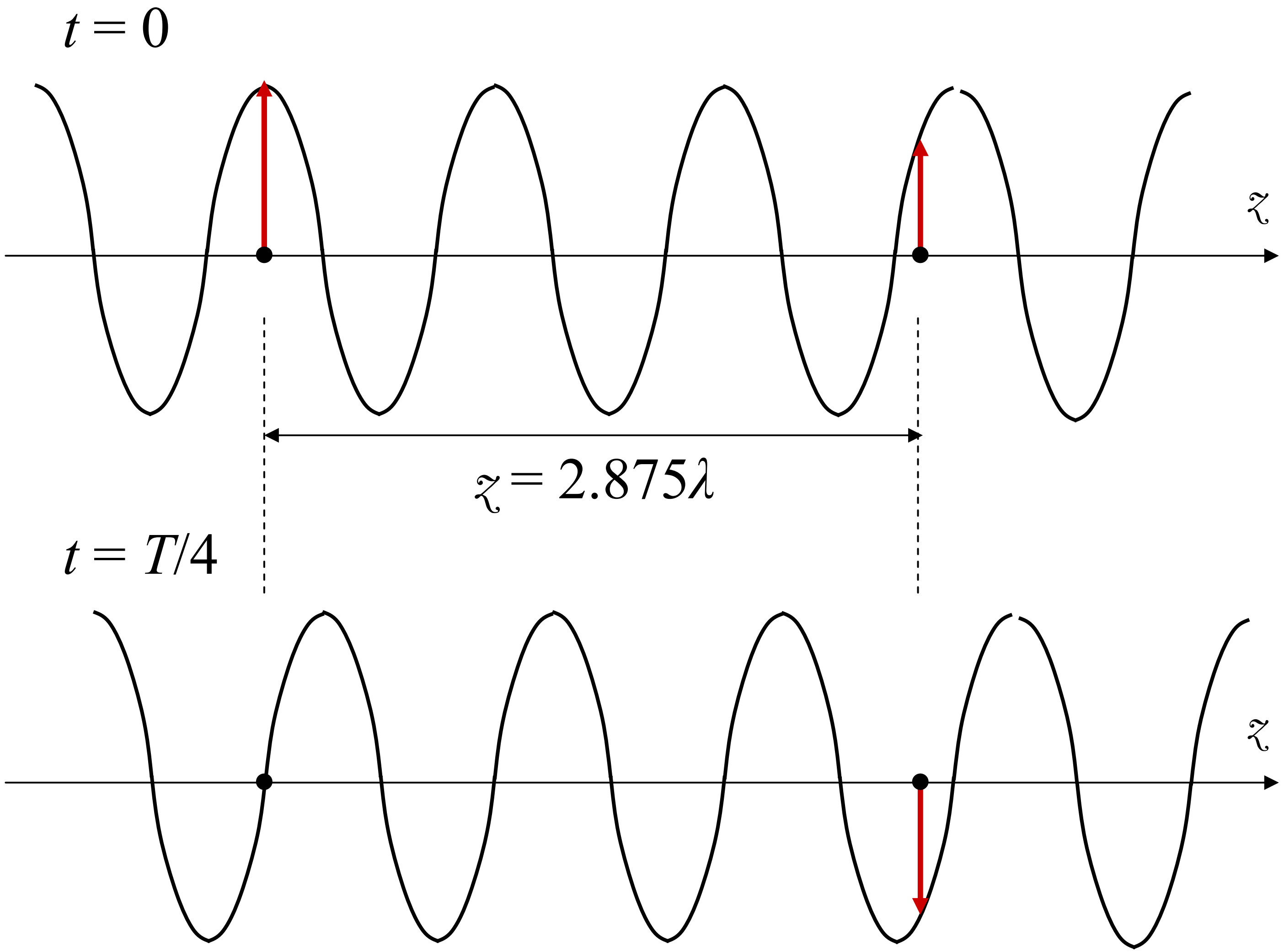
Today's summary

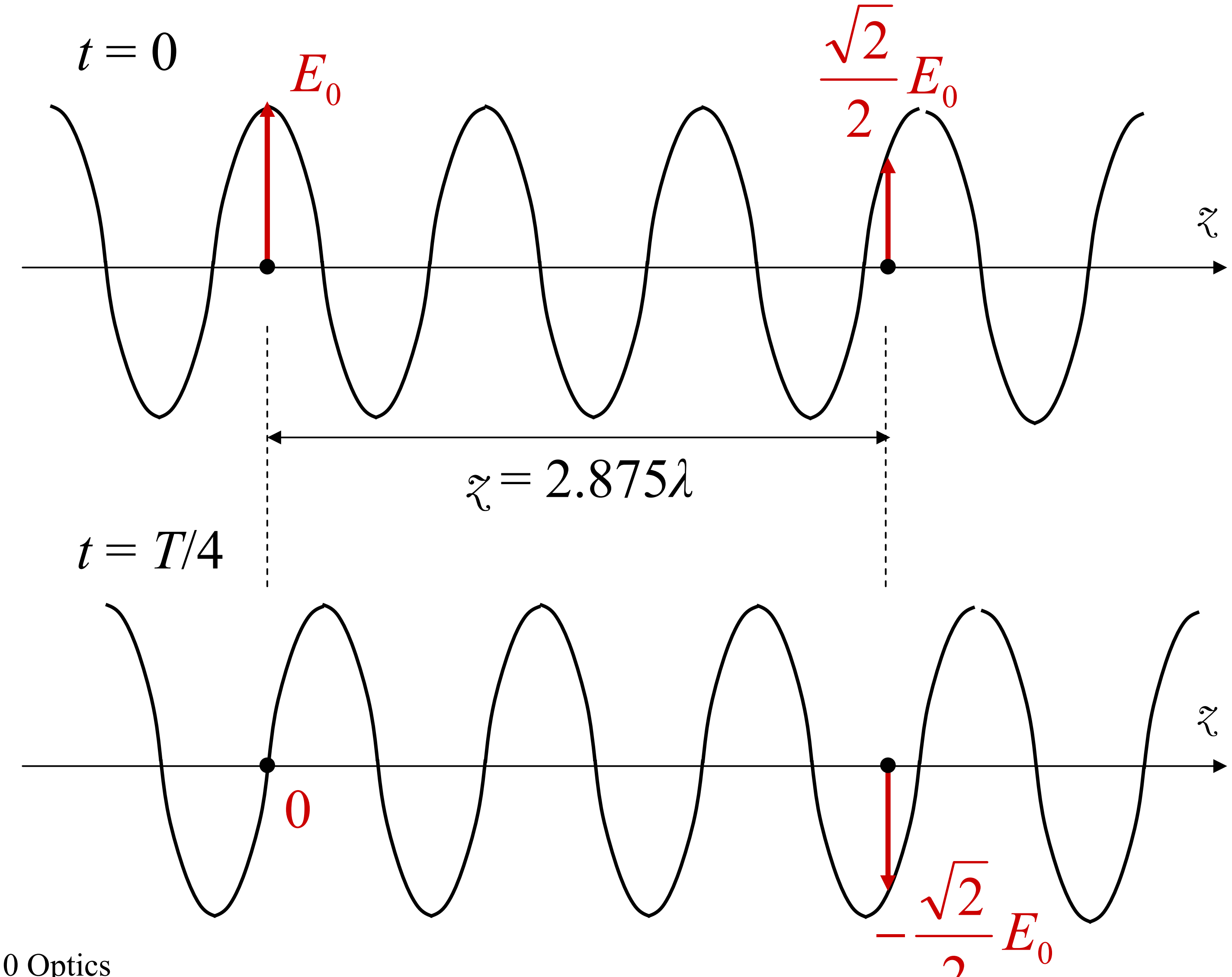
- A new look at propagation and phase delays
- Description of plane & spherical waves in terms of phase delay

- Interference
- Interferometers
  - Michelson
  - Mach-Zehnder
  - Young

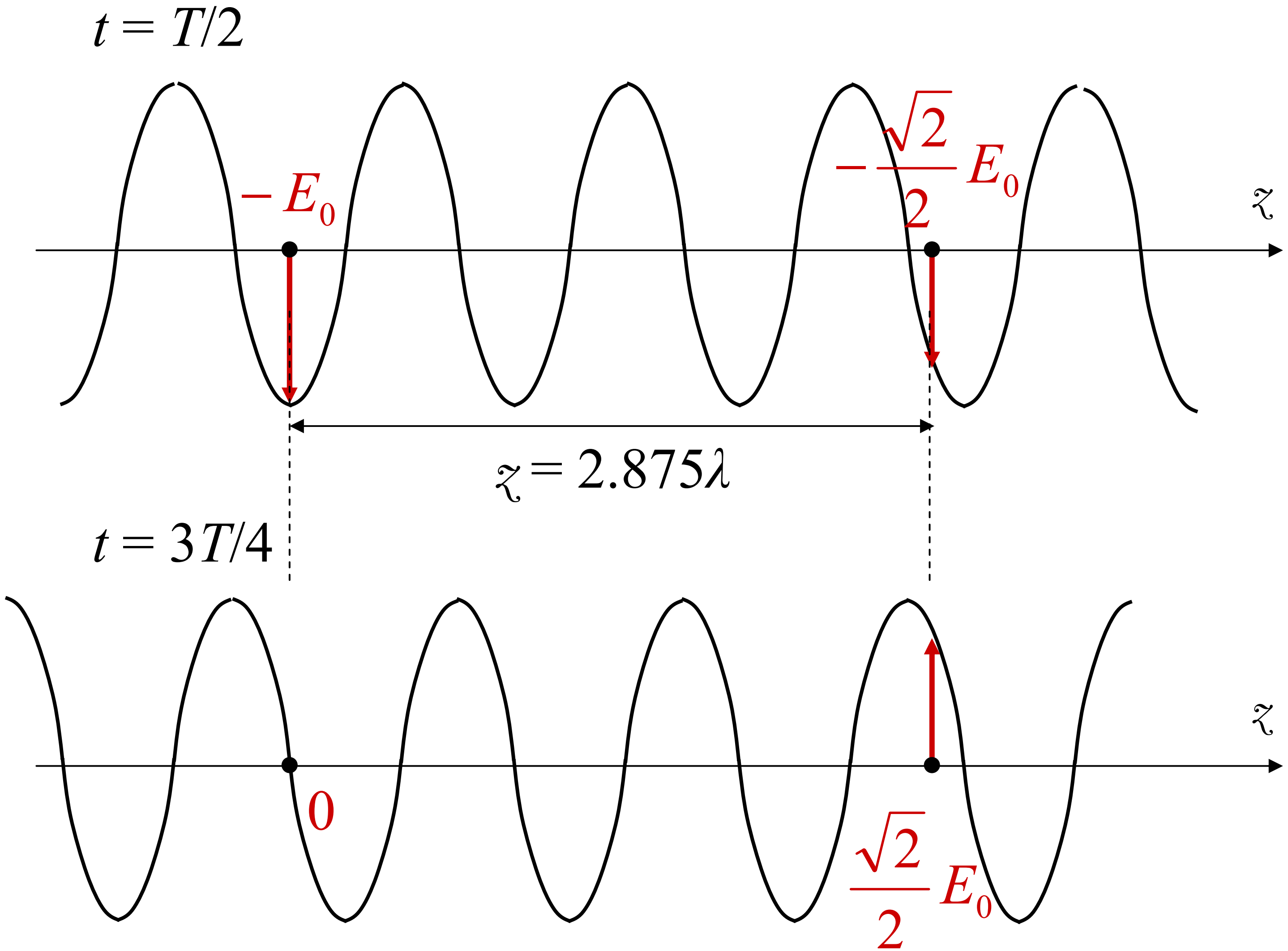
Optical path delay



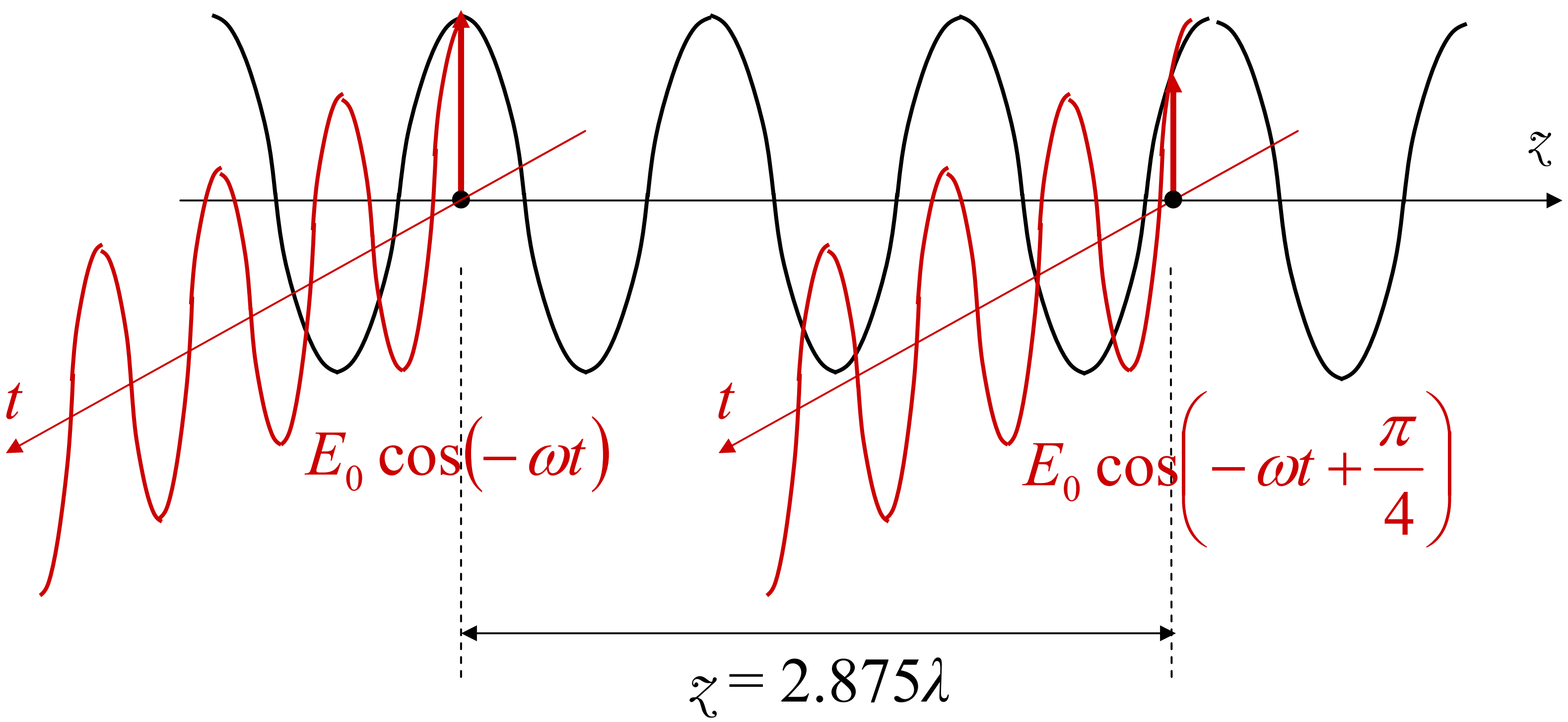
Optical path delay



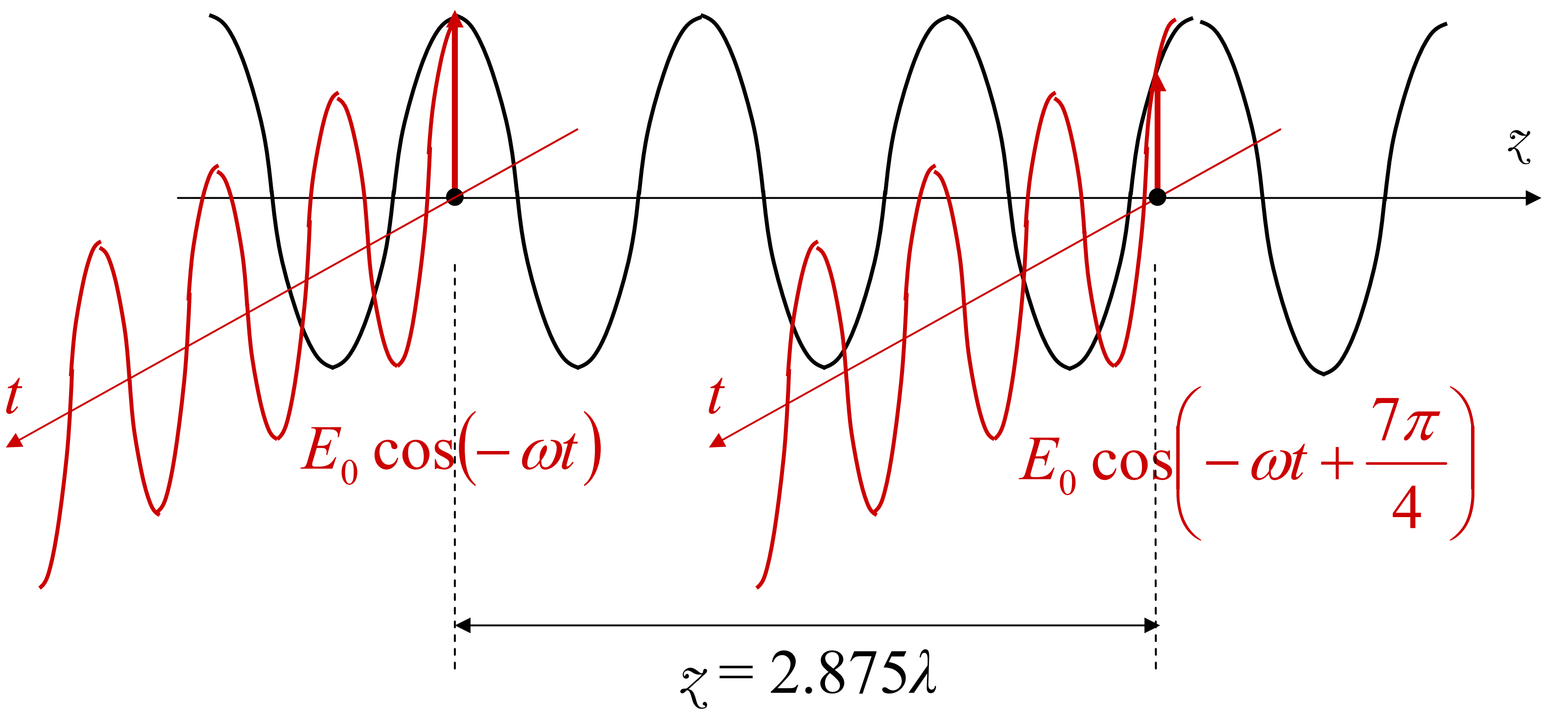
Optical path delay



Optical path delay

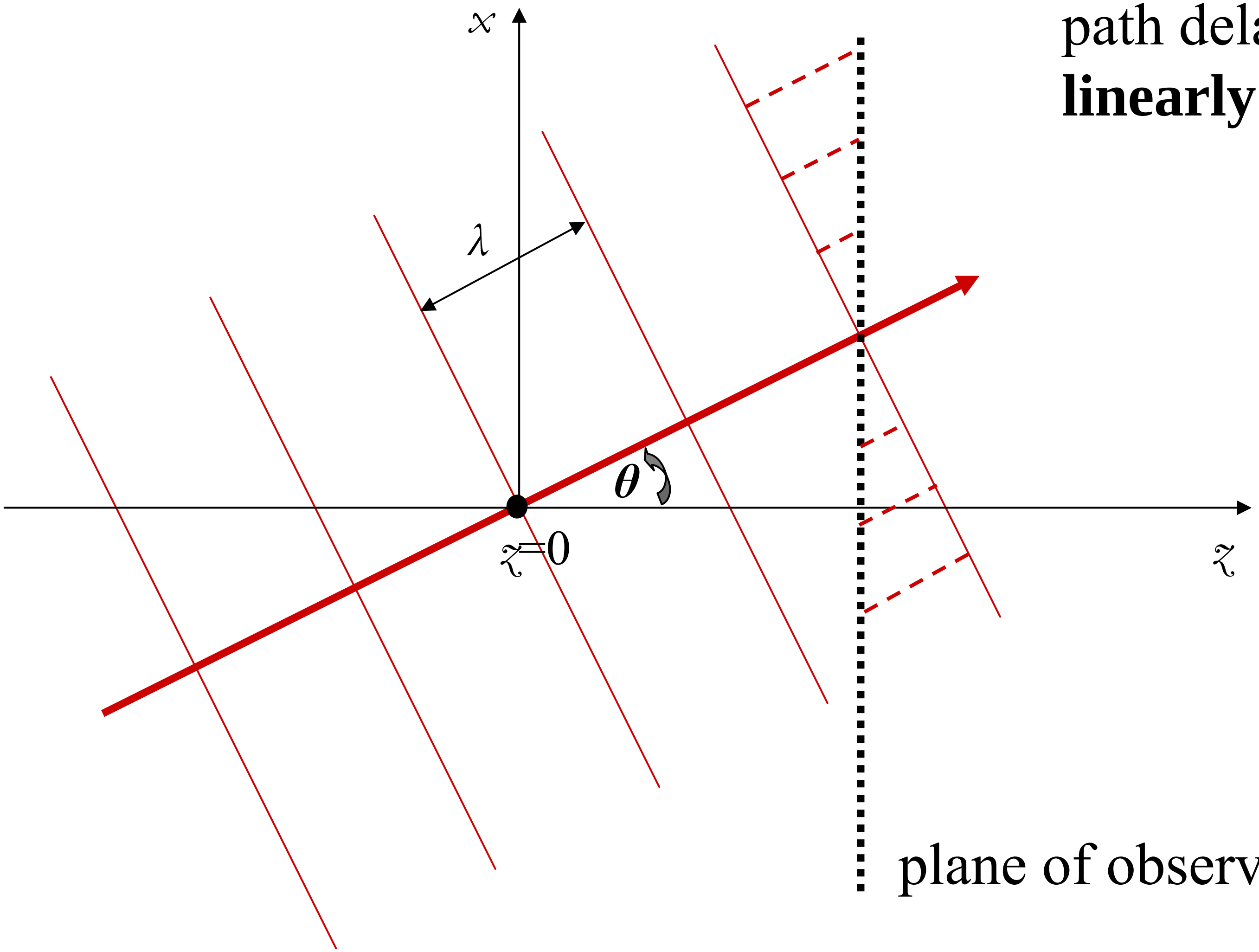


Optical path delay



In general, $E_0 \cos(kz - \omega t) \Rightarrow E_0 \exp\left(i 2\pi \frac{z}{\lambda}\right)$ phasor due to propagation (path delay)

Plane wave propagation



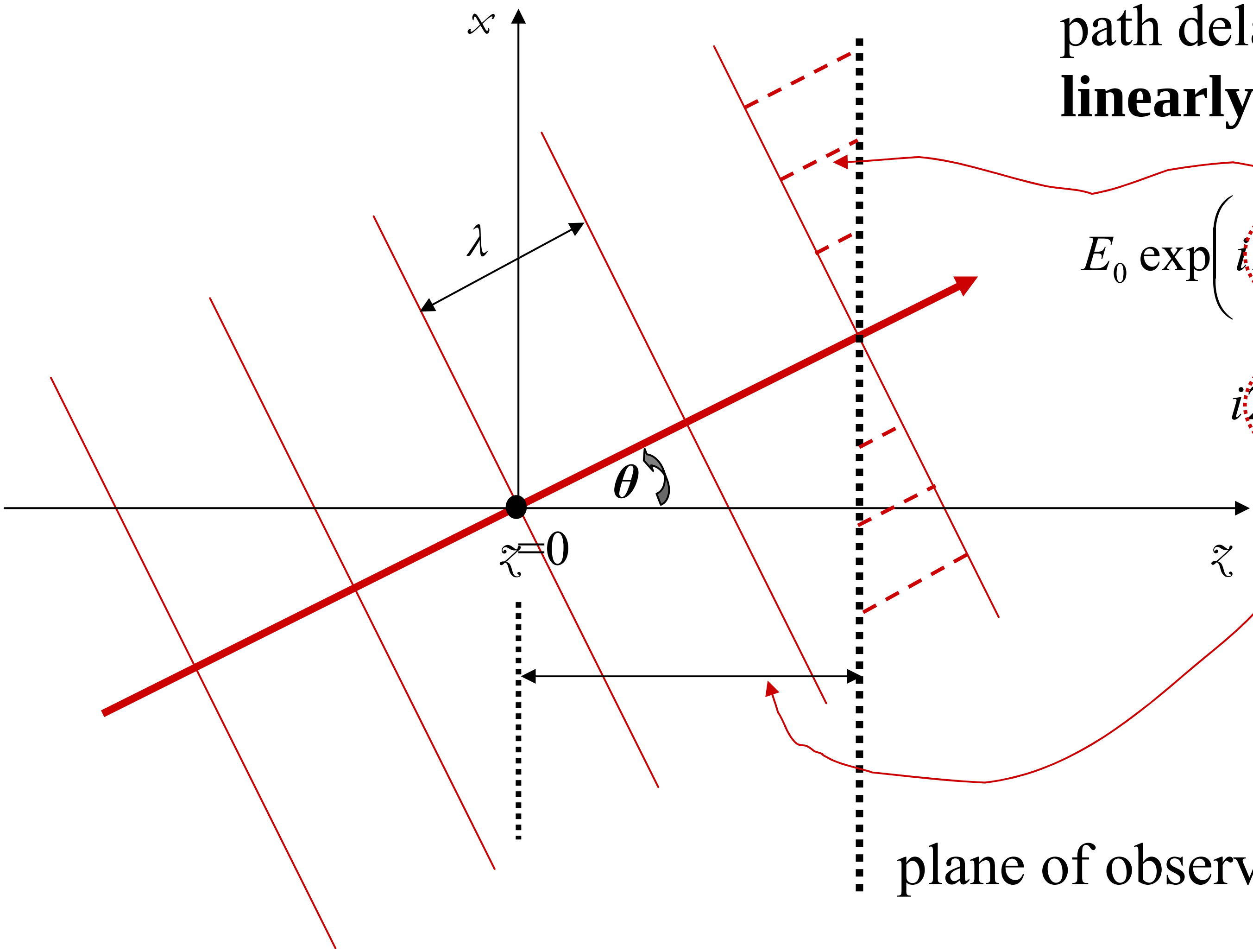
path delay increases **linearly** with x

plane of observation

Plane wave propagation

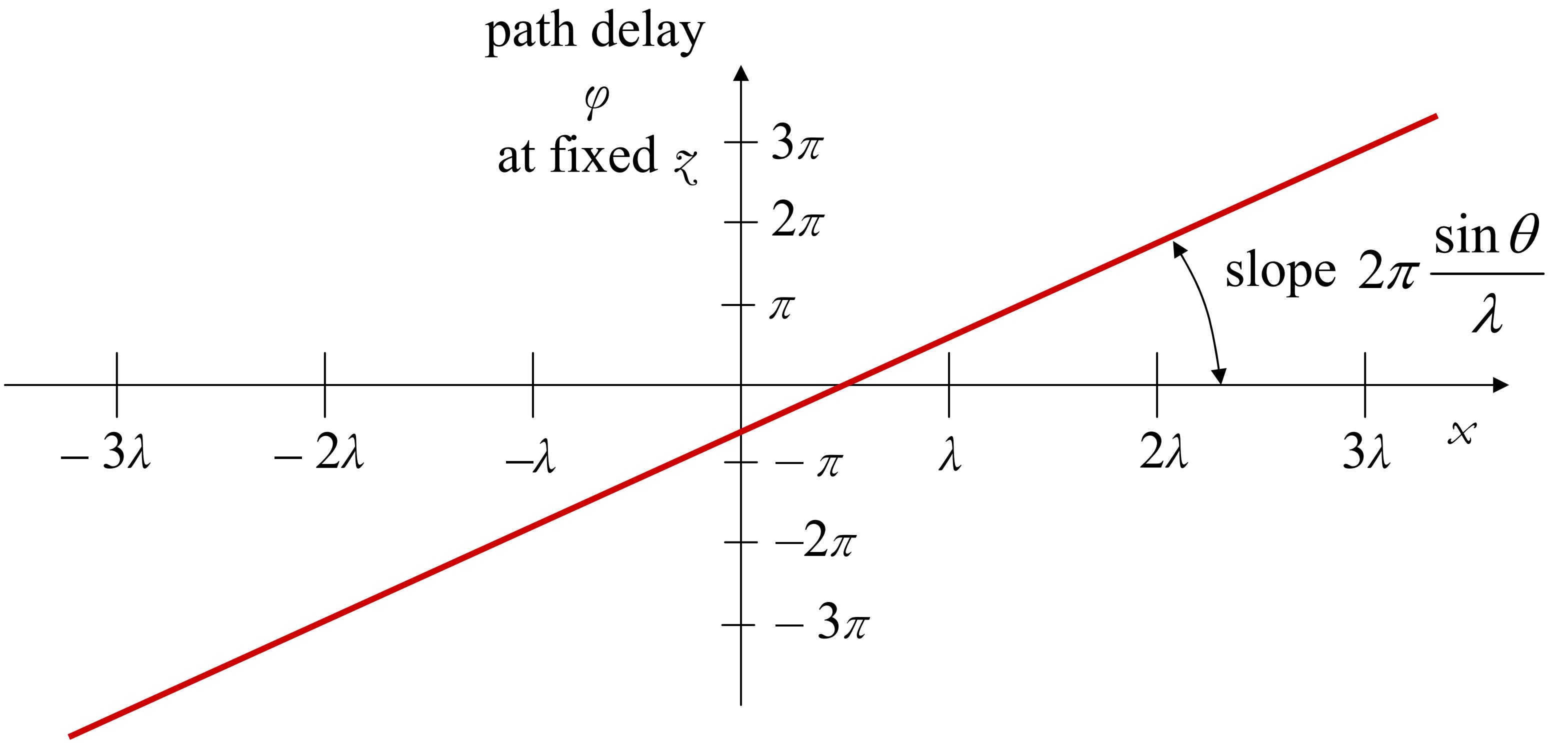
path delay increases **linearly** with x

$$E_0 \exp\left(i2\pi \frac{x}{\lambda}\sin\theta + i2\pi \frac{z}{\lambda}\cos\theta \right)$$



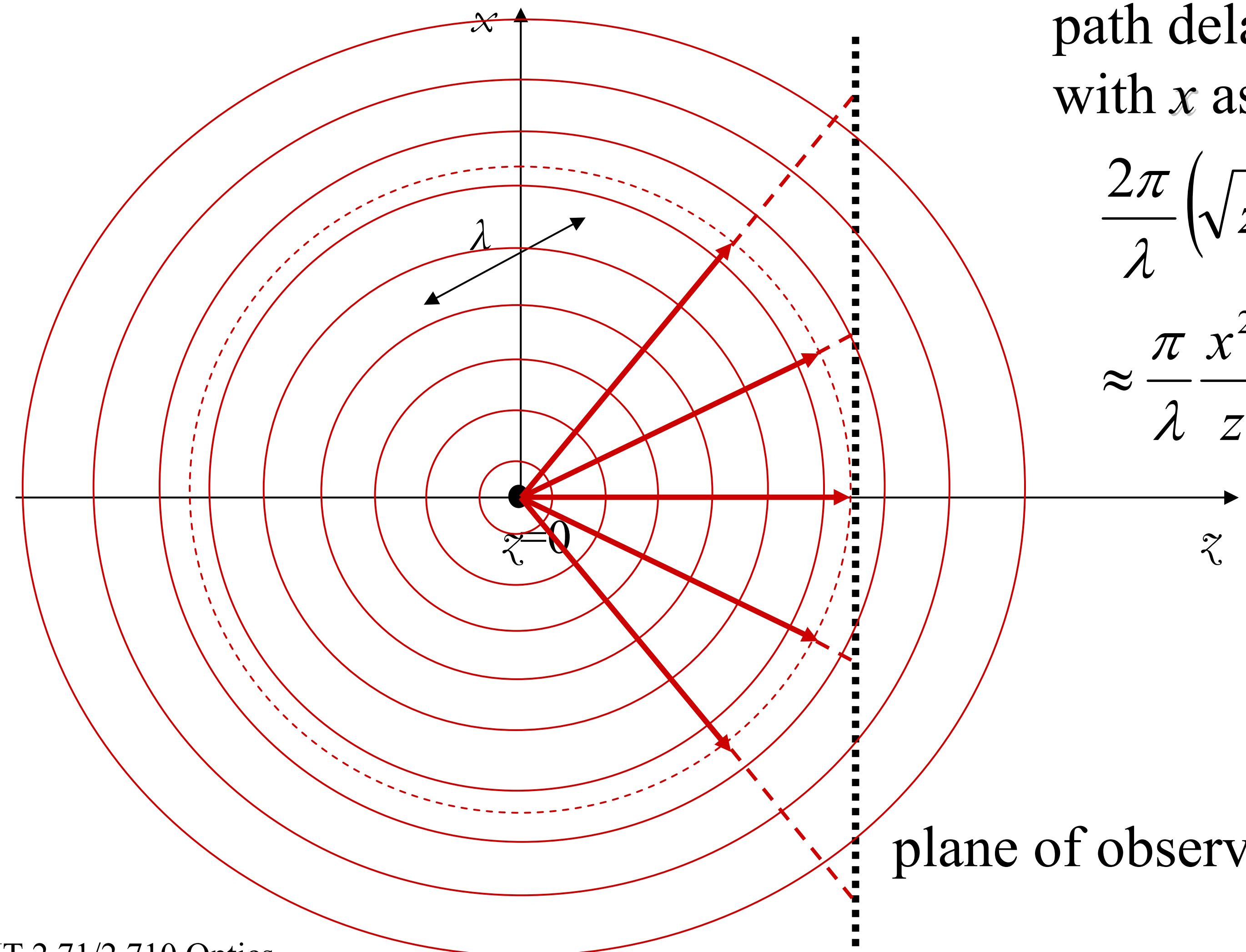
plane of observation

Plane wave propagation



$$E_0 \exp(i\varphi) \quad \text{where} \quad \varphi = 2\pi \frac{x}{\lambda} \sin\theta + 2\pi \frac{z}{\lambda} \cos\theta$$

Spherical wave propagation



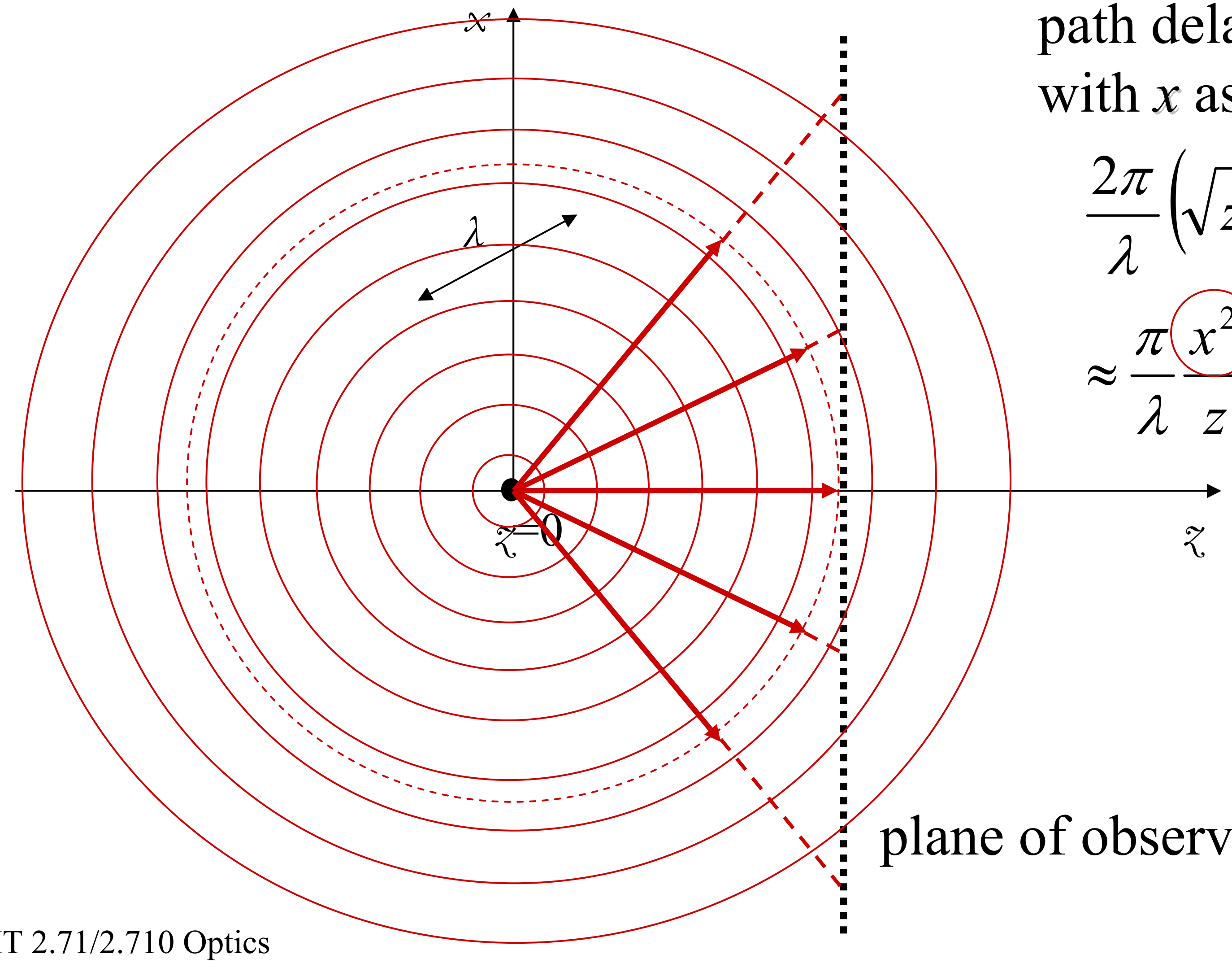
path delay increases with x as

$$\frac{2\pi}{\lambda}\left(\sqrt{z^2+x^2}-z\right)$$

$$\approx \frac{\pi}{\lambda}\frac{x^2}{z} \quad \text{for } x << z$$

plane of observation

Spherical wave propagation



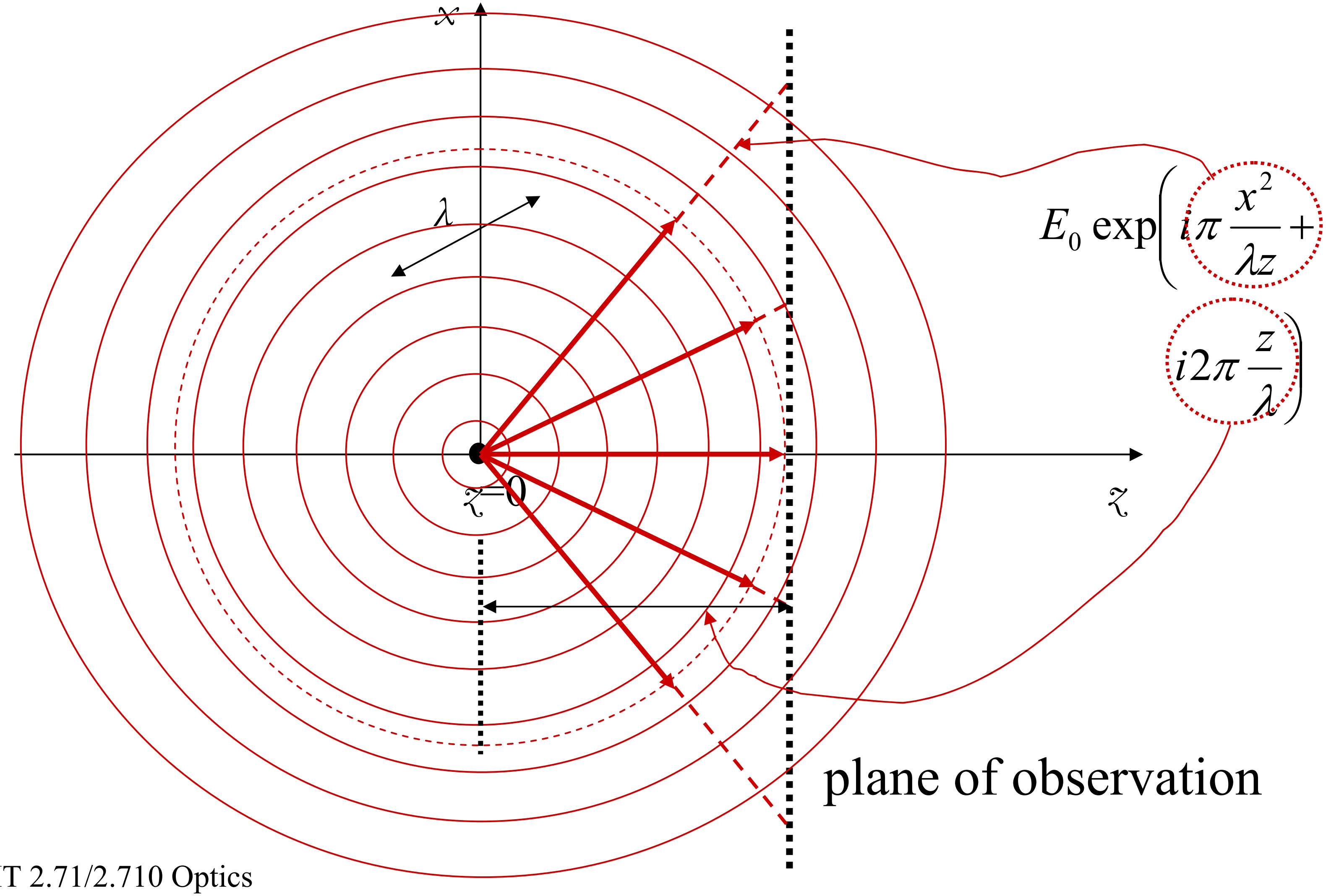
path delay increases with x as

$$\frac{2\pi}{\lambda}\left(\sqrt{z^2+x^2}-z\right)$$

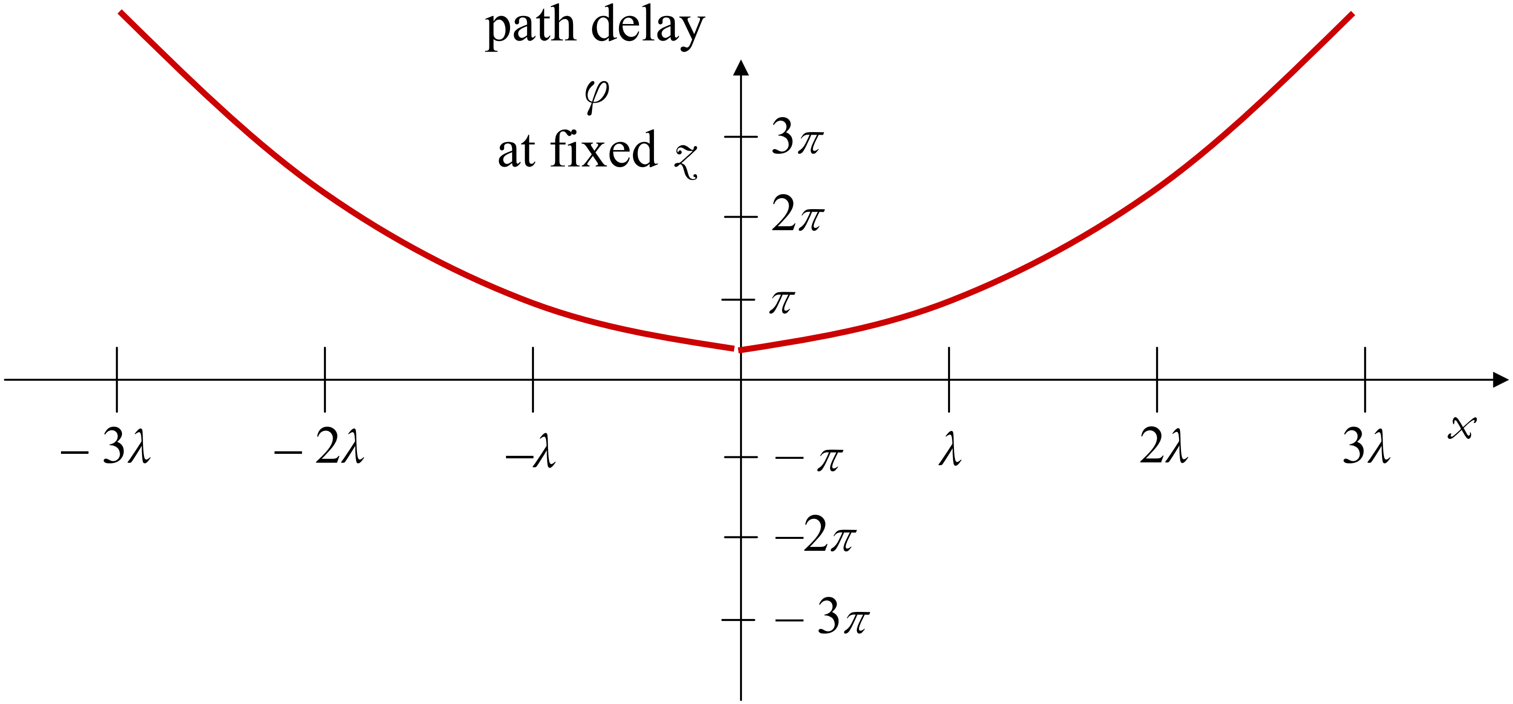
$$\approx \frac{\pi}{\lambda}\frac{x^2}{z}$$

quadratic near the z axis

Spherical wave propagation

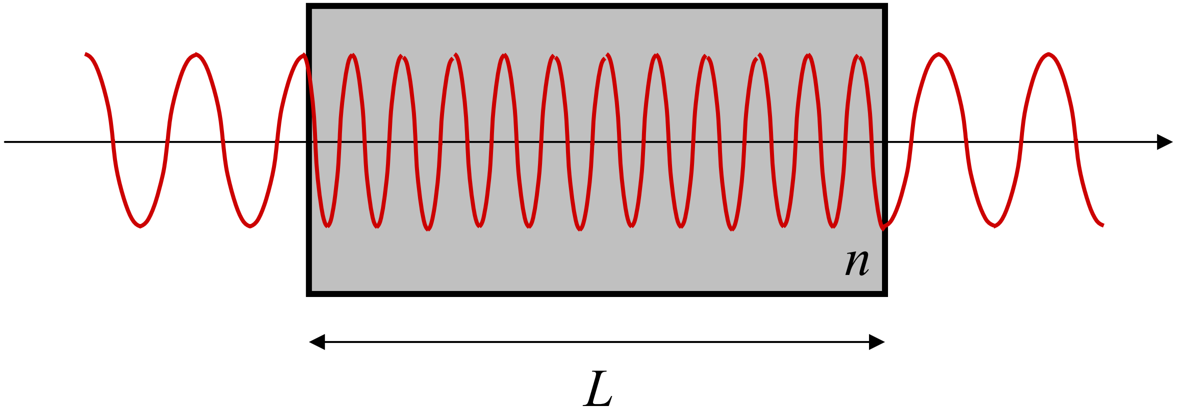


Spherical wave propagation



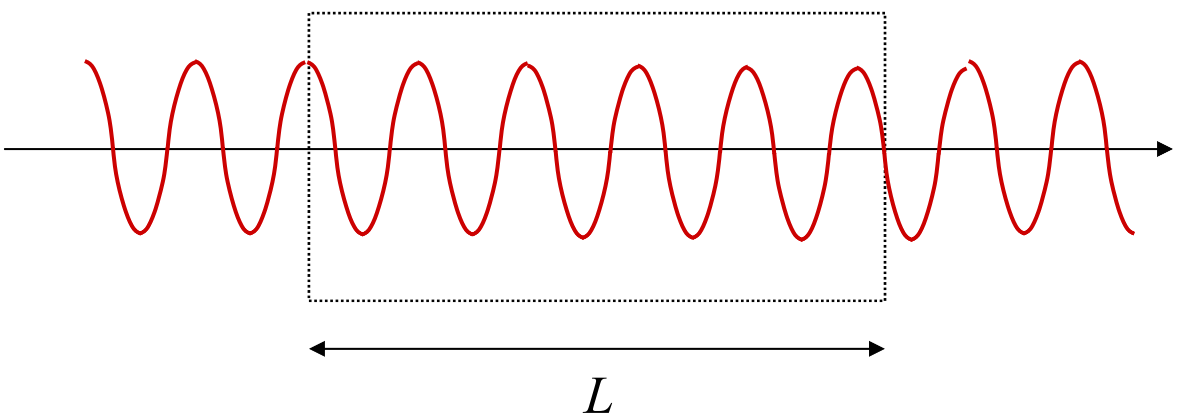
$$E_0 \exp(i\varphi) \quad \text{where} \quad \varphi = 2\pi\frac{z}{\lambda} + \pi\frac{x^2 + y^2}{\lambda z}$$

Optical path delays matter



path delay in material of index n :

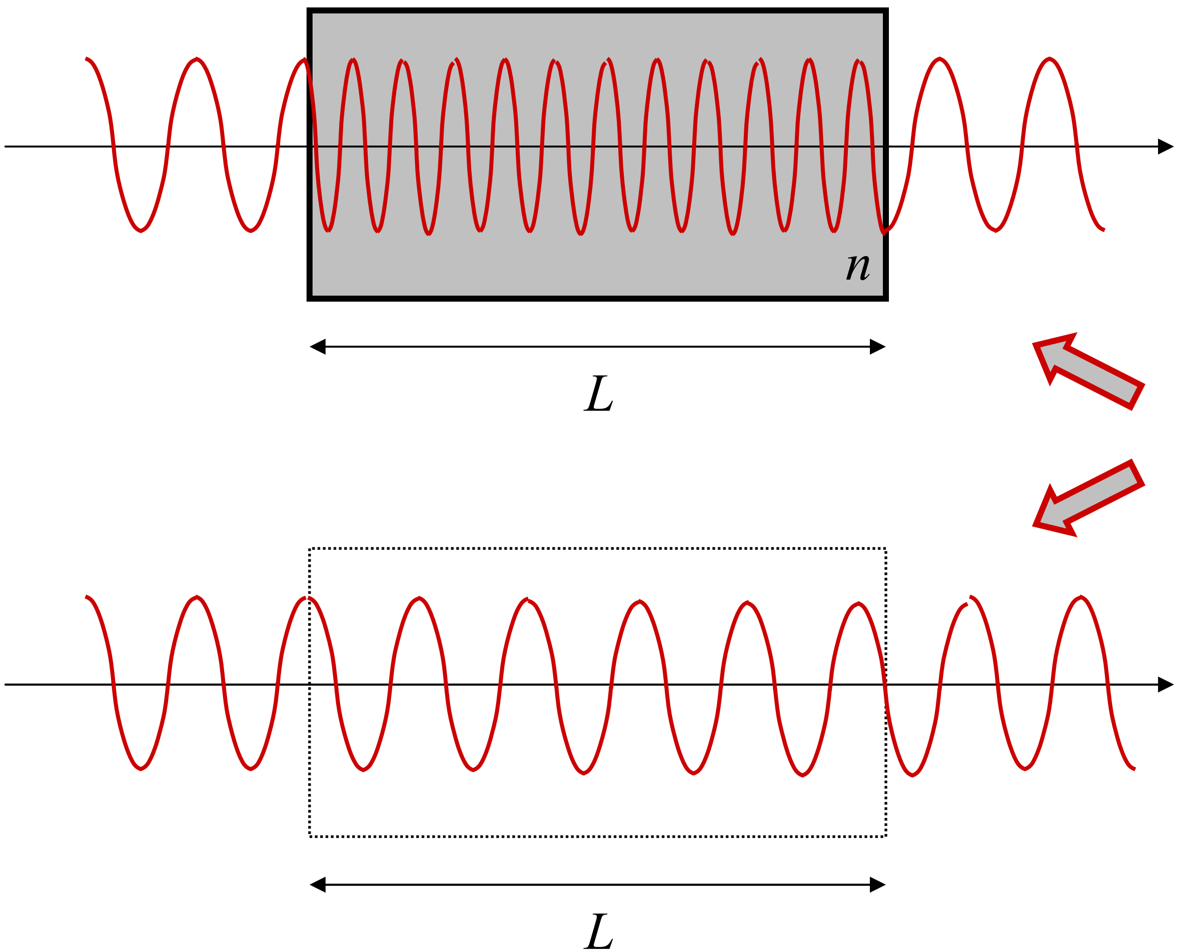
$$2\pi \frac{nL}{\lambda}$$



compare with free space propagation :

$$2\pi \frac{L}{\lambda}$$

Optical path delays matter



path
difference:

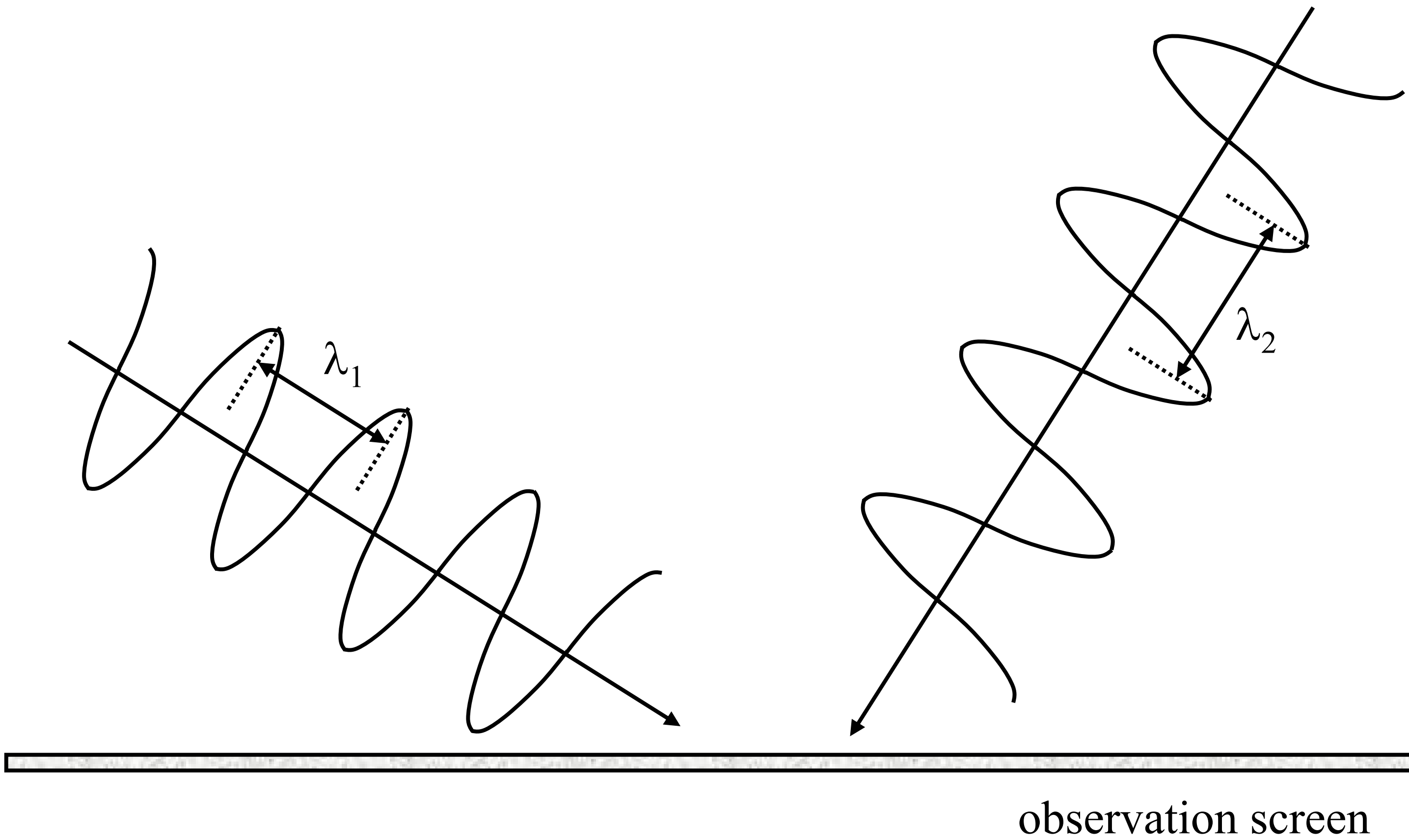
$$2\pi \frac{(n-1)L}{\lambda}$$

Phase delays matter
Can we measure them and how?

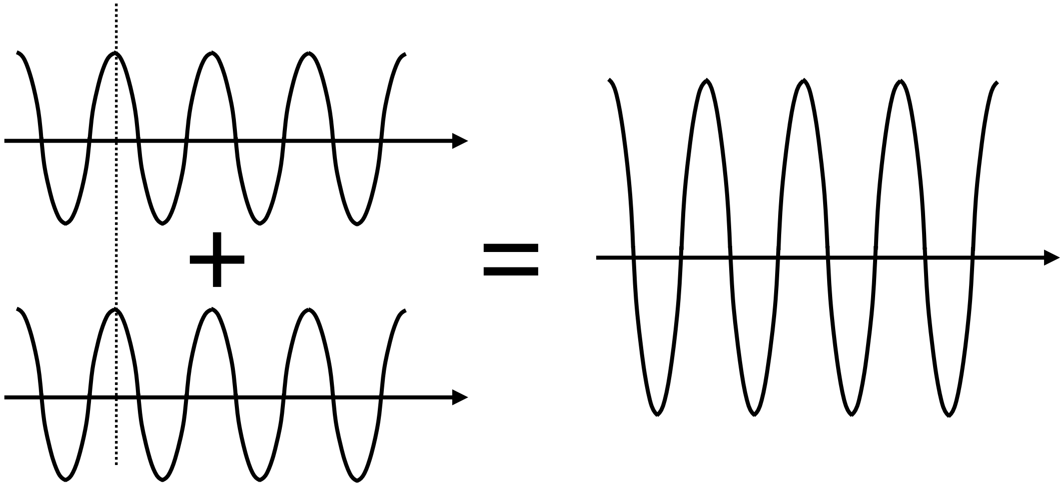
- Direct measurement does not work: light waves oscillate too fast for any instrument to follow
- We need an indirect method
- <u>Solution</u>: ***interferometers*** "map" phase onto light intensity which can be measured directly

Interference

Wave interference

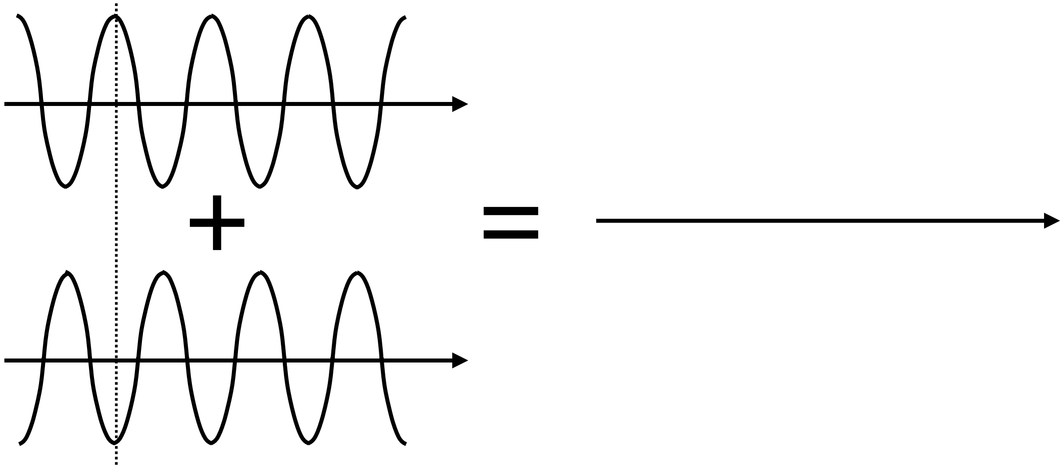


Interference: extreme cases



Waves **in-phase**

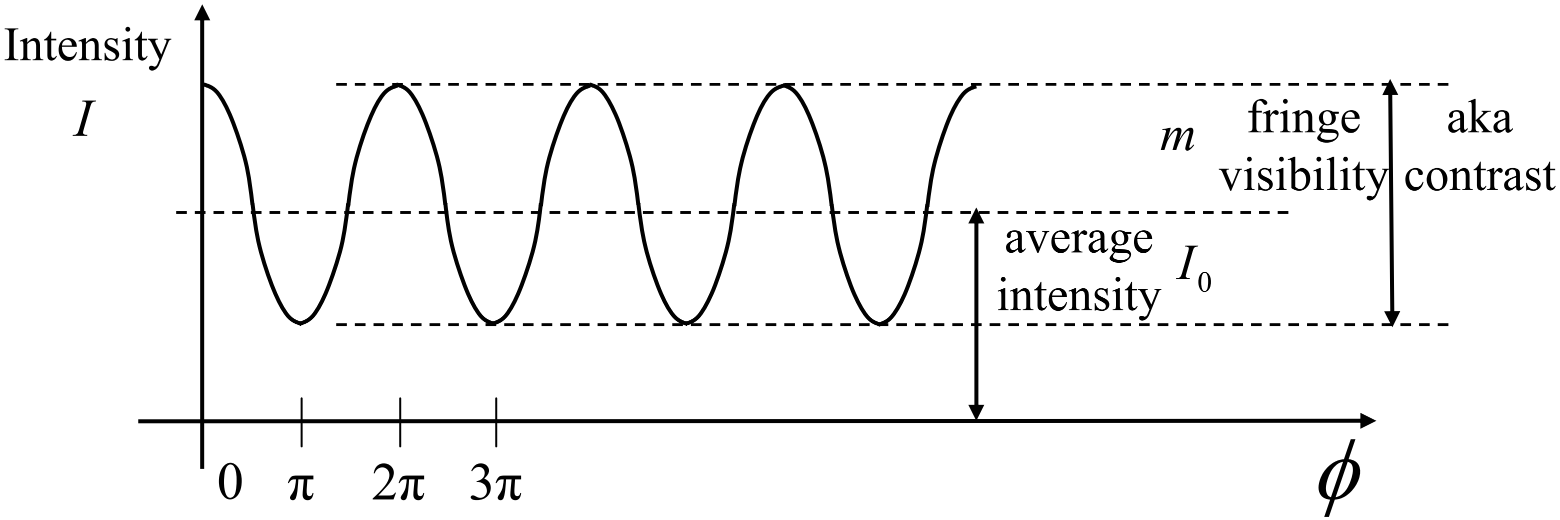
Constructive interference



Waves **out-of-phase**

Destructive interference

Interference *vs* phase delay & contrast



$$\text{Field } 1 = a_1 \cos(\omega t)$$
$$\text{Field } 2 = a_2 \cos(\omega t + \phi)$$

relative phase delay

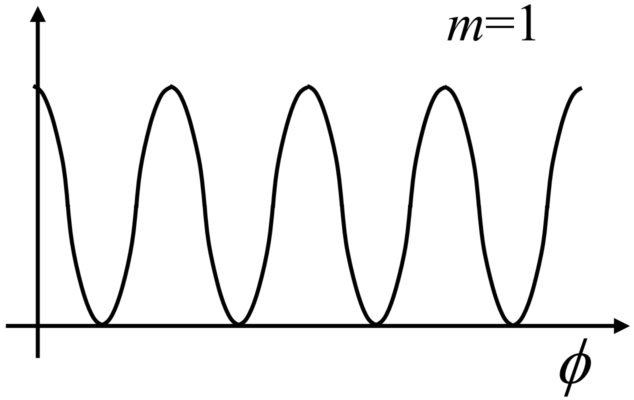
$$I(\phi) = I_0 (1 + m \cos\phi)$$

$$I_0 = a_1^2 + a_2^2 \qquad m = \frac{2a_1 a_2}{a_1^2 + a_2^2}$$

Interference *vs* phase delay & contrast

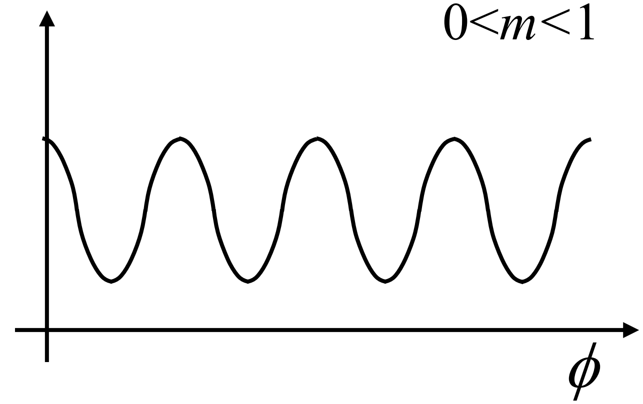
$a_1 = a_2$
perfect contrast
$m=1$

Intensity



$a_1 \neq a_2$
imperfect contrast
$0<m<1$

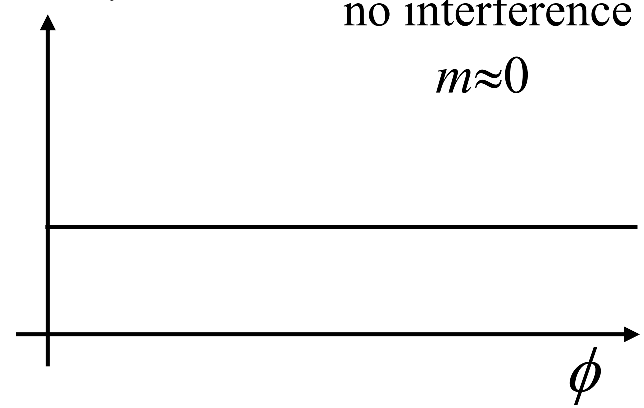
Intensity



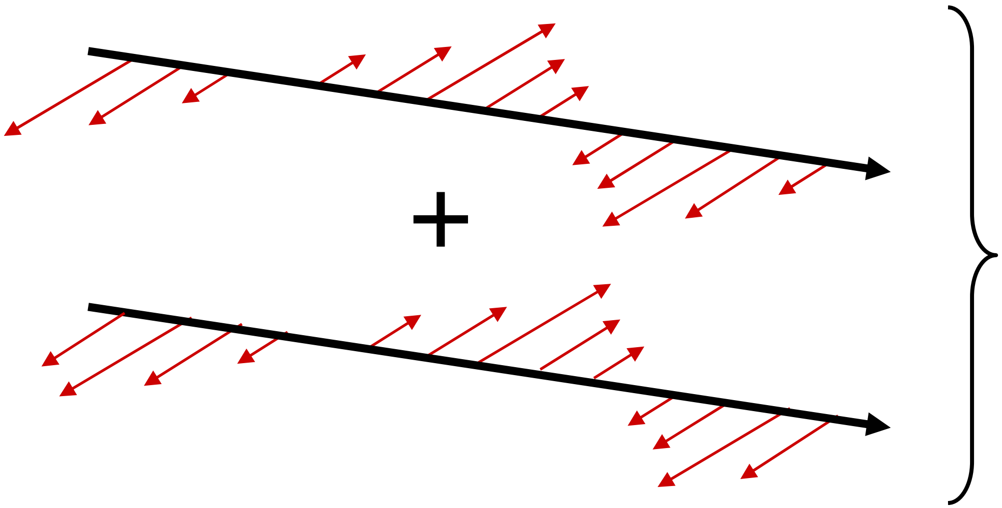
Highest contrast / fringe visibility is obtained by interfering beams of **equal** amplitudes

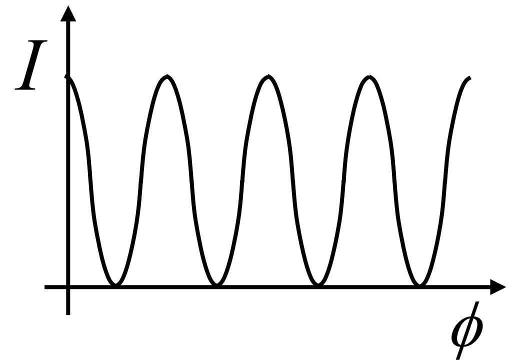
$a_1 << a_2$
no interference
$m \approx 0$

Intensity

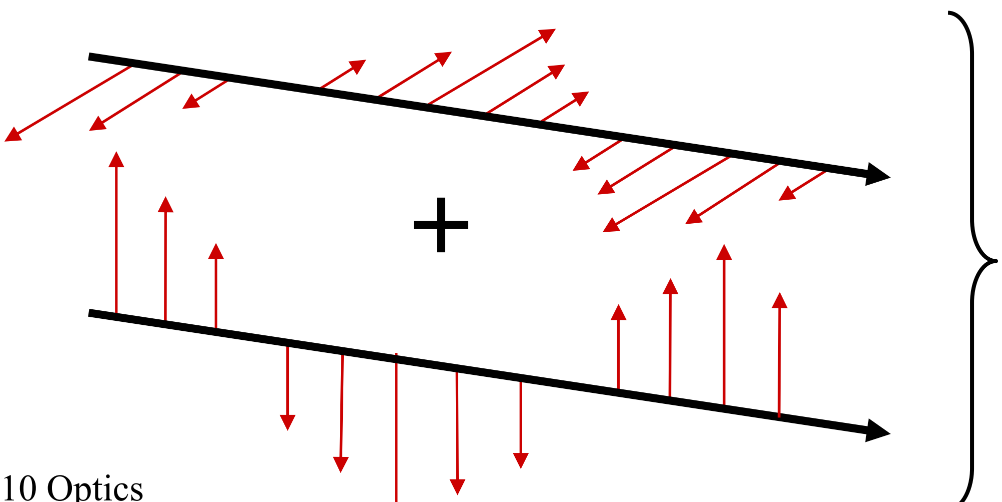


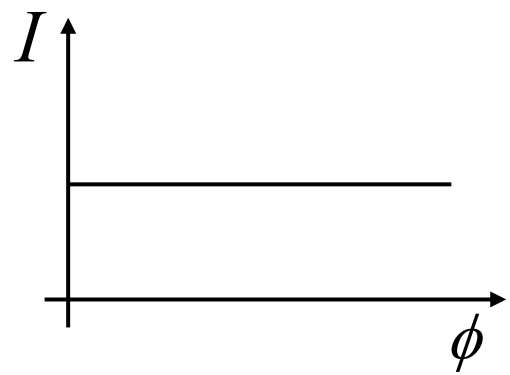
Polarization and interference





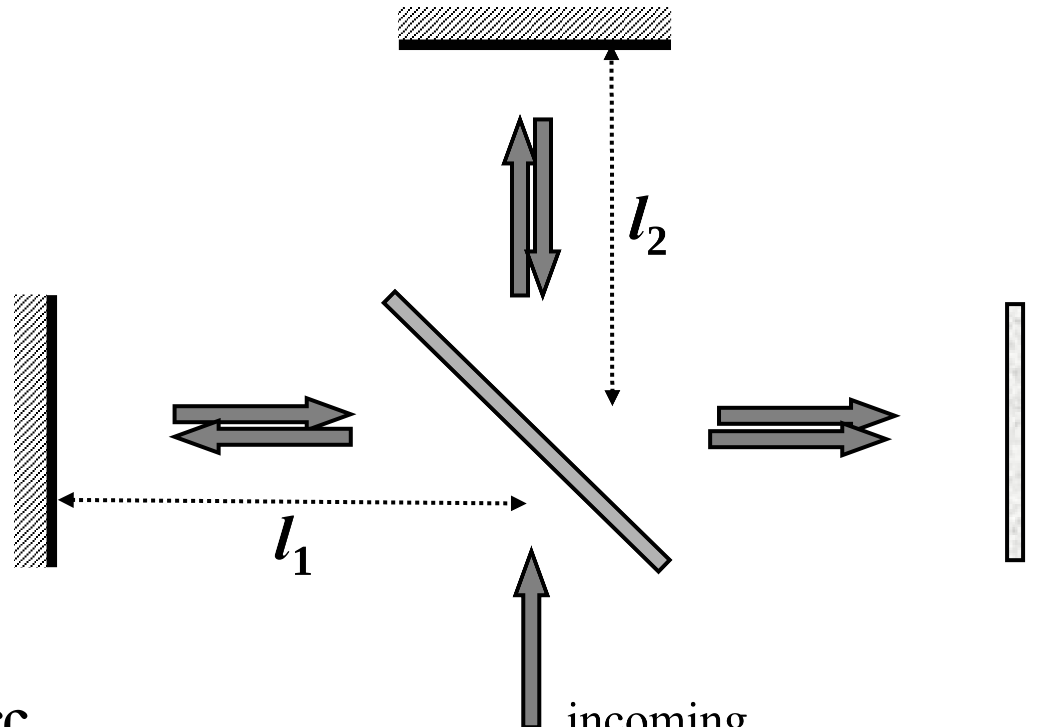
||-polarized waves
interfere





$\perp$-polarized waves
do not interfere

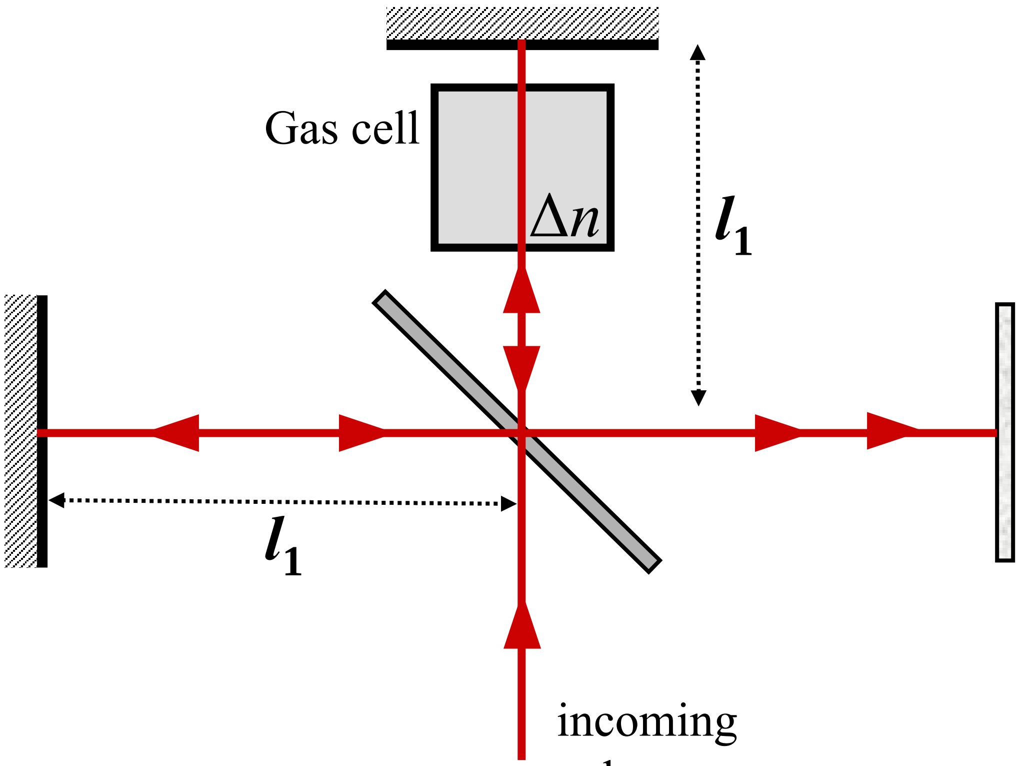
Michelson interferometer



path difference:

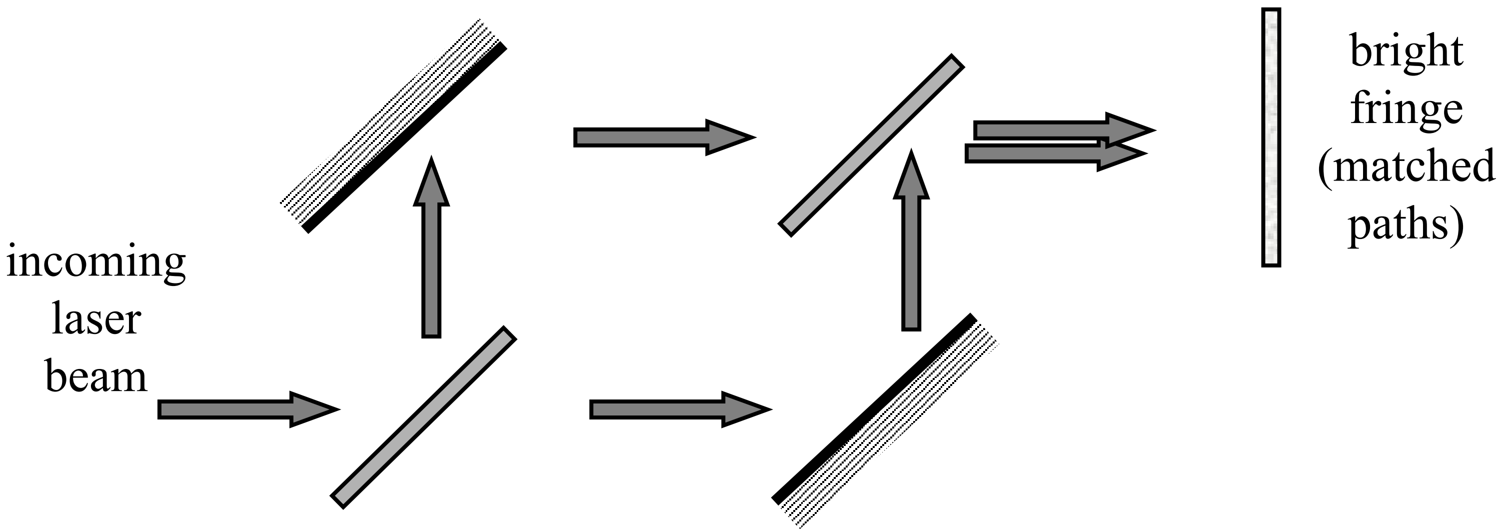
$$\phi = \frac{2\pi}{\lambda} 2\left(l_1 - l_2\right)$$

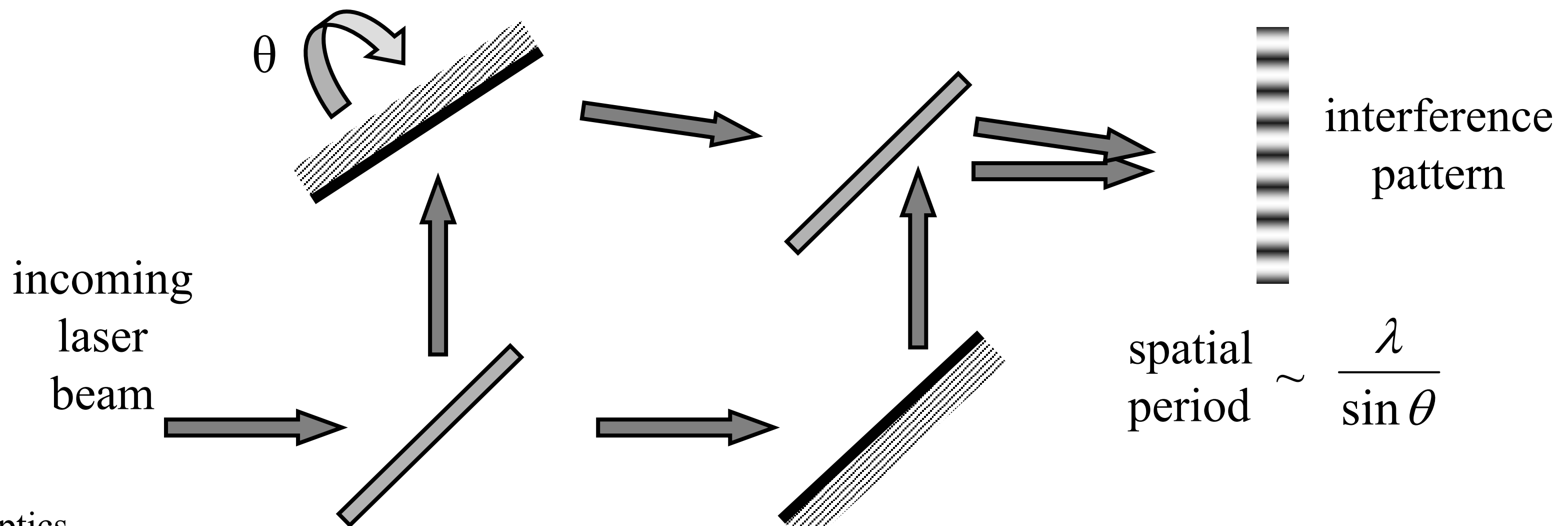
Michelson with variable phase-delay



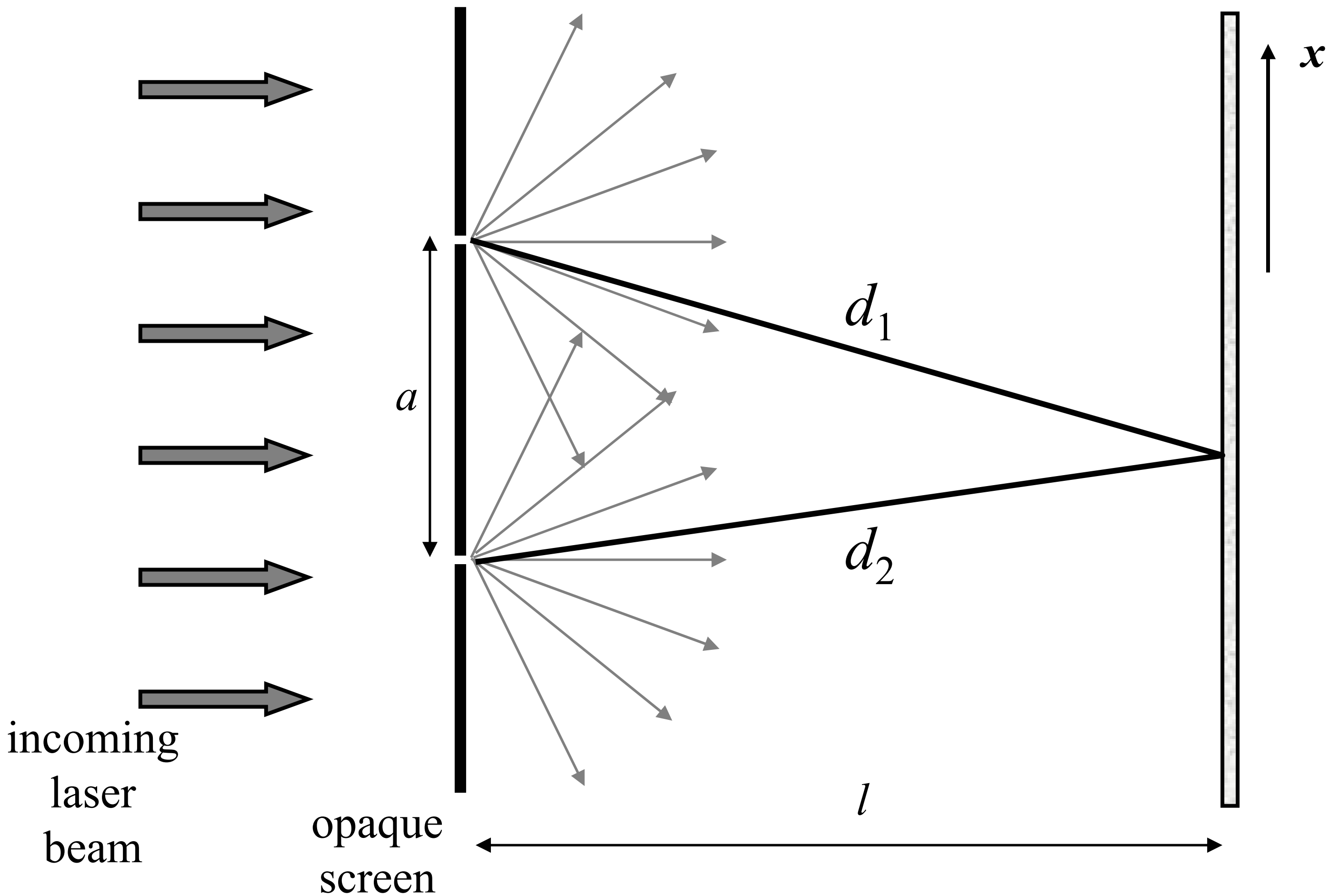
$$\phi = \frac{2\pi}{\lambda} 2\left(\Delta n - 1\right) t_{\text{cell}}$$

Mach-Zehnder interferometer

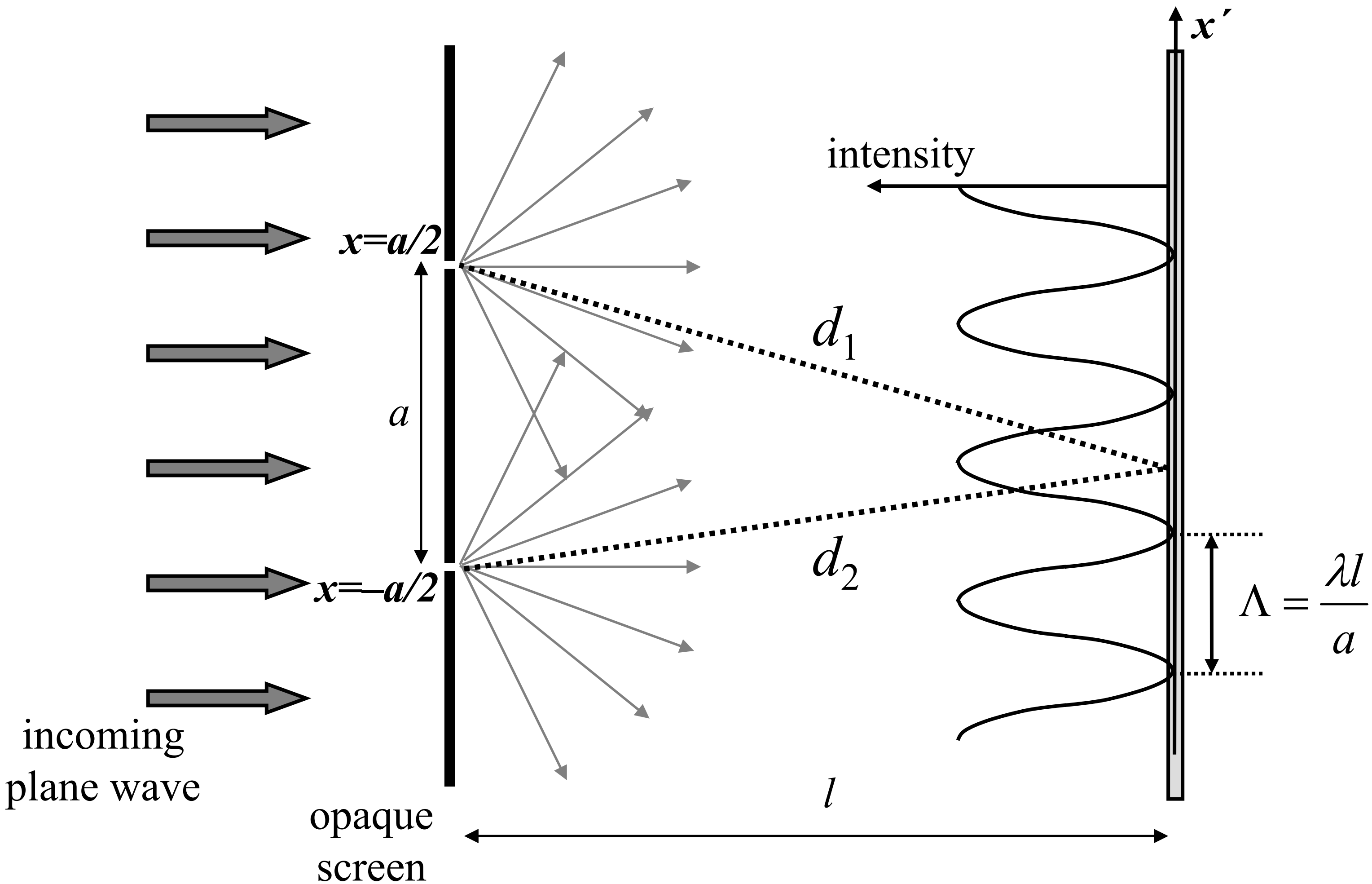




Young interferometer



Young interferometer



Two point sources interfering: math…

<u>Paraxial analysis:</u>

Amplitude:

$$e(x',y') = -\frac{1}{i\lambda l}\exp\left\{i2\pi\frac{l}{\lambda}\right\}\left[\exp\left\{i\pi\frac{\left(x'-a/2\right)^2+y'^2}{\lambda l}\right\}+\exp\left\{i\pi\frac{\left(x'+a/2\right)^2+y'^2}{\lambda l}\right\}\right]=$$

$$=\frac{1}{i\lambda l}\exp\left\{i2\pi\frac{l}{\lambda}+i\pi\frac{x'^2+\frac{a^2}{4}+y'^2}{\lambda l}\right\}\left(\exp\left\{-i2\pi\frac{ax'}{2\lambda l}\right\}+\exp\left\{i2\pi\frac{ax'}{2\lambda l}\right\}\right)$$

$$=\frac{2}{i\lambda l}\exp\left\{i2\pi\frac{l}{\lambda}+i\pi\frac{x'^2+\frac{a^2}{4}+y'^2}{\lambda l}\right\}\cos\left(\pi\frac{ax'}{\lambda l}\right).$$

$$\text{Intensity}: I(x',y')=\left|e(x',y')\right|^2=\left|\frac{2}{i\lambda l}\exp\left\{i2\pi\frac{l}{\lambda}+i\pi\frac{x'^2+\frac{a^2}{4}+y'^2}{\lambda l}\right\}\cos\left(\pi\frac{ax'}{\lambda l}\right)\right|^2=$$

$$=\frac{4}{(\lambda l)^2}\cos^2\left(\pi\frac{ax'}{\lambda l}\right)=\frac{2}{(\lambda l)^2}\left(1+\cos\left\{2\pi\left(\frac{a}{\lambda l}\right)x'\right\}\right).$$