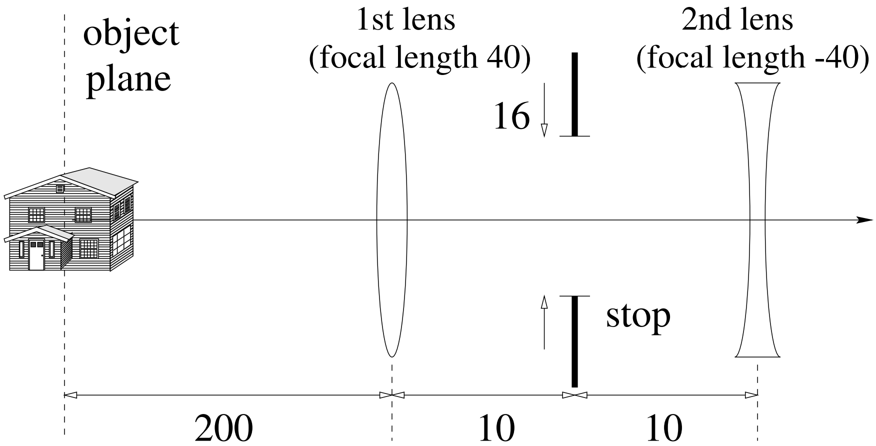
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.710 Optics **Fall '04**

Problem Set #4 Posted Oct. 06, 2004 — Due Wednesday, Oct. 13, 2004

1. Consider the two–lens system shown below (all distances in cm). Find the numerical aperture.



2. Consider two identical thin lenses of focal length $f = 5$ inches, spaced one focal length apart (see figure on next page.)

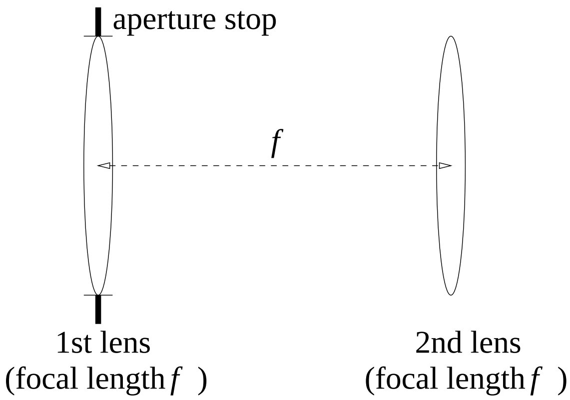
2.a) Calculate the focal length and back focal length of the combination.

2.b) Assuming the stop is located at the first lens and has a diameter of 1 inch, and the second lens is also 1 inch in diameter, calculate and draw the trajectory of the marginal and chief rays.

2.c) What is the field of view of the system?

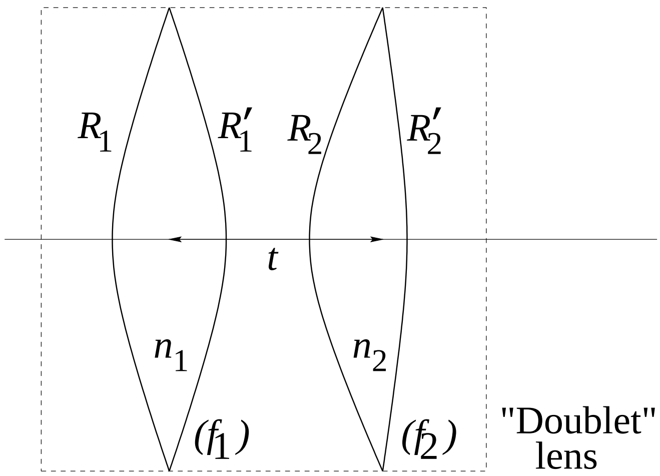
2.d) What is the $f/\#$ of the system?

2.e) How does the performance of the system change if the aperture stop is placed 1 focal length to the front (*i.e.*, to the left) of the first lens?



Optical system for Problem 2.

3. Achromatic doublet. A common method to reduce chromatic aberration is to combine *two* optical elements with different aberration coefficients and optical powers such that the optical power of the compound element equals the desired power and the chromatic aberration of the compound element equals zero. These elements are known as "achromats," or "achromatic doublets." (Note that the term "achromat" as a disability condition refers rather to persons who lack cones in their retina and hence are unable to perceive color.) Consider the achromat shown below, where the individual elements are thin spherical lenses and there is a distance t between them. We seek to minimize the chromatic aberration of the achromat between two wavelengths of interest λ_{a} and λ_{b}.



3.a) Let V_j ($j = 1, 2$) denote Abbe's V-number for the j-th element, and f_j the corresponding focal length. Show that the chromatic dispersion in the optical power of the j-th element is

$$\Delta\left(\frac{1}{f_j}\right) = \frac{1}{V_j f_j}.$$

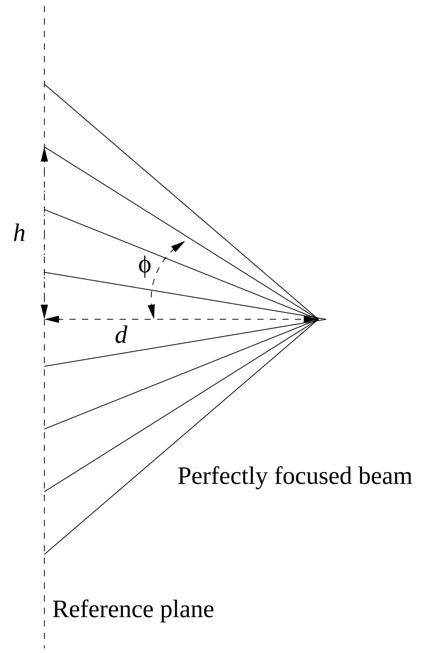
3.b) Derive the thickness t that is required to eliminate chromatic aberration from the composite element in terms of V_j, f_j ($j = 1, 2$.) What is then the optical power ϕ of the compound (*i.e.*, the doublet) ? What are t and ϕ if the lenses are identical?

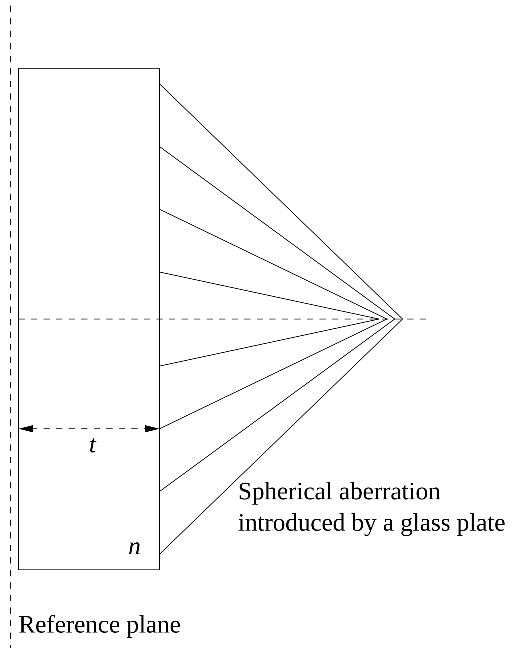
3.c) Practical doublets are often cemented (*i.e.*, "glued" together) in order to minimize the possibility of misalignments and other instabilities. Then $t = 0$ and $R_1{}' = R_2$. Derive the condition that V_j, f_j ($j = 1, 2$) must satisfy in order to eliminate chromatic aberration in this case. What is then the composite optical power ϕ?

3.d) Design a cemented achromatic doublet for $\phi = 10D$. (You should select the appropriate materials and curvatures for the individual lens components.)

3.e) Plot the focal length of the doublet you designed for all wavelengths between $\lambda_a = 0.2\mu$m and $\lambda_b = 0.7\mu$m and specify the maximum chromatic aberration within this range and the wavelength(s) for which it occurs.

4. A beam is perfectly focused in free space so that all the rays converge towards the axis at distance d from a reference plane (see left figure below). We insert a plate of thickness t and refractive index n in the path of the beam, just beyond the reference plane (see the right figure below). Compute the primary (Seidel) longitudinal spherical aberration introduced by the plate. Ignore reflections from the plate surfaces. [Hint: calculate the distance from the plate where the rays intersect the axis as the function of the ray incidence angle ϕ and the height h on the reference plane; expand to Taylor series in the small number (h/d), making sure that powers up to $(h/d)^2$ survive in the final result.]





5. A concave mirror of radius $|R|$ is centered at C. An erect object is located distance $1.5|R|$ from the convex of the mirror. Locate the image and calculate the magnification.