MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.710 Optics **Fall '04**

Problem Set #1 Posted Sept. 8, 2004 — Due Wednesday, Sept. 15, 2004

1. **Optics jargon** Below is a list of buzzwords that currently dominating the headlines of optics research. Pick one, then do a quick Web search and write briefly (5-10 sentences) what it means and why it is considered important.

   a) hyperspectral imaging,

   b) X-ray lithography,

   c) long-baseline radio telescope,

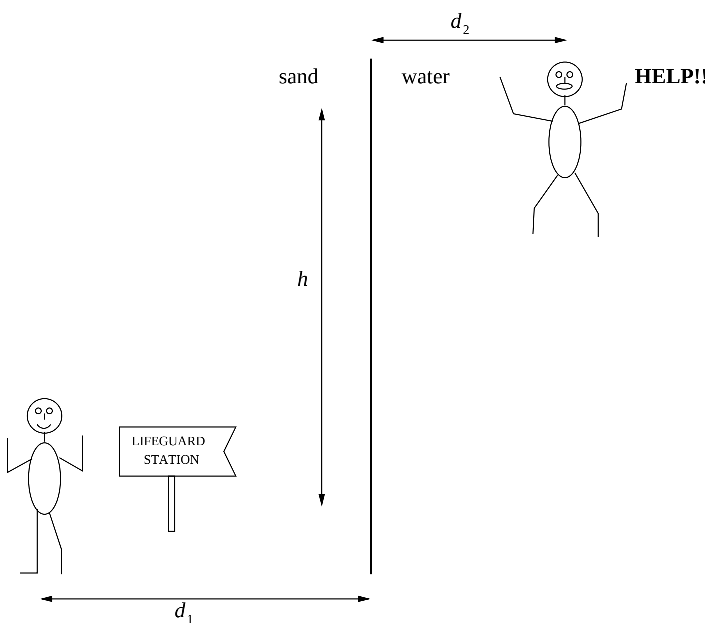
   d) photonic bandgap crystals,

   e) negative refractive index,

   f) Bose-Einstein condensate

   g) gravitational lensing.

2. **The lifeguard problem.** The lifeguard shown below can run at speed c on the sand, and swim at speed c/n ($n > 1$) in water. He is standing distance d_1 from the beach, and notices a swimmer in danger at distance h along the coastline and d_2 away from the coastline. Plan the lifeguard's path that reaches the swimmer in minimum time.



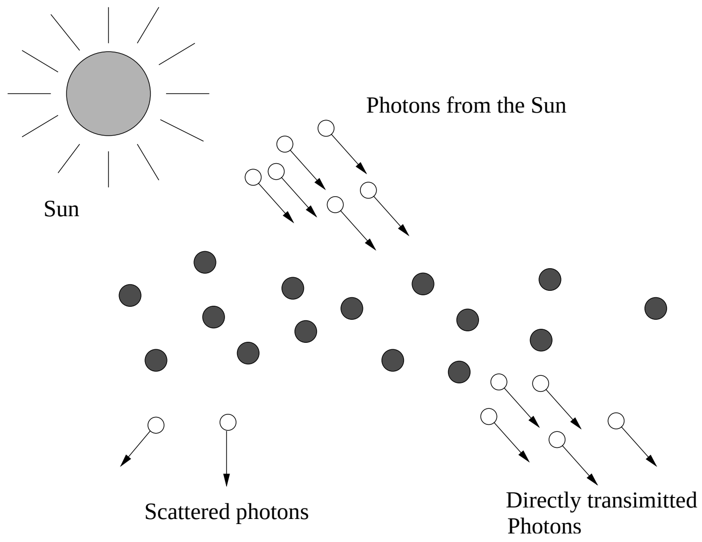


Figure 3.a

3. Why is the sky blue? In 1899, Lord Rayleigh observed that when we look at the sky, we see two kinds of light: "direct" light from the sun, which forms a painfully bright image of the sun on our retina, and "scattered" light which appears as the diffuse blue sky. Rayleigh attributed the blueness of the sky to the scattering of photons from sun by particles in the atmosphere, primarily nitrogen. See Figure 3.a above. In this problem, we will walk through Rayleigh's derivation and arrive to the famous law of scattering which explains why blue light is scattered more strongly than other wavelengths.

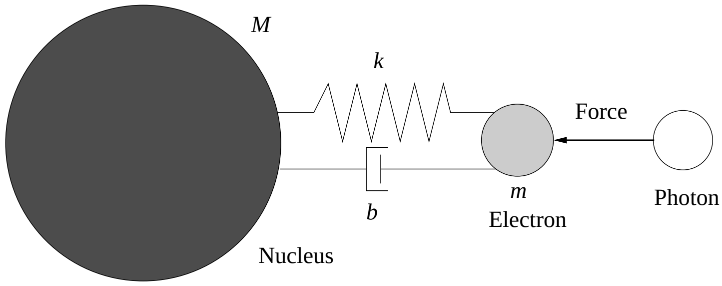


Figure 3.b

Consider the model shown in Figure 3.b, where a photon from the sun is incident on a nitrogen atom. The atom is modeled as a nucleus of mass M and an electron of mass $m \ll M$ bound to the nucleus with a spring with spring constant k and friction coefficient b. The electron is set in motion by the incident electron, whereas the massive nucleus is assumed not to move at all. The force applied by the photon on the electron as function of time t is proportional to $\cos(\omega t)$, where ω is the photon frequency (this is because the force is proportional to the photon's oscillating electric field). The result of this "excitation" of the nitrogen atom by incident light results in the scattering of a new photo, as shwon in Figure 3.c. How strong is the scattering? Rayleigh assumed that the scattering strength, sometimes referred to as "scattering cross-section," is proportional to the modulus squared of the acceleration of the electron that results form excitation.

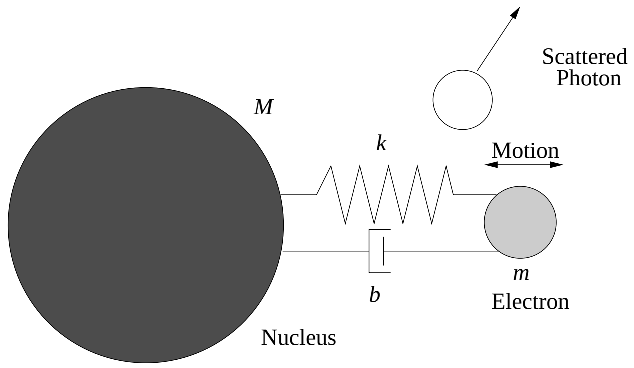


Figure 3.c

a) Derive the transfer function $A(\omega)$ of the electron acceleration with the force by the photon as input.

b) Identify the resonance frequency and the damping ratio of the system from the transfer function, and sketch the modulus-squared $|A(\omega)|^2$ of the transfer function that you obtained.

c) The nitrogen atom has spring constant $k \approx 140 N/m$ and the mass of the electron is $m = 9.11 \times 10^{-31} kg$. Show that the resonance frequency of this system is higher than all optical frequencies within the visible spectrum of light.

d) Show that $|A(\omega)|^2$ behaves approximately as ω^4 in the visible region; hence higher frequencies (shorter wavelengths) are scattered more than lower frequencies. This is known as " Rayleigh's law of scattering."