MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.710 Optics **Fall '04**

Problem Set #8 Posted Wednesday, Nov. 03, 2004 — Due Wednesday, Nov. 10, 2004

1. **Paserval's theorem.** Let $f(x)$ denote a square integrable and sufficiently smooth function and $F(u)$ its Fourier transform.

   1.a) Show that
$$\int_{-\infty}^{\infty} |f(x)|^2 \, \mathrm{d}x = \int_{-\infty}^{\infty} |F(u)|^2 \, \mathrm{d}u$$

   1.b) Argue that this result expresses energy conservation in the context of an optical system.

2. **Telescopes and magnification.** A 4-F system (*i.e.* a telescope) is constructed with two lenses L1 and L2 of focal lengths f_1 and f_2, respectively, so that $f_1 > f_2$. Light enters L1 first, and then L2.

   2.a) Is the system operating as a magnifier or demagnifier in the lateral coordinate?

   2.b) Is the system operating as a magnifier or demagnifier in the angular coordinate?

   2.c) Are the two answers above consistent and why? (Discuss as thoroughly as you can)

3. The Fourier transform may be regarded as a mapping of functions into their transforms and therefore satisfies the definition of a system.

   3.a) Is this system linear?

   3.b) Can you specify a transfer function for this system? If yes, what is it? If not, why not?

4. **Spatial filtering.** Figure A (next page) shows a 4-F optical imaging system with 1:1 magnification. Lens L1 and L2 are identical with focal length $f = 10\text{cm}$. Figure B shows the amplitude transmittivity $t(x)$ of the thin transparency which is located at the input plane. The modulation of the transparency is periodic with period $L = 20\mu\text{m}$. The transparency is illuminated by a plane wave, which is generated by a laser at wavelength $\lambda = 0.5\mu\text{m}$, and is incident in a direction parallel to the optical axis. At the Fourier plane of L1 there is another transparency which transmits the optical field within distances $d_i = 1.5\text{mm}$ and $d_o = 8.5\text{mm}$ from the optical axis, and is opaque everywhere else (see also Figure A.) Compute the simplest possible expression for the amplitude distribution $a(x')$

at the image plane of the system. Treat the system as 1D and ignore the effect of the finite lens apertures.

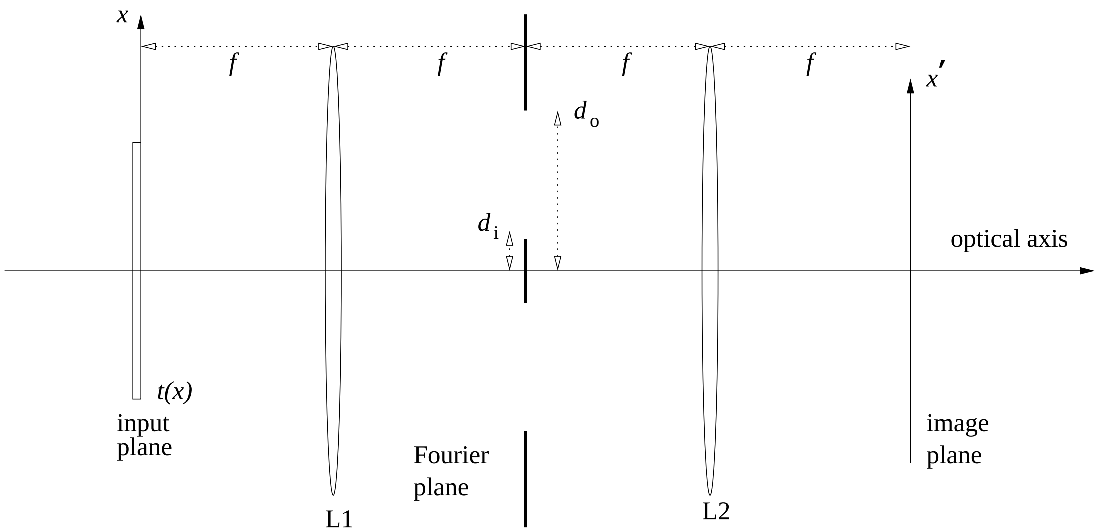

Figure A: 4F imaging system with Fourier plane filter.

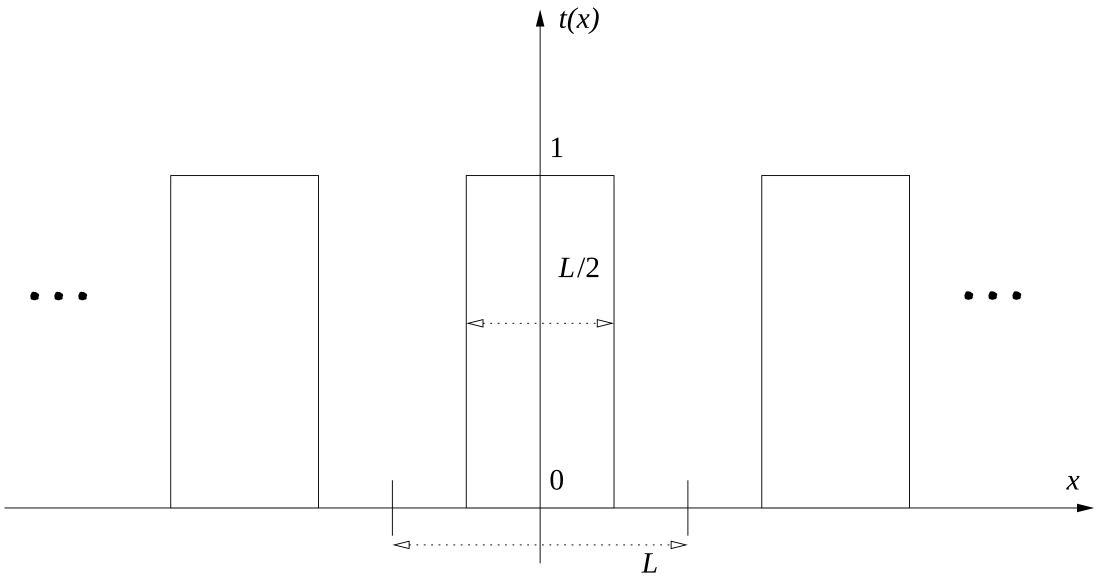

Figure B: Amplitude transmittance of the input transparency.

5. **Image processing.** Download an image from your favorite image-intensive website (e.g., imdb.com), convert it to grayscale by adding the R, G, B color values at each pixel, and crop its central portion $g(x, y)$ so that it have square shape (e.g., 128×128.) Please do not use any images that might be considered offensive.

5.a) Plot your image next to the amplitude $|G(u, v)|$ of its Fourier transform $G(u, v)$ (more details may be visible if instead you plot $\log_1 0|G(u, v)|$.)

5.b) Select a 5×5 square region $\mathcal{S}$ around the origin of the Fourier transform domain and define the new function $G_1(u, v)$ such that

$$G_1(u,v) = \begin{cases} G(u,v) & \text{outside } \mathcal{S} \\ 0 & \text{inside } \mathcal{S} \end{cases}$$

Plot the inverse Fourier transform $g_1(x, y)$ of $G_1(u, v)$.

5.c) Define the new function $G_2(u, v)$ such that

$$G_2(u,v) = \begin{cases} G(u,v) & \text{inside } \mathcal{S} \\ 0 & \text{outside } \mathcal{S} \end{cases}$$

Plot the inverse Fourier transform $g_2(x, y)$ of $G_2(u, v)$.

5.d) Comment on the appearances of $g_1(x, y)$, $g_2(x, y)$ and how these appearances are affected by the size of the region $\mathcal{S}$.

If you use MATLAB to solve this problem, you will find the following functions useful: (i) `fft2` computes the 2D Fourier transform of an image and returns it with some quadrants swapped, (ii) `fftshift` rearranges the quadrants of the Fourier transform in their proper order, (iii) `ifft2` computes the inverse 2D Fourier transform (iv) `imagesc; colormap gray` displays a real grayscale image, (v) `print -dps [filename]` prints a figure into a postscript file which you can then print at any server printer using the `lpr` command.