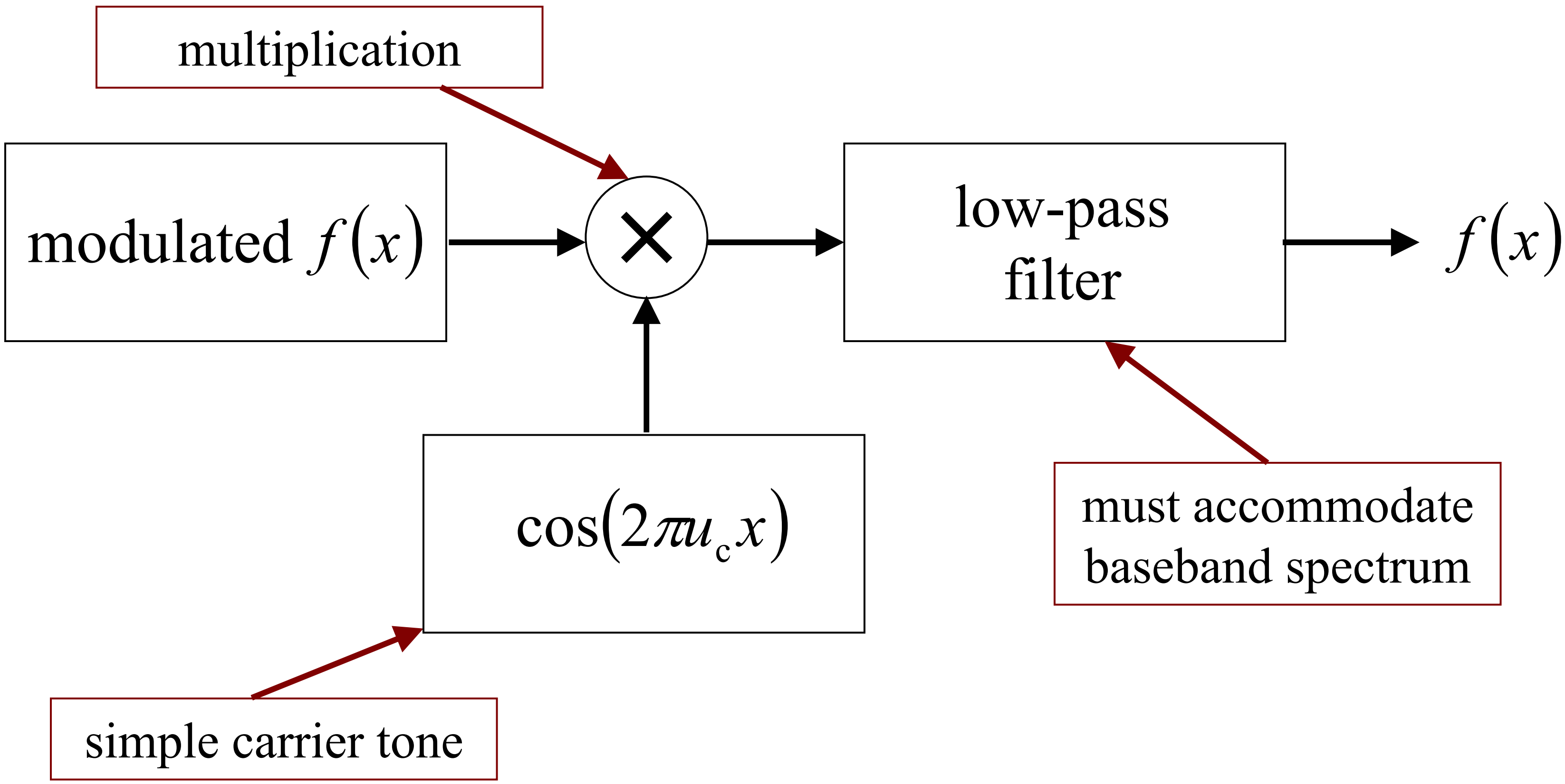
$$= \frac{1}{2}\left[F(u - u_c) + F(u - u_c)\right]$$

modulation in the frequency domain: two replicas of the baseband spectrum, centered on the carrier frequency

Modulation

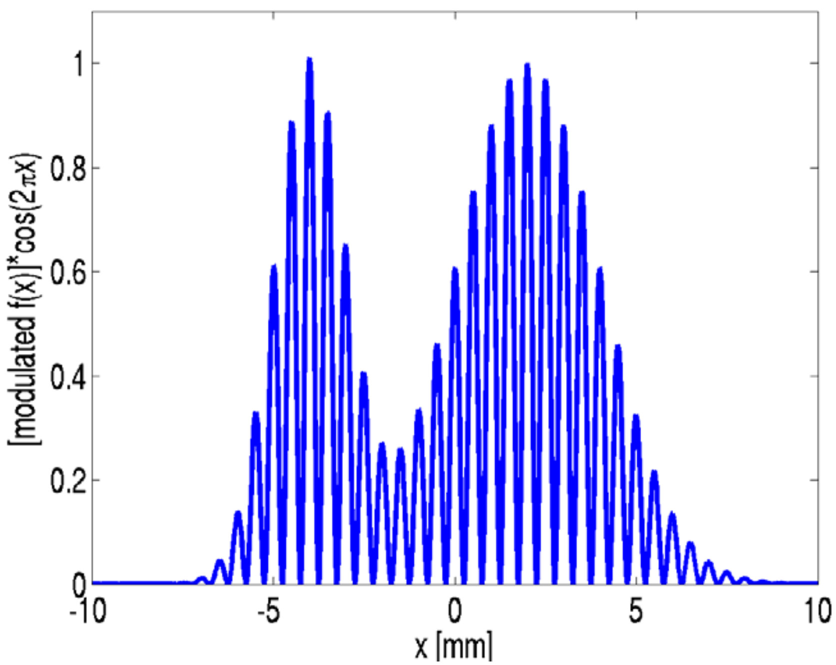

multiplication
$f(x)$
$\times$
modulated $f(x)$
$\cos(2\pi u_c x)$
simple carrier tone

Demodulation

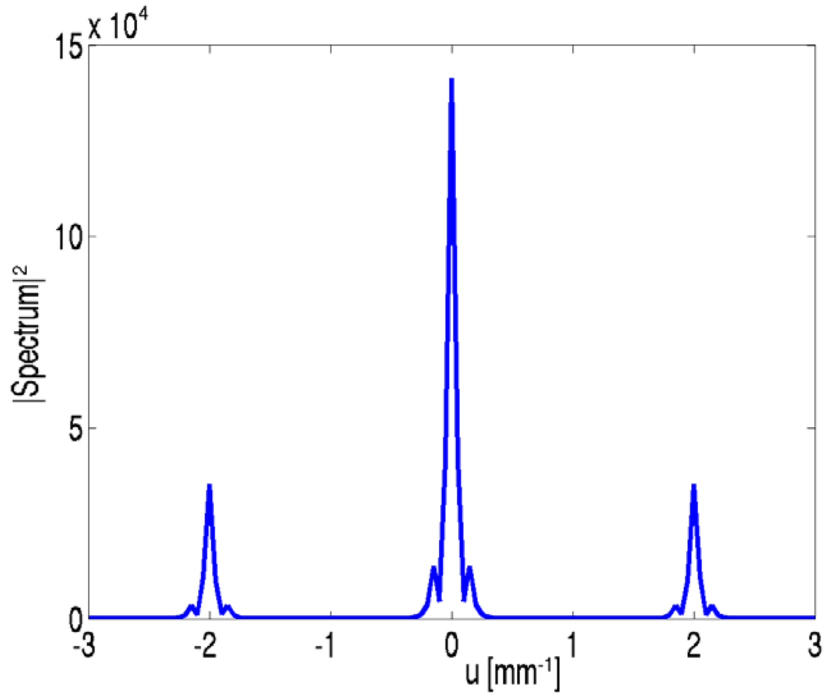


Demodulation

$f(x) \times \cos^2(2\pi u_c x)$

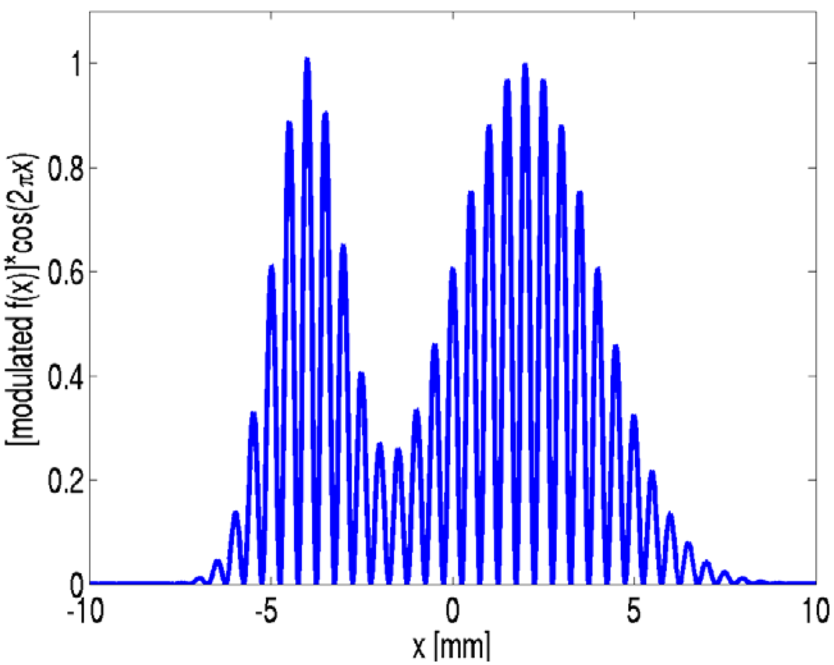


spectrum of $f(x) \times \cos^2(2\pi u_c x)$

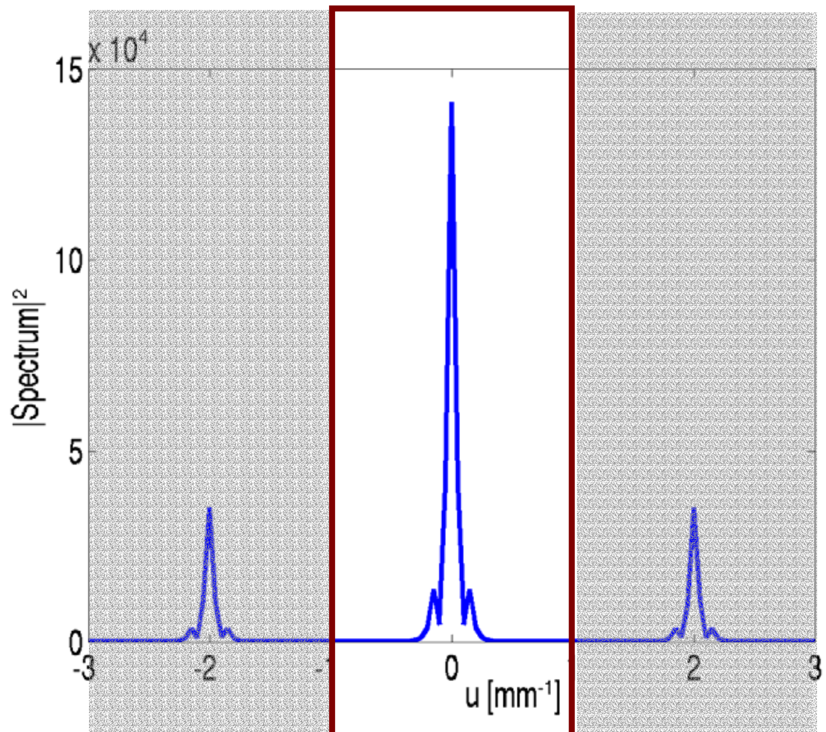


Demodulation

$f(x) \times \cos^2(2\pi u_c x)$



spectrum of $f(x) \times \cos^2(2\pi u_c x)$



LP filter pass-band

The wavefront reconstruction problem

- Wavefront is the amplitude (i.e. magnitude and phase) of the electric field as function of position
- Traditional coherent imaging results in intensity images (because detectors do not respond fast enough at optical frequencies) → magnitude information is recovered but phase information is lost
- Can we imprint ***intensity*** information on an optical wave? YES → **photography** (known since the 1840's)
- Can we imprint ***wavefront*** information on an optical wave? YES → **holography** (Gabor, late 1940s)

Photography: recording

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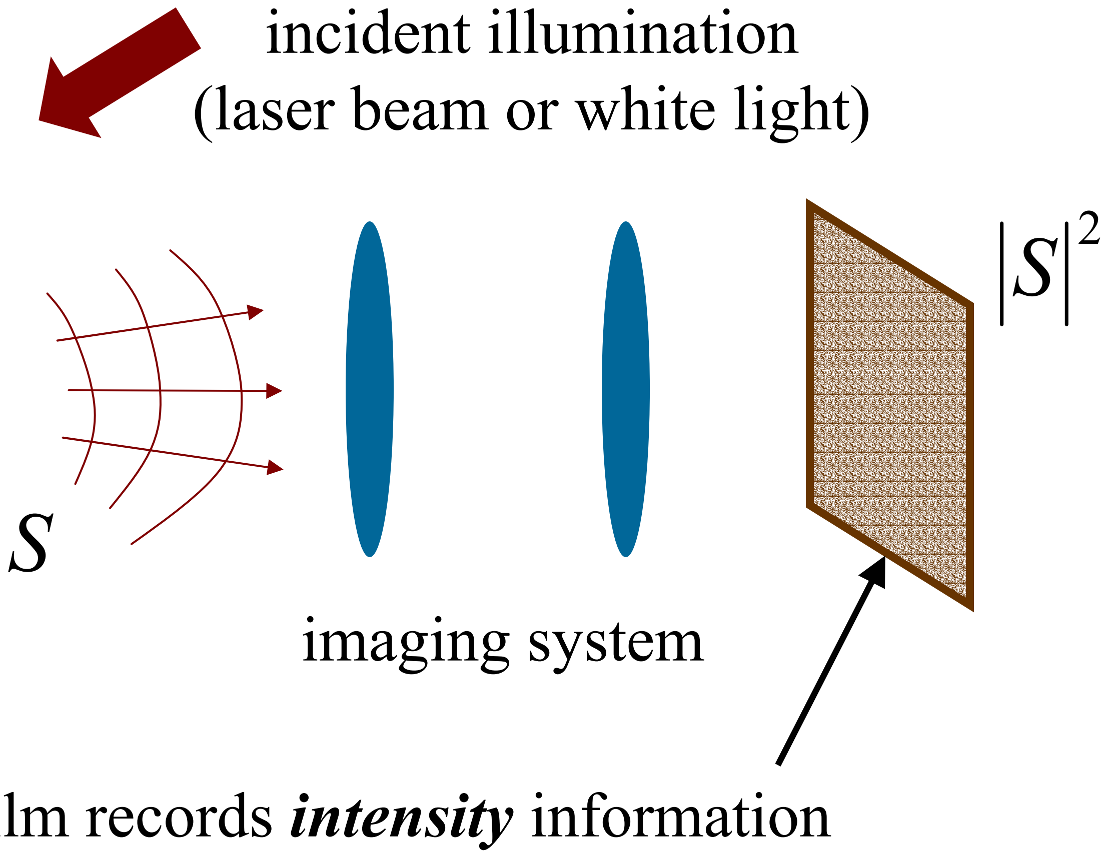
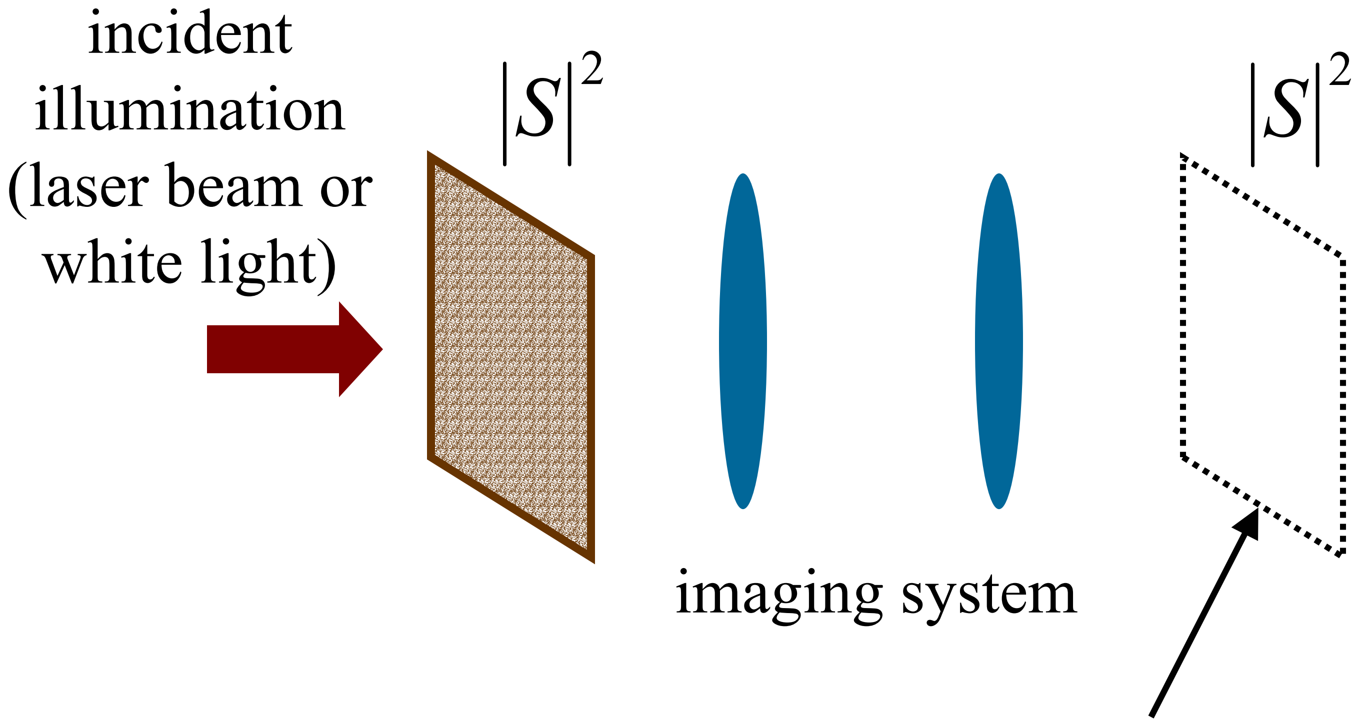


Photography: reconstructing the <u>intensity</u>



at the image plane, an intensity pattern is formed that replicates the originally recorded intensity

Holography: recording

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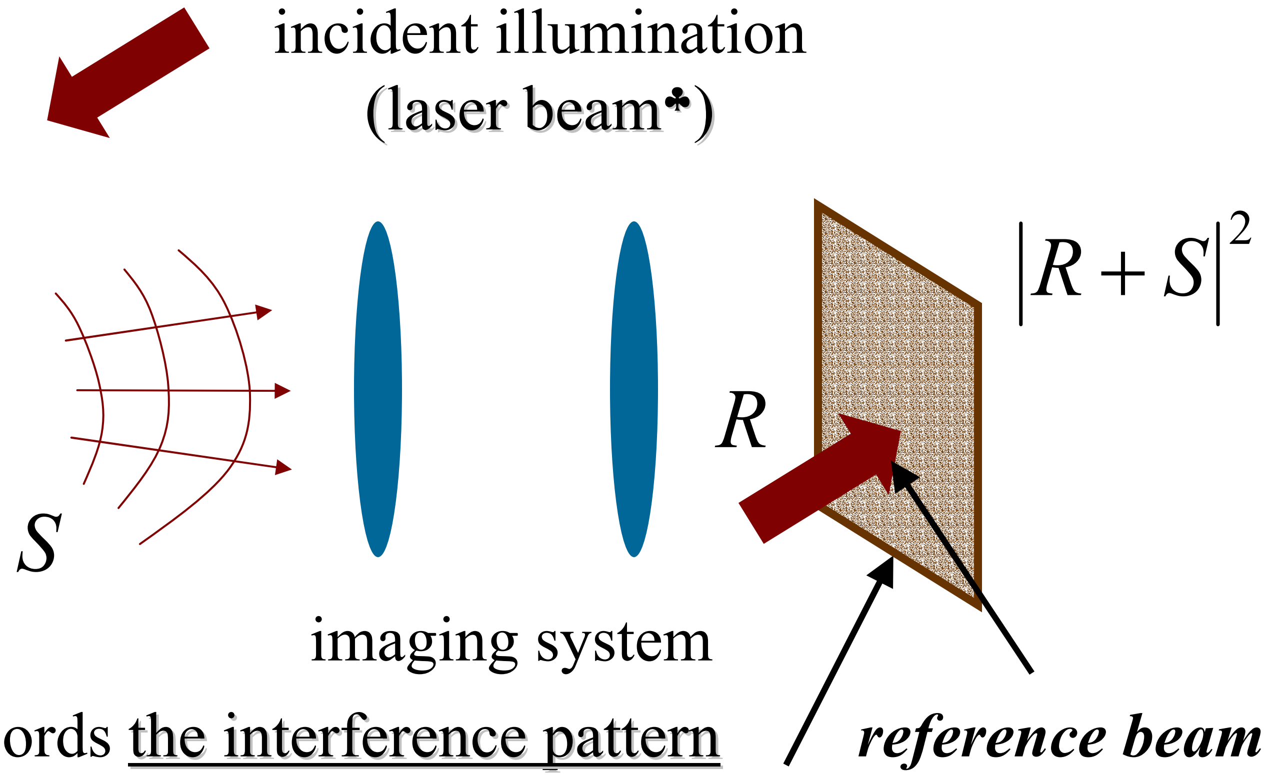
incident illumination (laser beam♣)

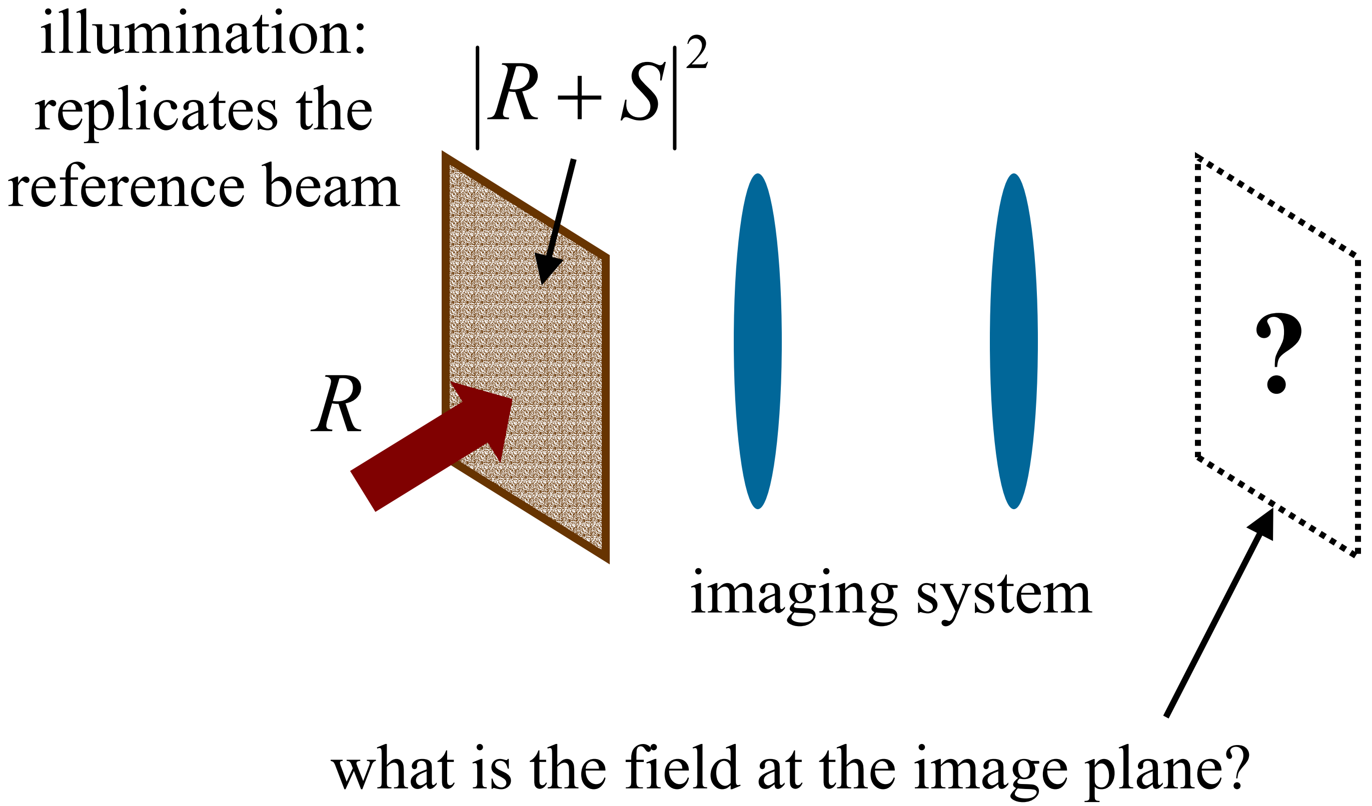


film records <u>the interference pattern</u> ***(interferogram)*** of the object wavefront and the reference wavefront

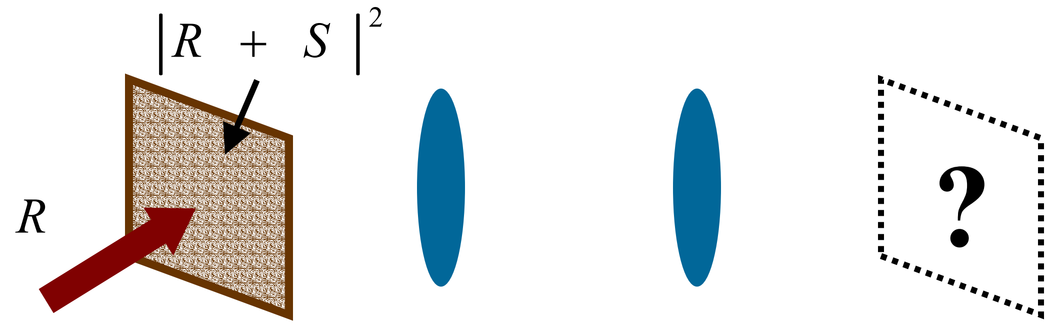
reference beam (split from the same laser)

♣in general, the illumination must be quasi-monochromatic, and spatially mutually coherent with the reference beam throughout the wavefront

Holography: reconstructing the <u>wavefront</u>



Holography: reconstructing the <u>wavefront</u>



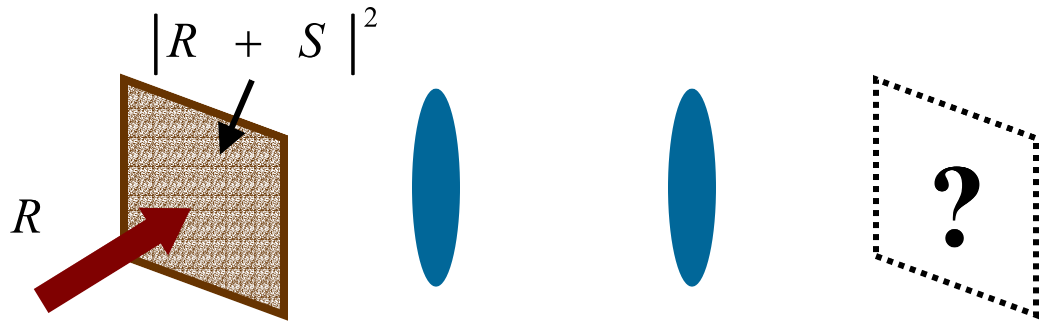
The field being imaged is:

$$R \times |R+S|^2 = R \times \left(|R|^2 + |S|^2 + R^* S + R S^*\right) =$$

$$= R \times \left(|R|^2 + |S|^2\right) + |R|^2 S + R^2 S^*$$

❶ ❷ ❸

Holography: reconstructing the <u>wavefront</u>



take the simplest possible reference wave, a **plane wave**:

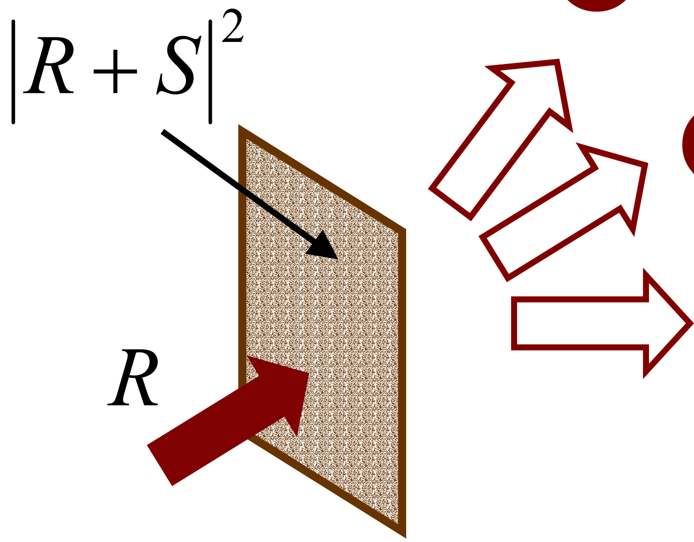
$$R = \mathrm{e}^{i2\pi u_0 x} \quad , \quad \text{spatial frequency} \quad u_0 = \frac{\sin\theta_0}{\lambda}$$

then the reconstructed field is:

$$= \underbrace{\mathrm{e}^{i2\pi u_0 x} \times \left(1 + |S|^2\right)}_{❶} + \underbrace{S}_{❷} + \underbrace{\mathrm{e}^{i2\pi(2u_0)x}\, S^*}_{❸}$$

Holography: reconstructing the <u>wavefront</u>

propagates at angle:



❸ $e^{i2\pi(2u_0)x}\, S^* \longrightarrow 2\theta_0$

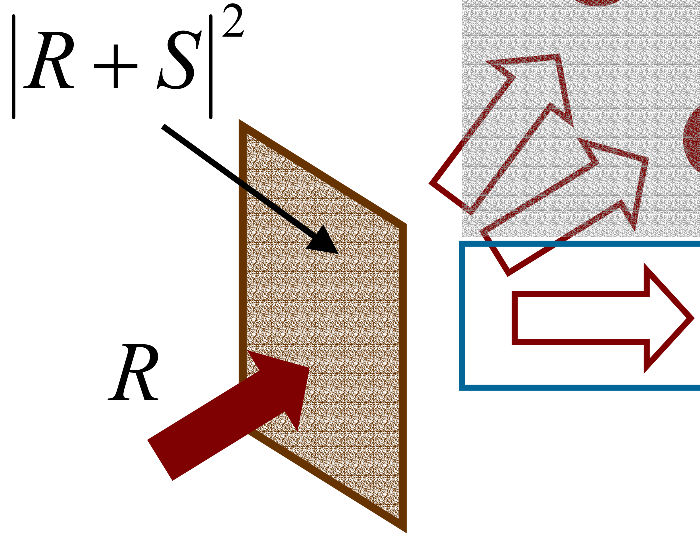
❶ $e^{i2\pi u_0 x}\times\left(1+|S|^2\right) \longrightarrow \theta_0$

❷ $S \longrightarrow$ on-axis

fields departing from the hologram

Holography: reconstructing the <u>wavefront</u>

not wanted



③ $e^{i2\pi(2u_0)x} S^* \longrightarrow 2\theta_0$

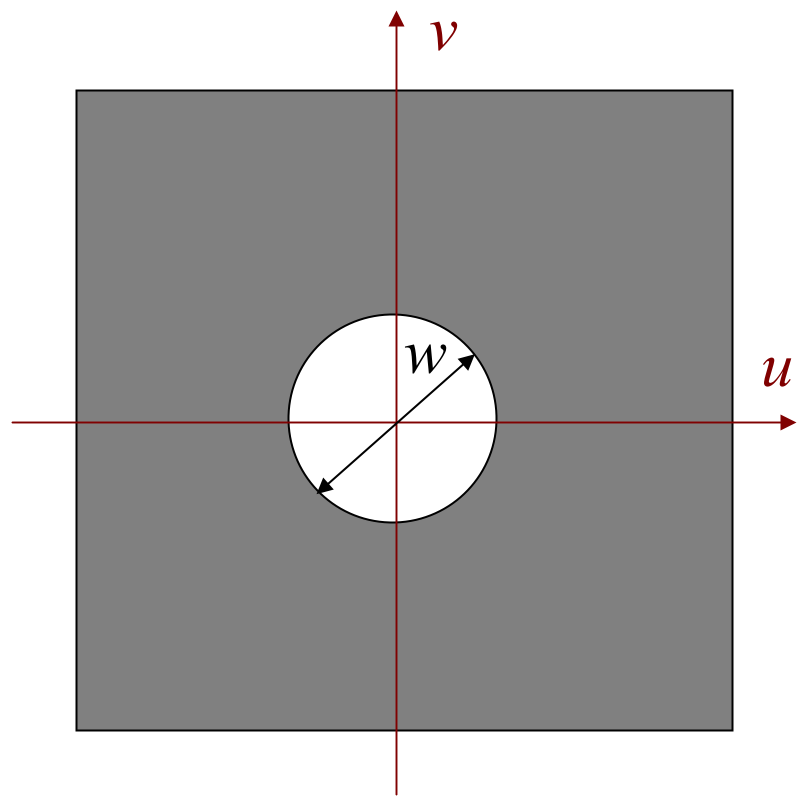
① $e^{i2\pi u_0 x} \times \left(1+|S|^2\right) \longrightarrow \theta_0$

② $S \longrightarrow$ on-axis

wanted

fields departing from the hologram

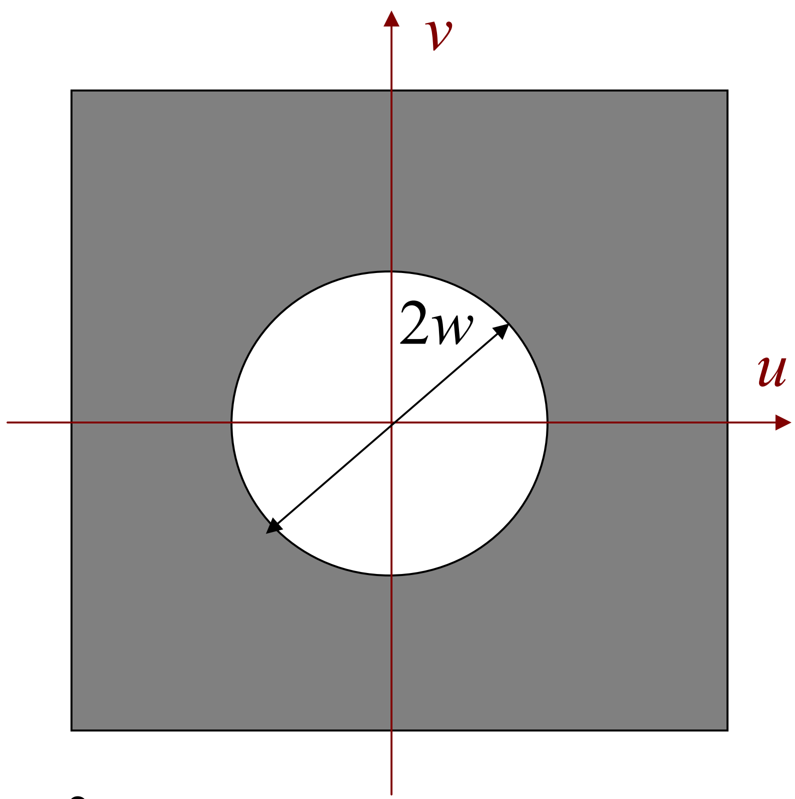
Filtering the wavefront: bandlimited signal



S has bandwidth w, i.e.

$\Im\{S\} \neq 0$ within circle of radius w

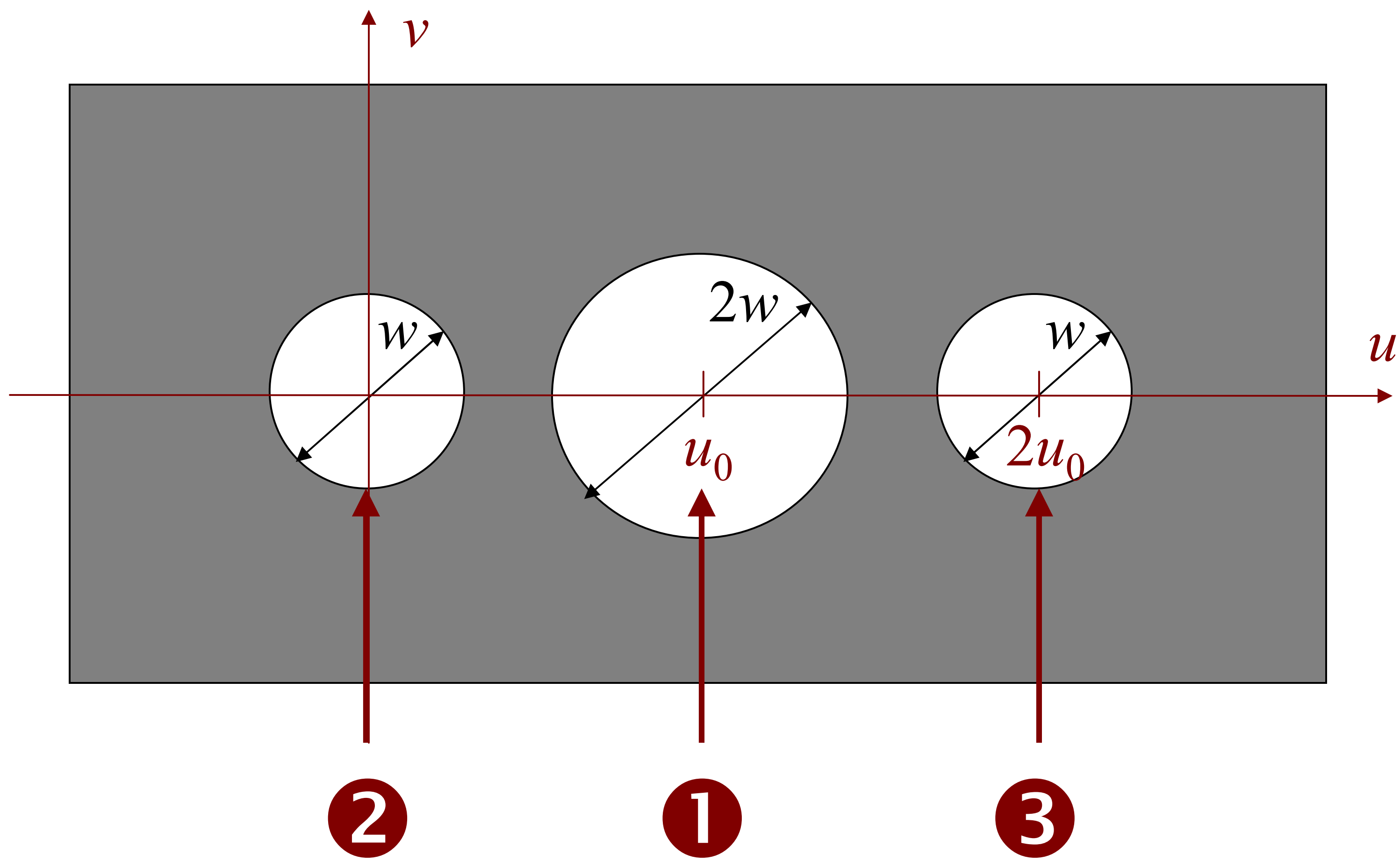
Filtering the wavefront: bandlimited signal



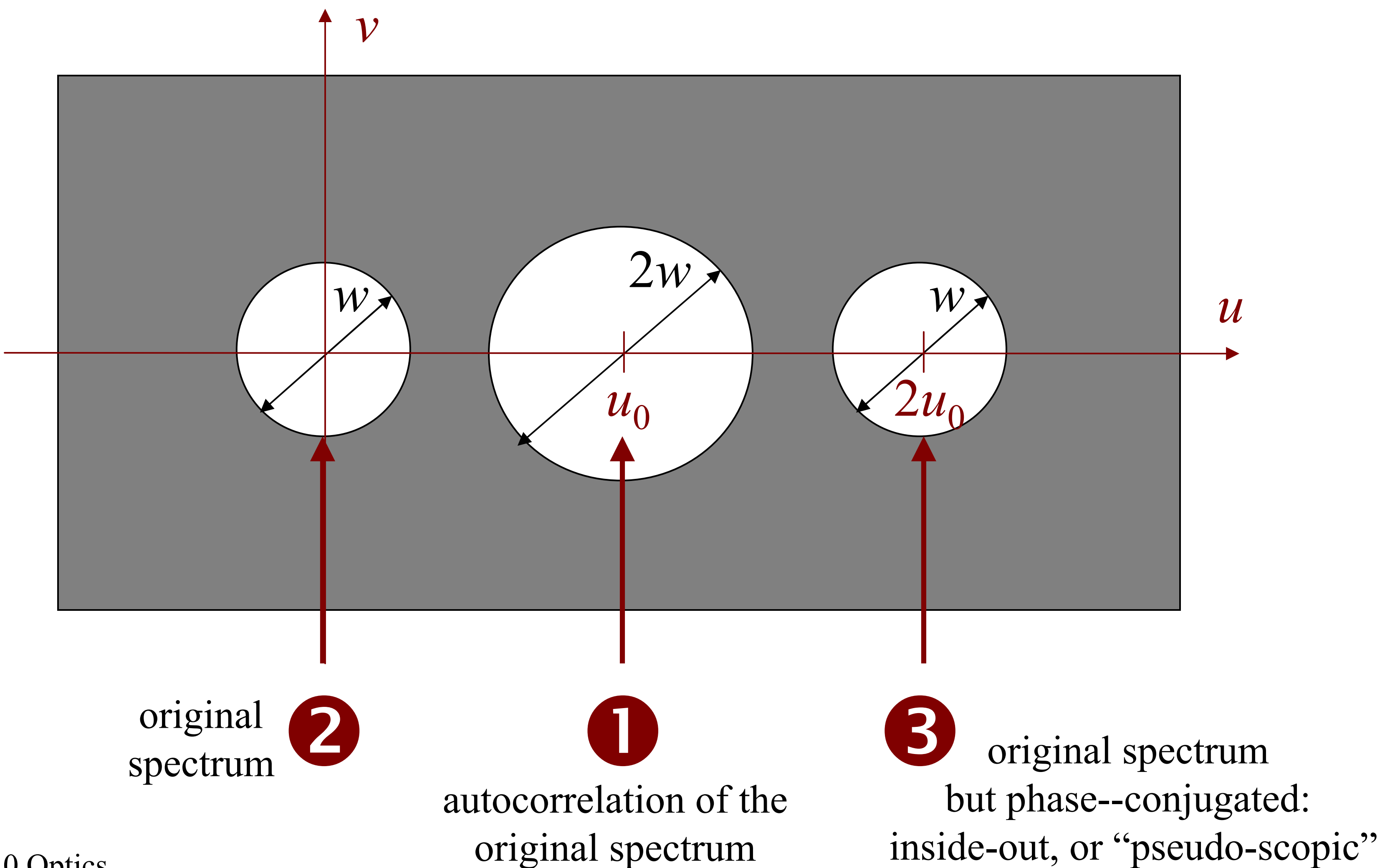
Term ❶ → $1+|S|^2$ has bandwidth $2w$, because

$$\Im\left\{|S|^2\right\}=\Im\{S\}*\Im\left\{S^*\right\}=\Im\{S\}\otimes\Im\{S\}$$

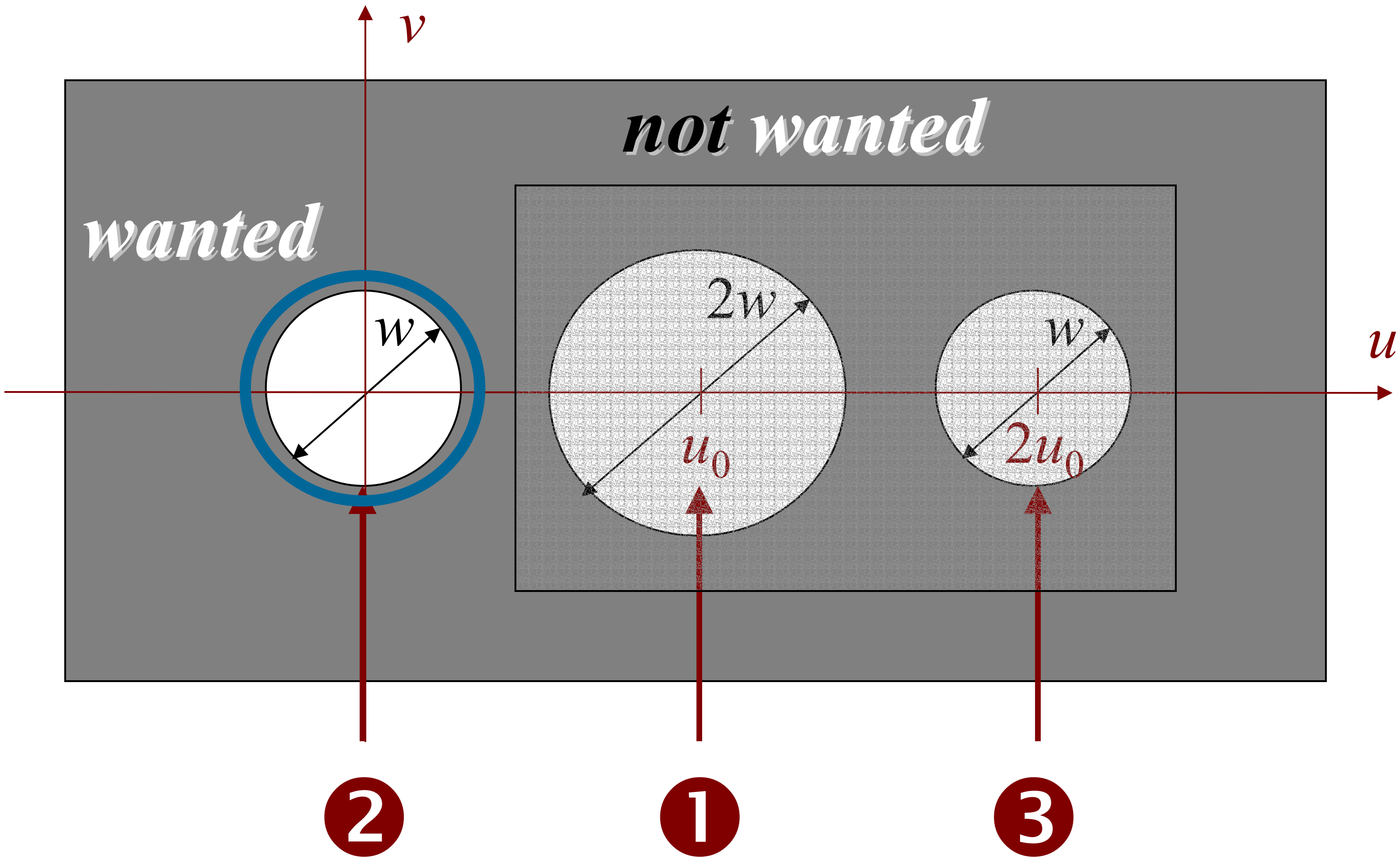
Filtering the wavefront: Fourier transform description



Filtering the wavefront: Fourier transform description

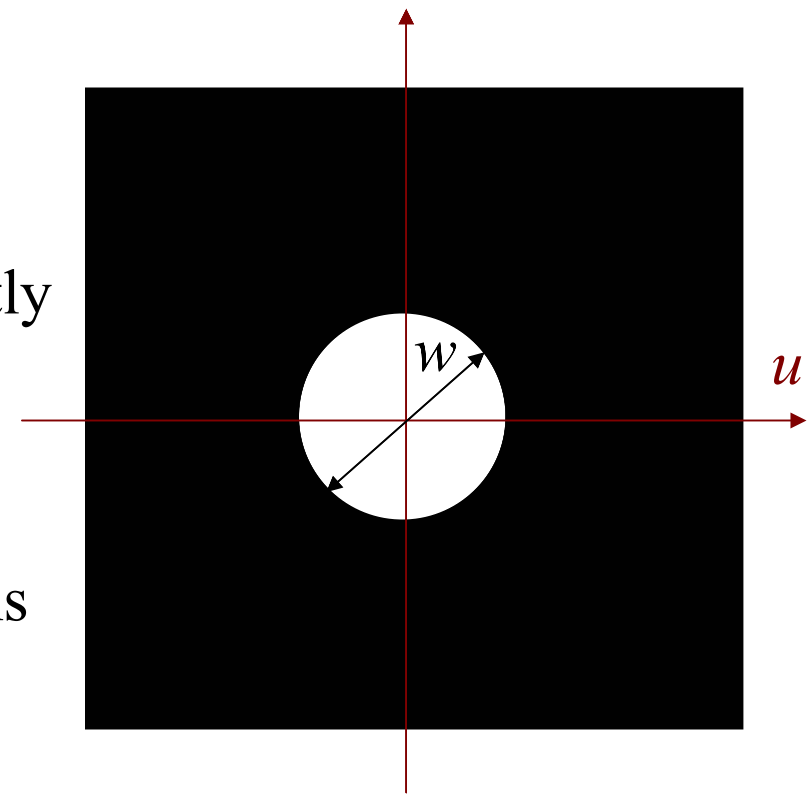


Filtering the wavefront: Fourier transform description

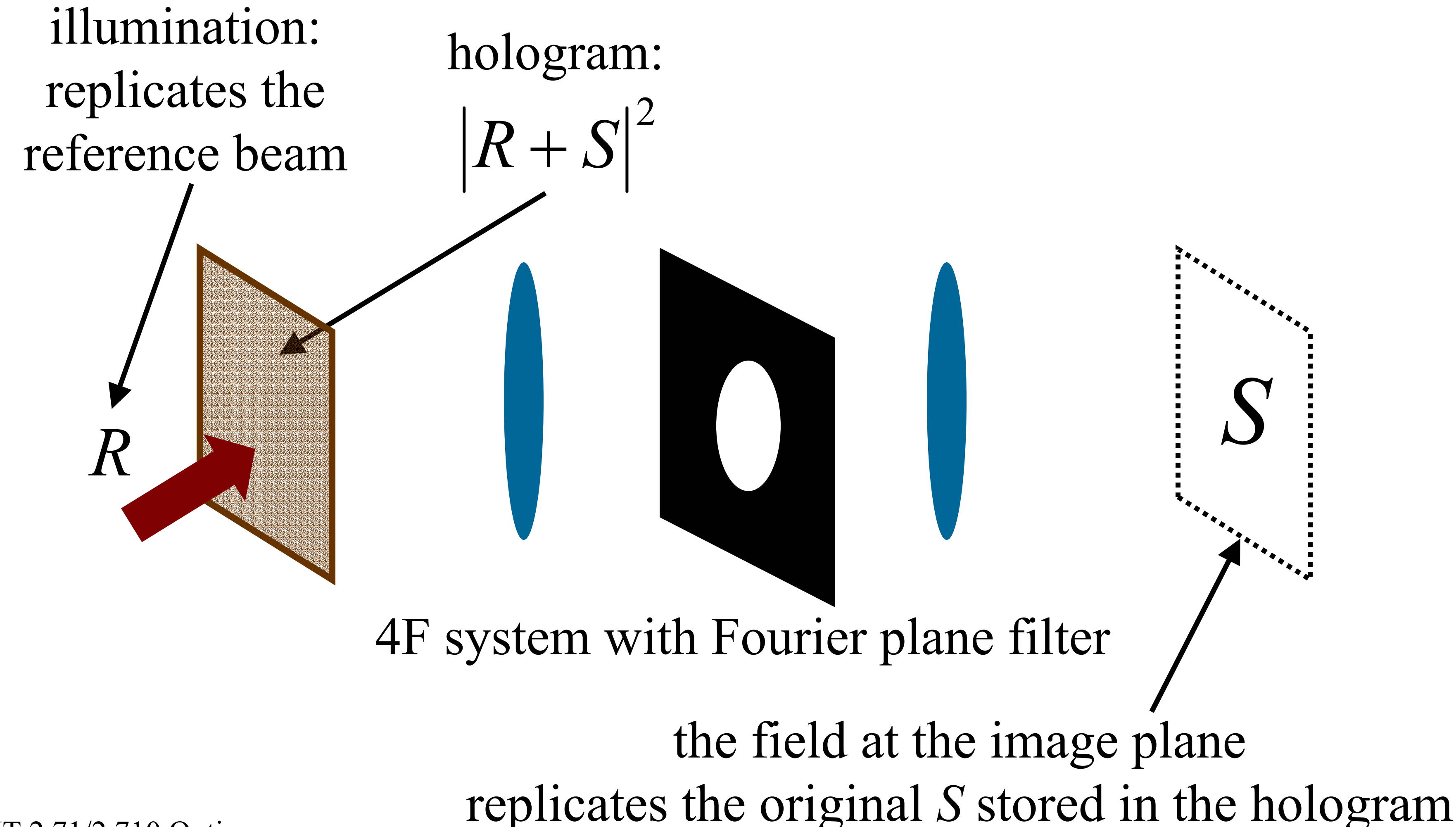


Filtering the wavefront: Fourier transform description

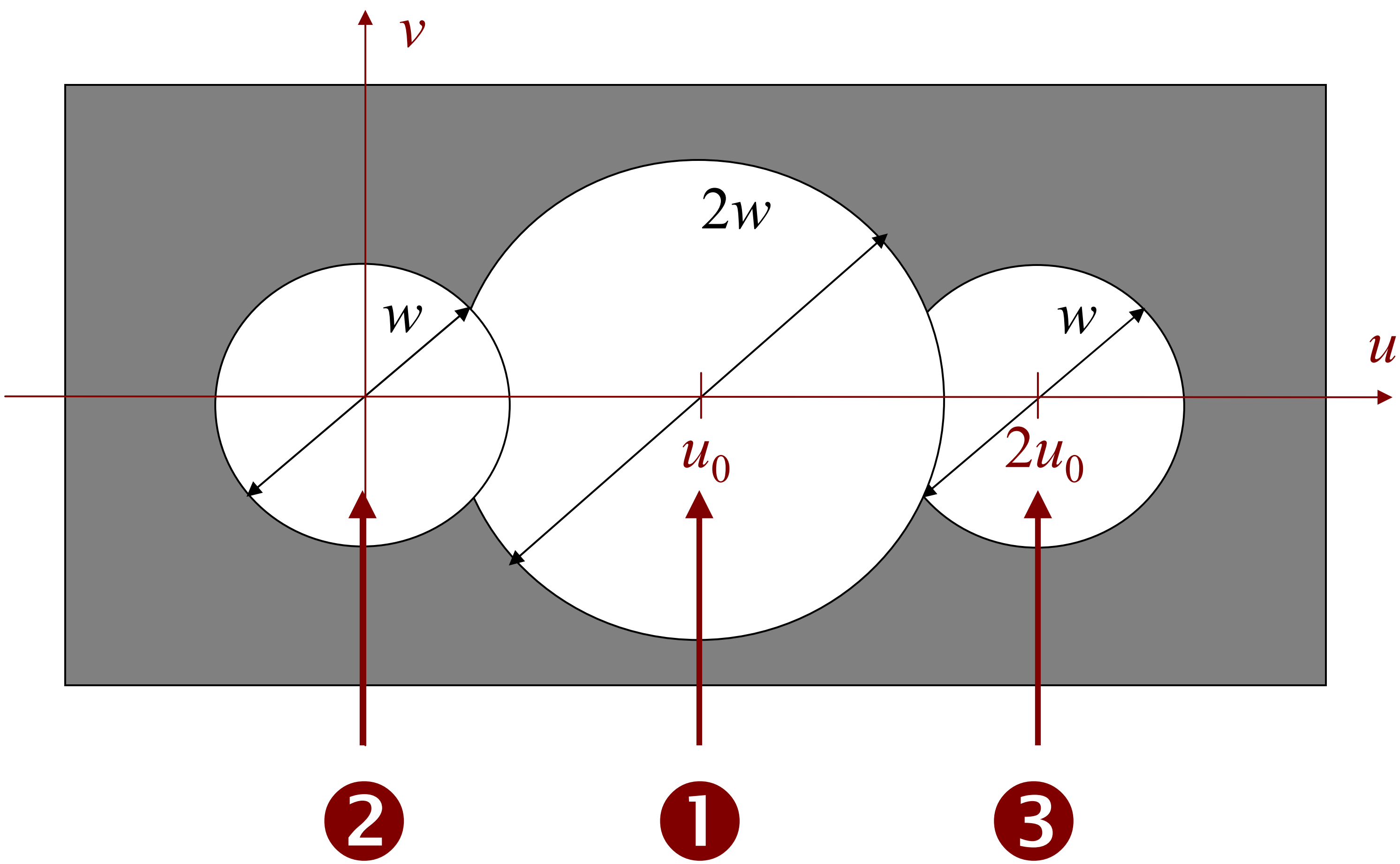
a low-pass filter of passband w or slightly greater permits the desired term ❷ to pass, and eliminates the undesirable terms ❶ and ❸.



Holography: reconstructing the <u>wavefront</u>

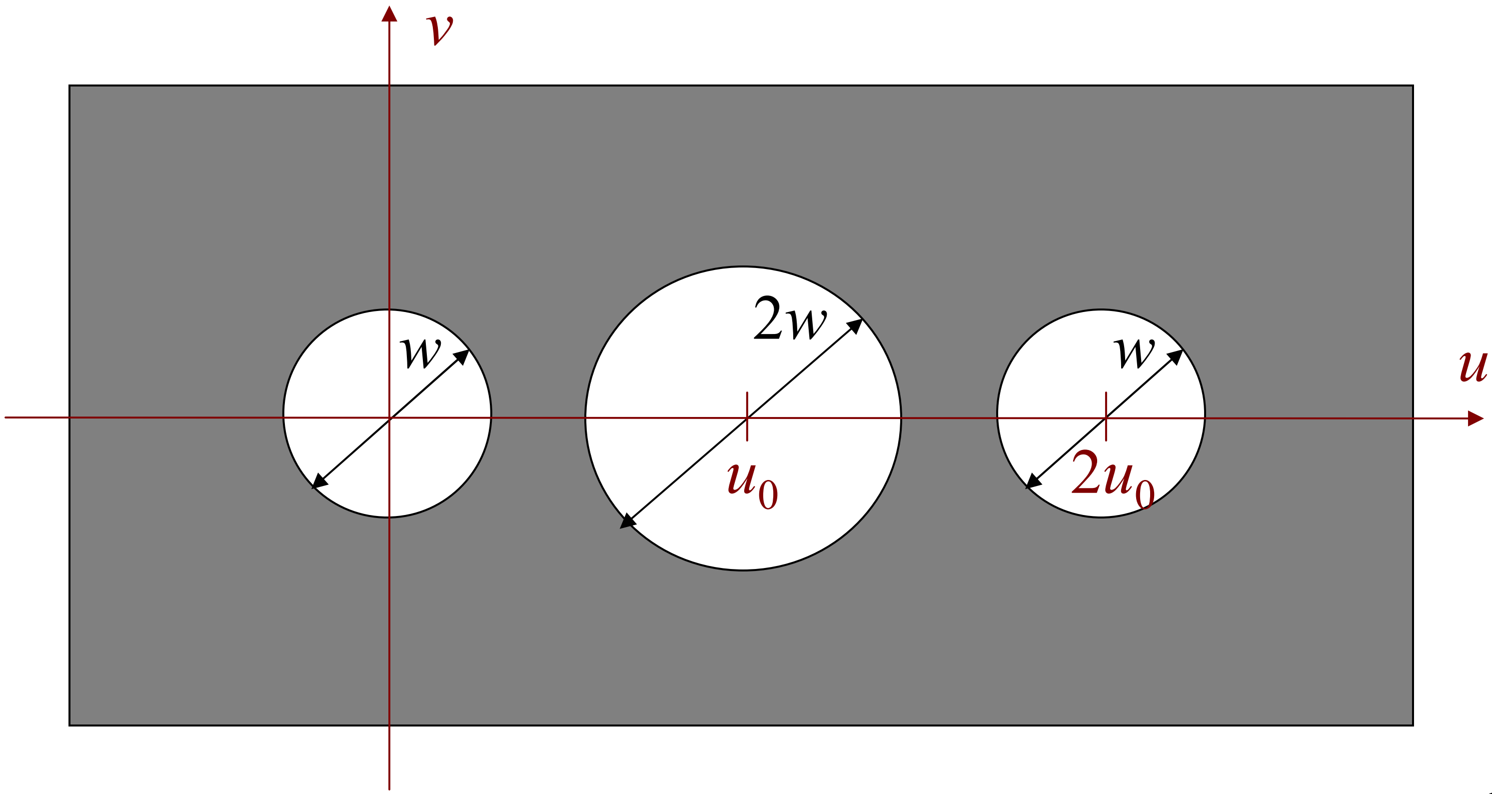


Filtering the wavefront: Fourier transform description



Potential problem: spectra overlap!

Filtering the wavefront: Fourier transform description



Spectra should not overlap, i.e. $u_0 - w > \frac{w}{2} \Leftrightarrow u_0 > \frac{3}{2} w$

Leith-Upatnieks *vs* Gabor hologram

Leith-Upatnieks

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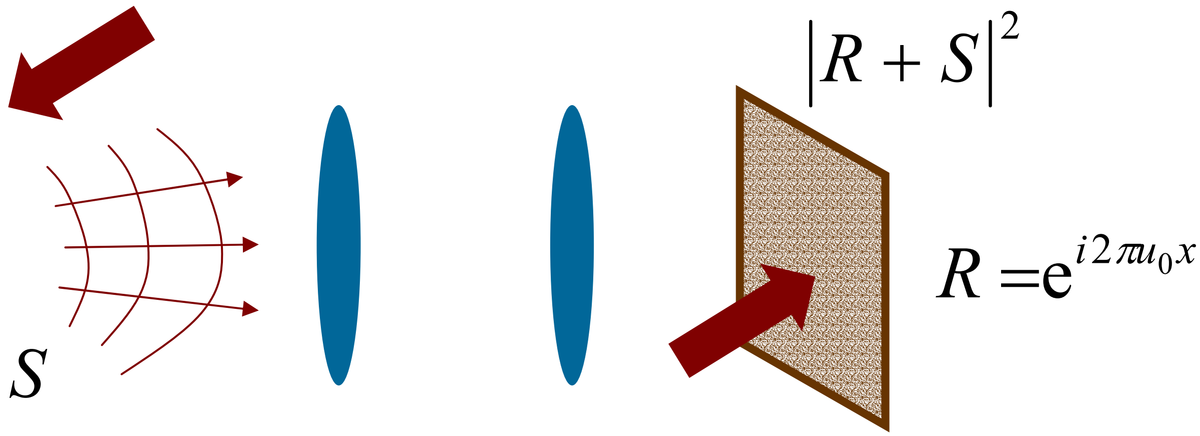

Gabor

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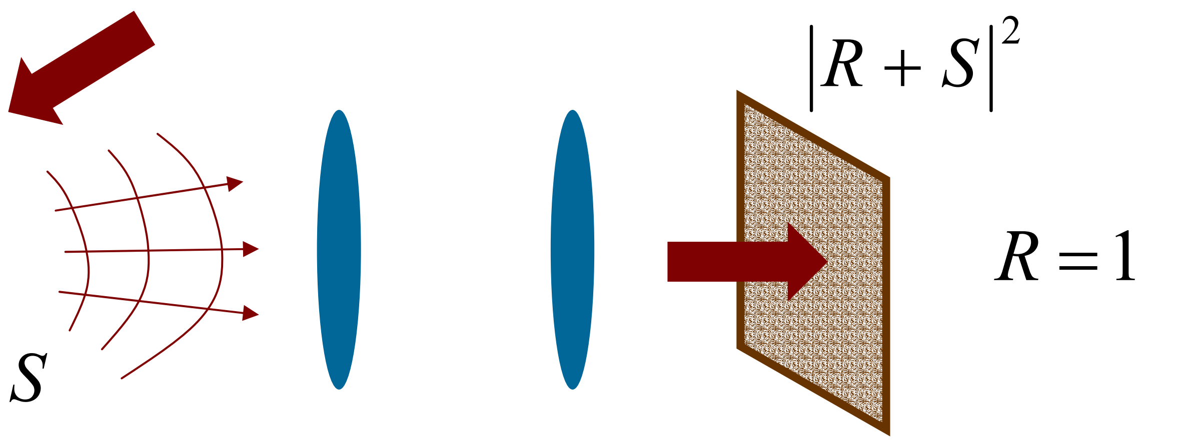
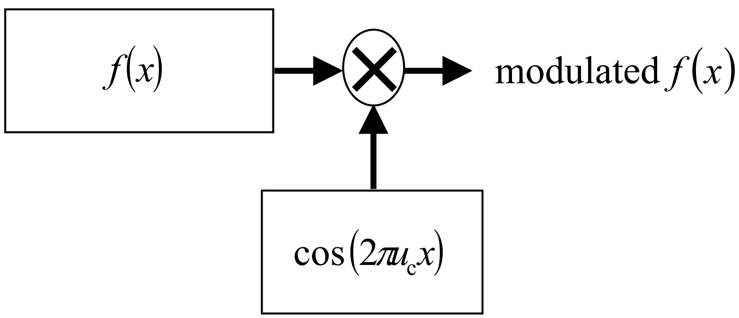


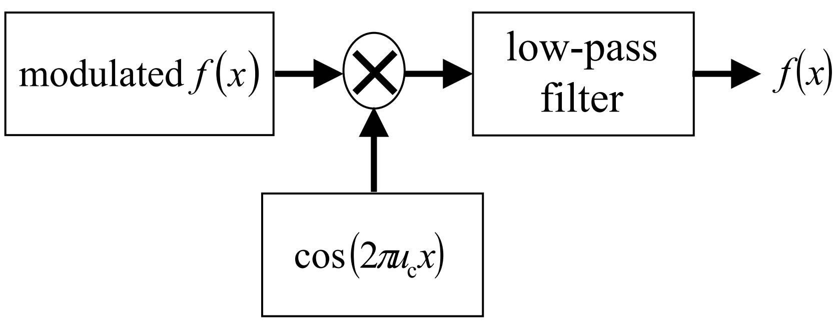
Analogy between the Leith-Upatnieks hologram and amplitude modulation (AM)

AM Radio

Modulation

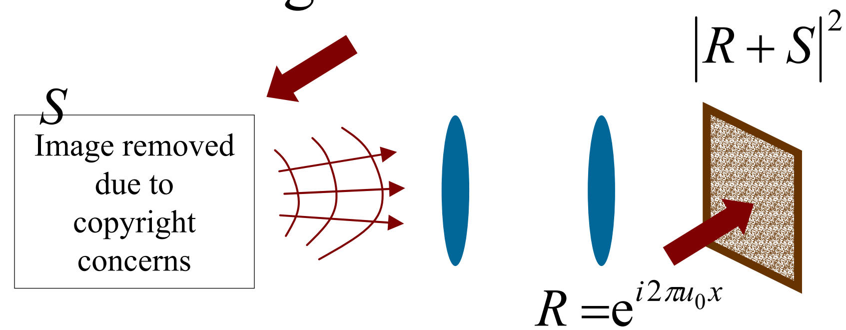


Demodulation



Holography

Recording



Reconstruction

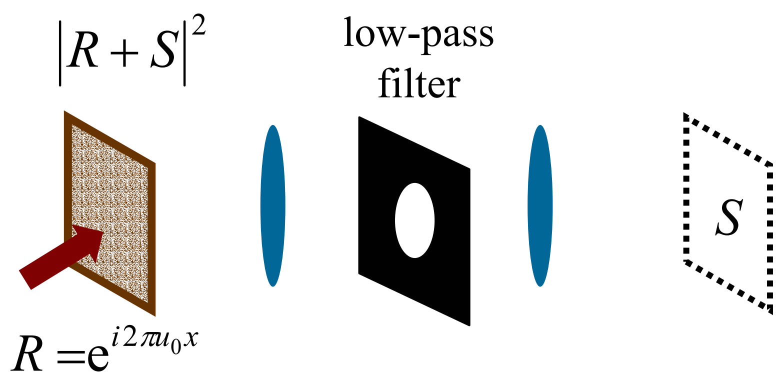


Image locations and magnification

Image removed due to copyright concerns

aaa

Image locations and magnification

$$z_i = \left(\frac{1}{z_p} \pm \frac{\lambda_2}{\lambda_1 z_r} \mp \frac{\lambda_2}{\lambda_1 z_0} \right)^{-1}$$

$$x_i = \mp \frac{\lambda_2 z_i}{\lambda_1 z_0} x_0 \pm \frac{\lambda_2 z_i}{\lambda_1 z_0} x_r + \frac{z_i}{z_p} x_p$$

$$y_i = \mp \frac{\lambda_2 z_i}{\lambda_1 z_0} y_0 \pm \frac{\lambda_2 z_i}{\lambda_1 z_0} y_r + \frac{z_i}{z_p} y_p$$

Transverse Magnification

$$M_t = \left| \frac{\partial x_i}{\partial x_0} \right| = \left| \frac{\partial y_i}{\partial y_0} \right| = \left| \frac{\lambda_2 z_i}{\lambda_1 z_0} \right| = \left| 1 - \frac{z_0}{z_r} \mp \frac{\lambda_1 z_0}{\lambda_2 z_p} \right|^{-1}$$

Axial Magnification

$$M_a = \left| \frac{\partial z_i}{\partial z_0} \right| = \left| \frac{\partial}{\partial z_0} \left(\frac{1}{z_p} \pm \frac{\lambda_2}{\lambda_1 z_r} \mp \frac{\lambda_2}{\lambda_1 z_0} \right)^{-1} \right| = \frac{\lambda_1}{\lambda_2} M_t^2$$

Holography of Three-Dimensional Scenes

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Orthoscopic and Pesudoscopic

Images removed due to copyright concerns

Holography of Three-Dimensional Scenes

Images removed due to copyright concerns

Transmission and Reflection Holograms

Images removed due to copyright concerns

Transmission and Reflection Holograms

Images removed due to copyright concerns

Rainbow hologram (Record)

Images removed due to copyright concerns

Rainbow hologram (Reconstruct)

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