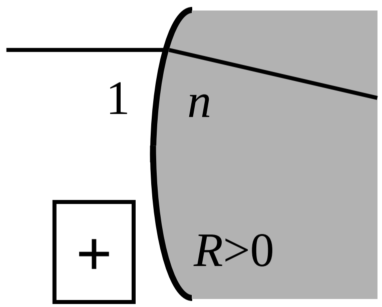
Lenses and Imaging (Part II)

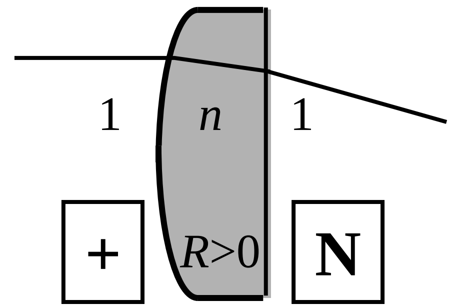
- Reminders from Part I
- Surfaces of positive/negative power
- Real and virtual images
- Imaging condition
- Thick lenses
- Principal planes

The power of surfaces

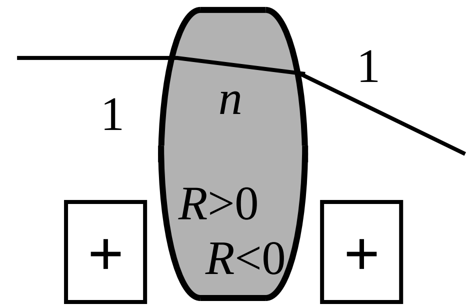
- Positive power : exiting rays converge



Simple spherical refractor (positive)

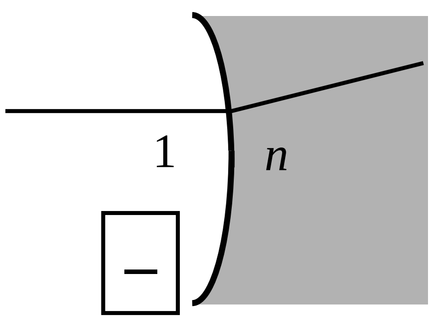


Plano-convex lens

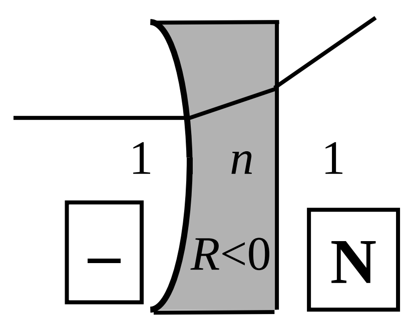


Bi-convex lens

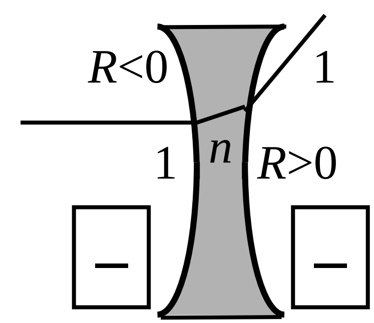
- Negative power : exiting rays diverge



Simple spherical refractor (negative)

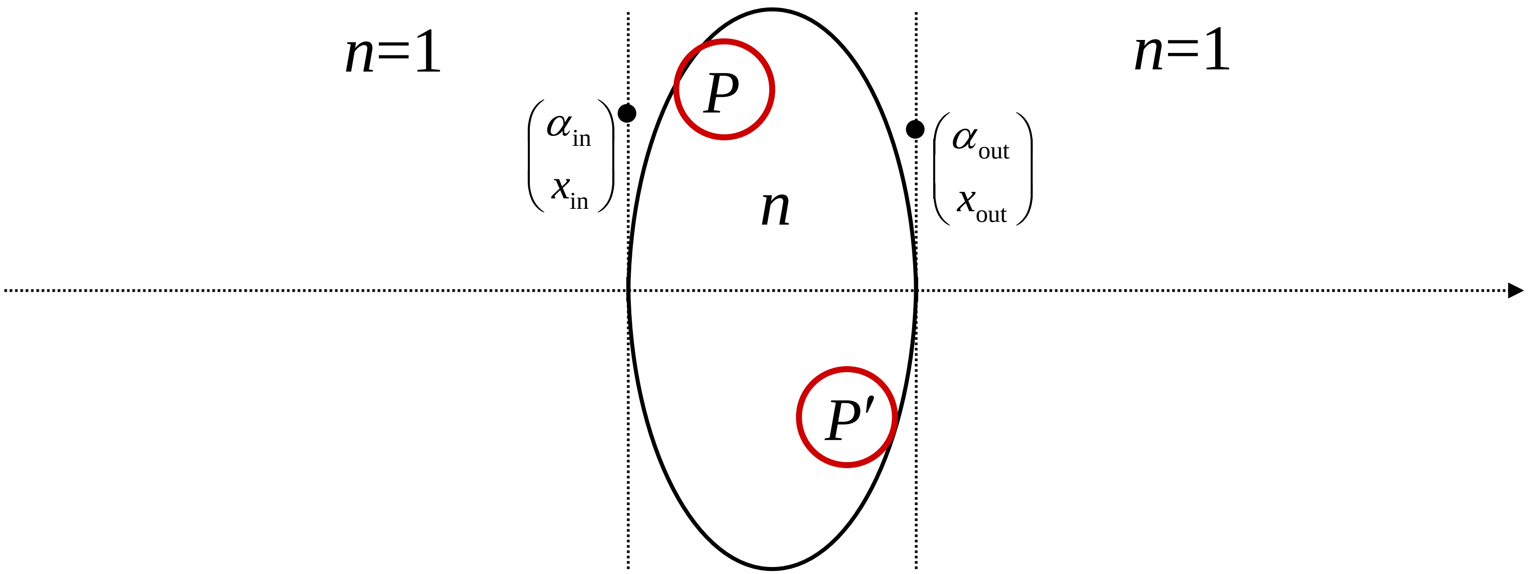


Plano-concave lens



Bi-concave lens

Thin lens in air

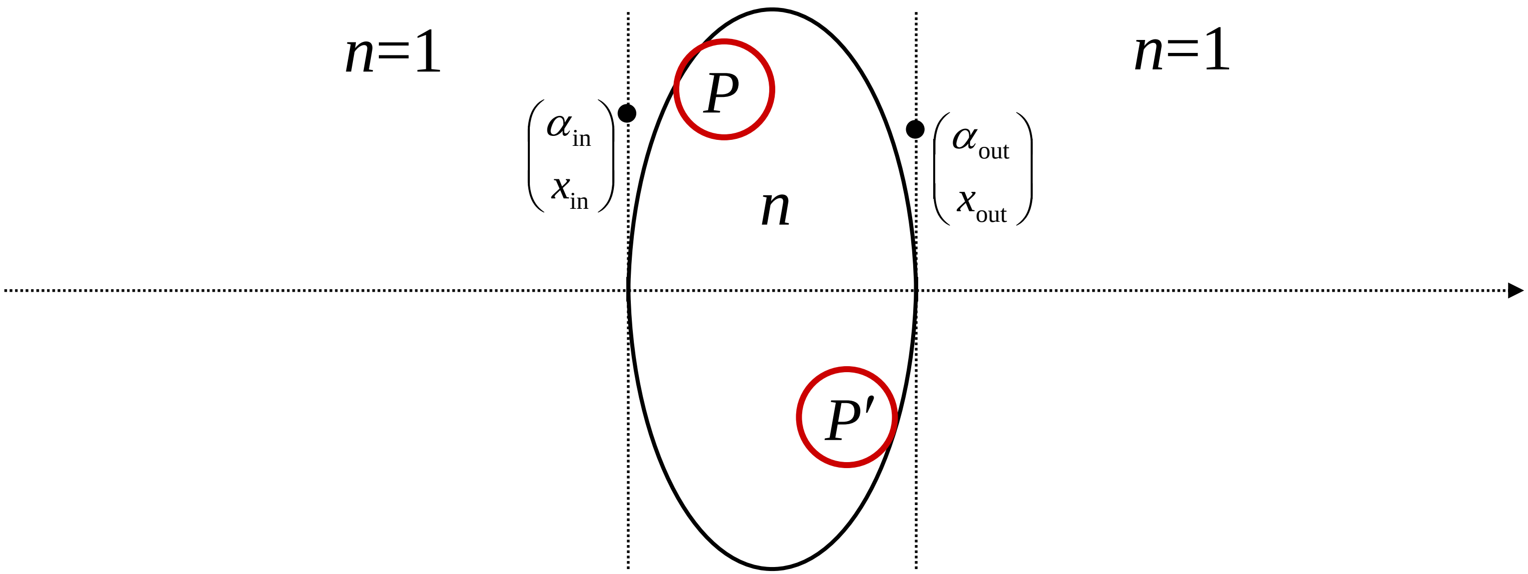


$$\begin{pmatrix}\alpha_{\text{out}}\\ x_{\text{out}}\end{pmatrix}=\begin{pmatrix}1 & -P_{\text{thin lens}}\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha_{\text{in}}\\ x_{\text{in}}\end{pmatrix}$$

$$P_{\text{thin lens}}=P+P'=\frac{n-1}{R}+\frac{1-n}{R'}=\boxed{(n-1)\left(\frac{1}{R}-\frac{1}{R'}\right)}$$

Lens-maker's formula

Thin lens in air



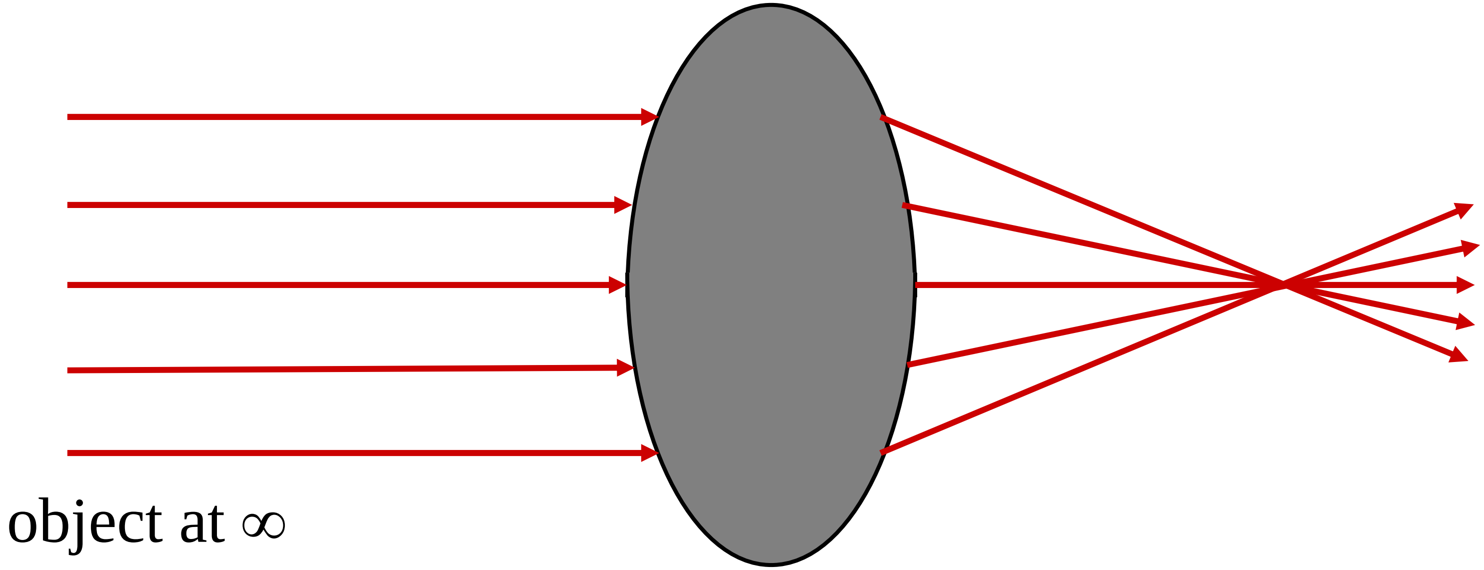
$$\begin{pmatrix} \alpha_{out} \\ x_{out} \end{pmatrix} = \begin{pmatrix} 1 & -P_{\text{thin lens}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_{in} \\ x_{in} \end{pmatrix} \Rightarrow$$

$$\alpha_{out} = \alpha_{in} - P_{\text{thin lens}}\, x_{in}$$

$$x_{out} = x_{in}$$

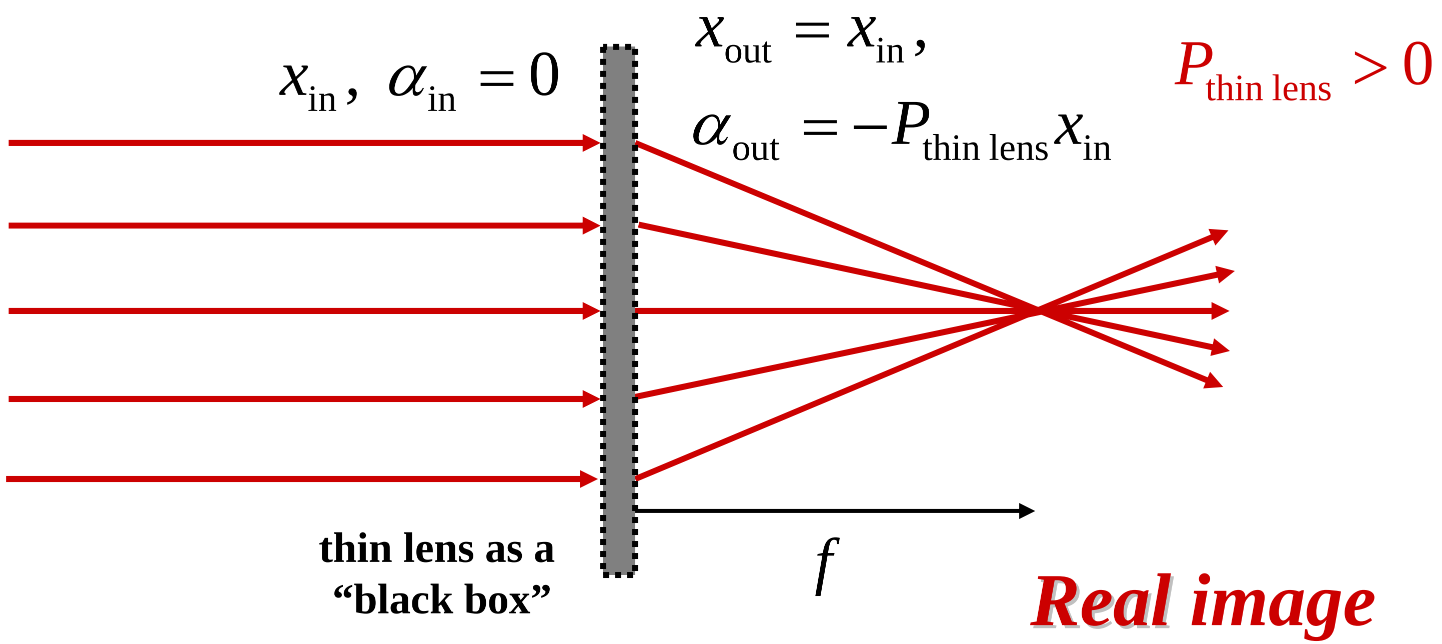
Ray bending is proportional to the distance from the axis

Positive thin lens in air



Ray bending is proportional to the distance from the axis

Positive thin lens in air

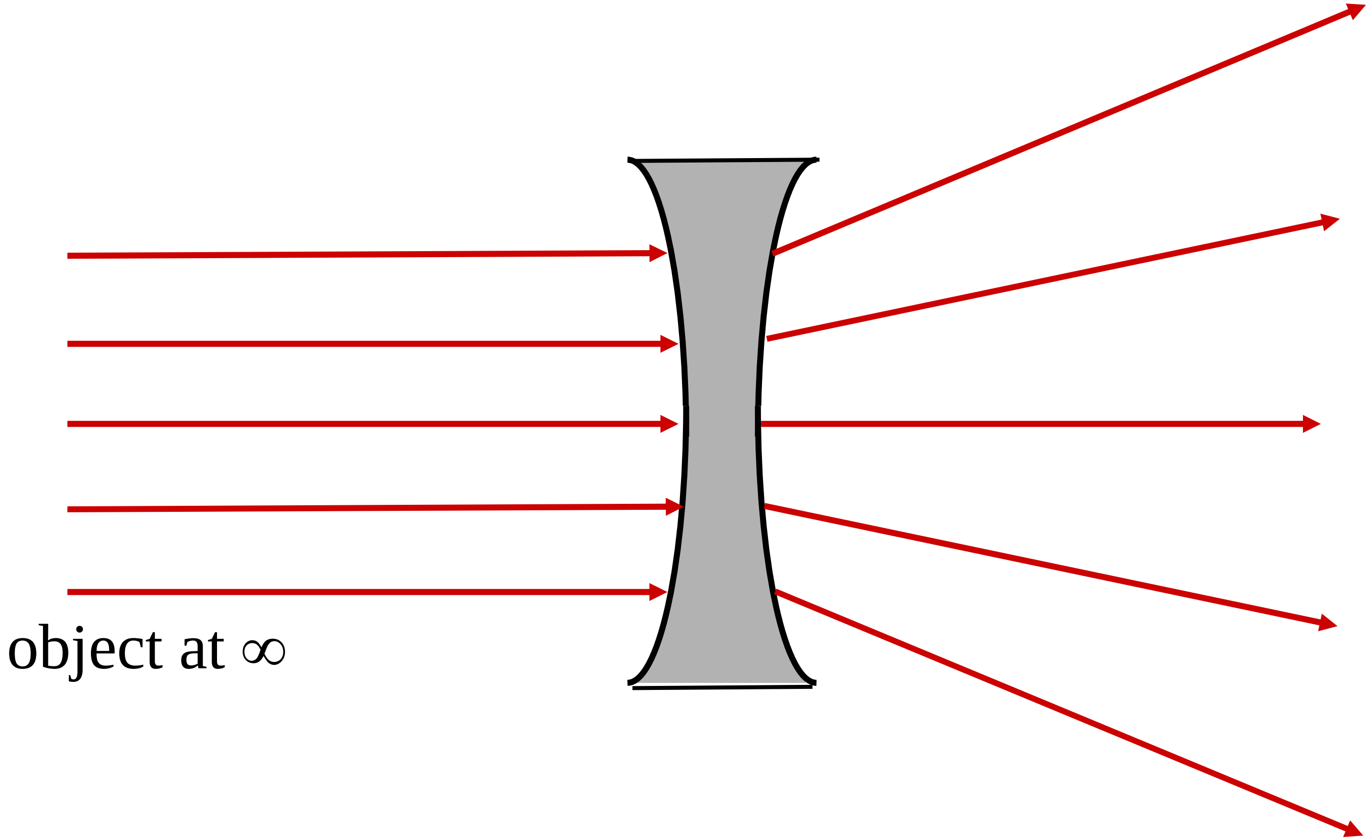


$$f = -\frac{x_{\text{in}}}{\alpha_{\text{out}}} \Rightarrow \boxed{f = \frac{1}{P_{\text{thin lens}}}}$$

Focal point = image of an object at ∞

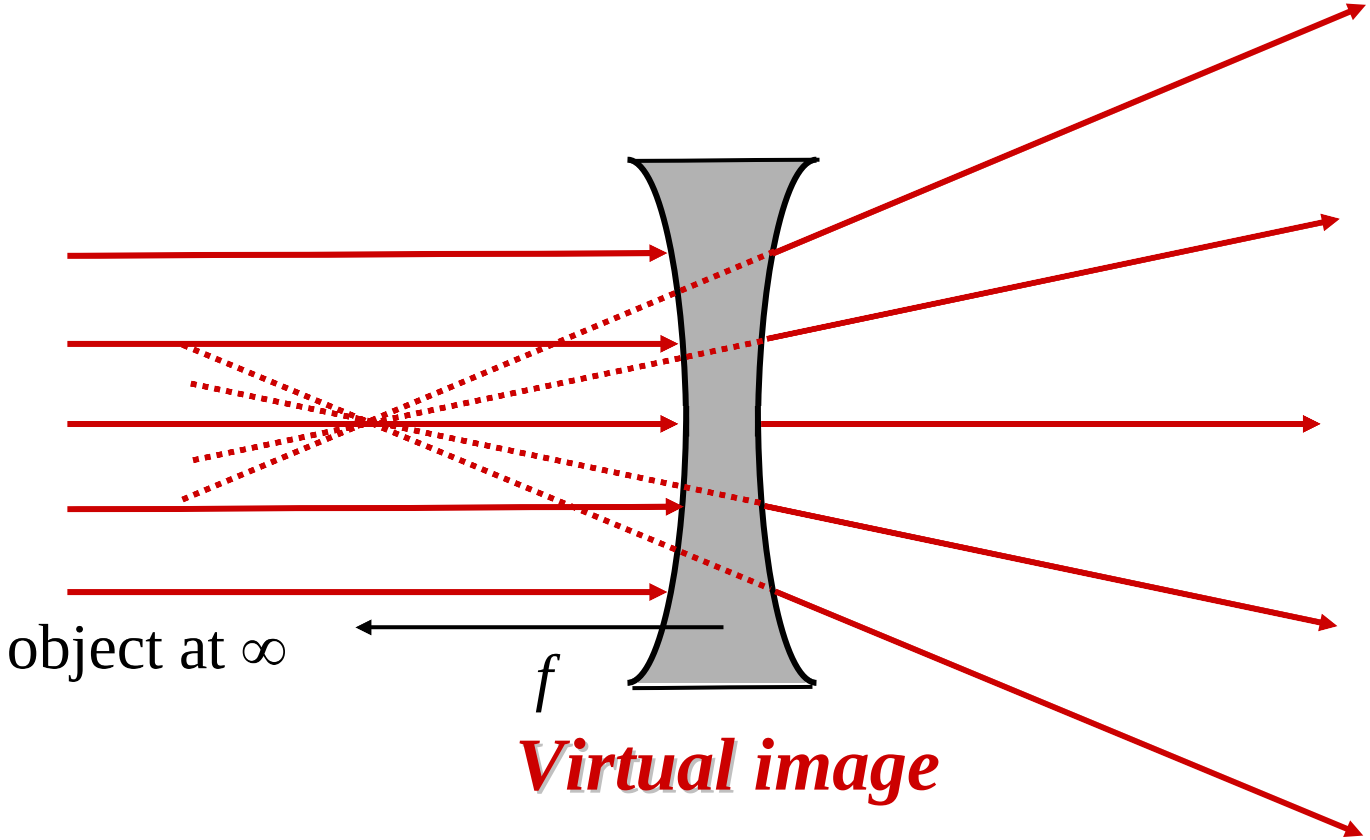
Focal length = distance between lens & focal point

Negative thin lens in air



Ray bending is proportional to the distance from the axis

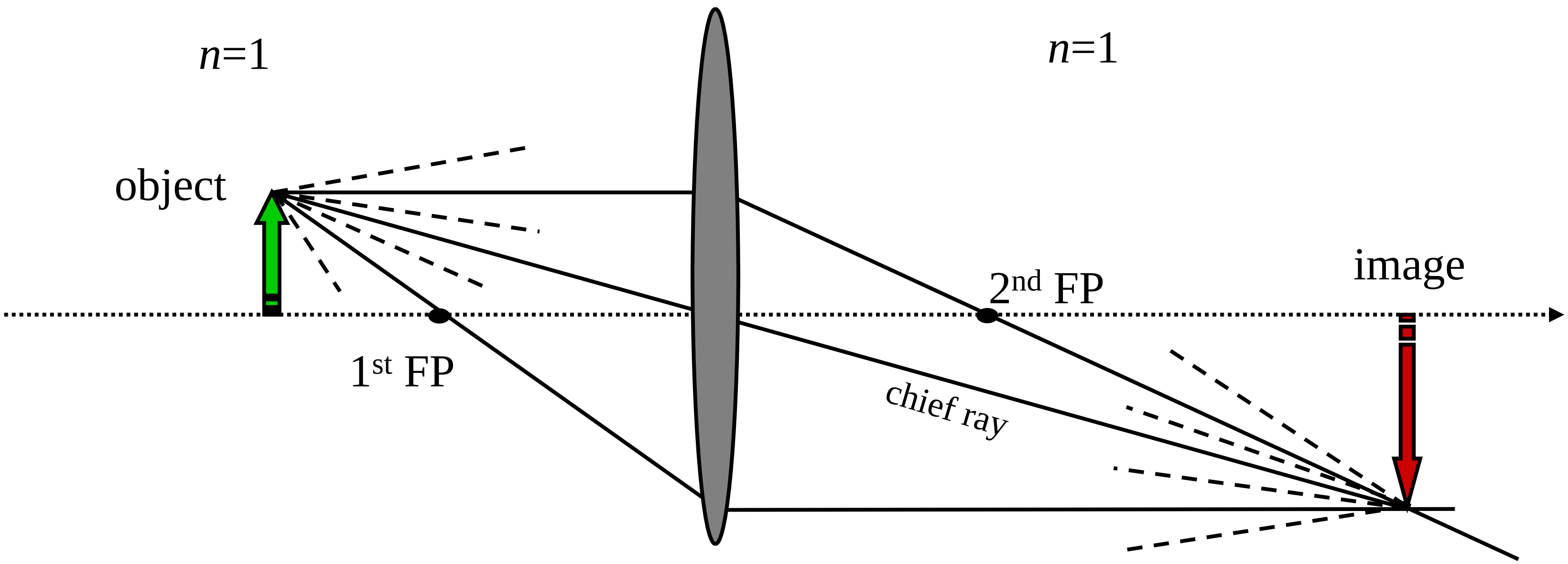
Negative thin lens in air



$$f = \frac{1}{P_{\text{thin lens}}}$$ still applies, now with $P_{\text{thin lens}} < 0 \Rightarrow$

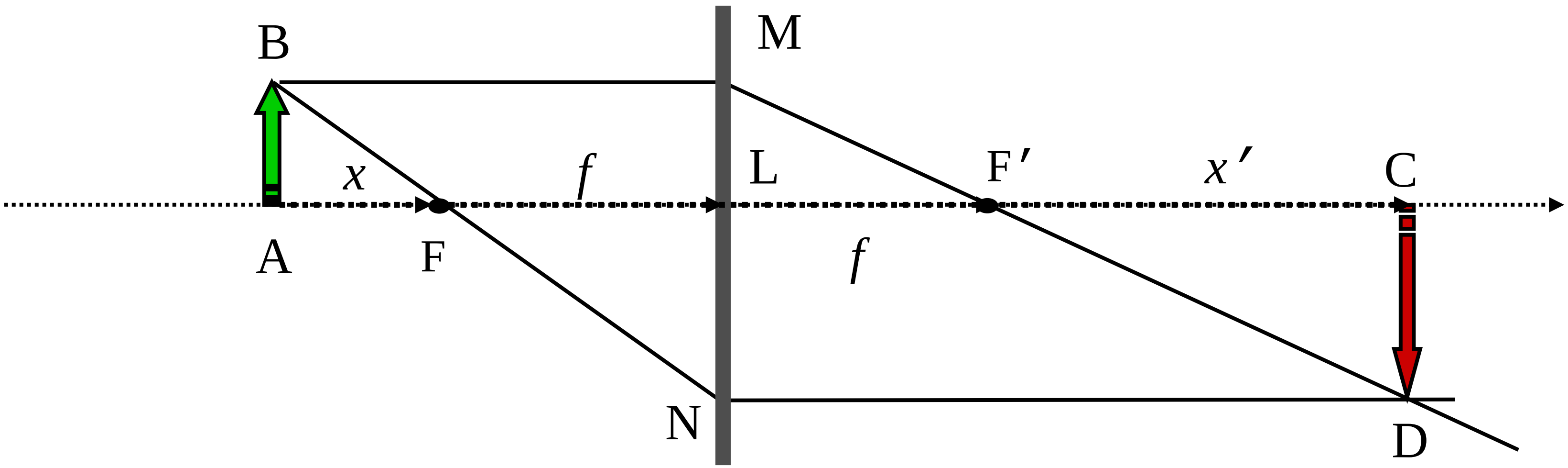
$\Rightarrow \quad f < 0$ (to the "*left*")

Imaging condition: ray-tracing



- Image point is located at the common intersection of **all** rays which emanate from the corresponding object point
- The two rays passing through the two focal points and the chief ray can be ray-traced directly

Imaging condition: ray-tracing

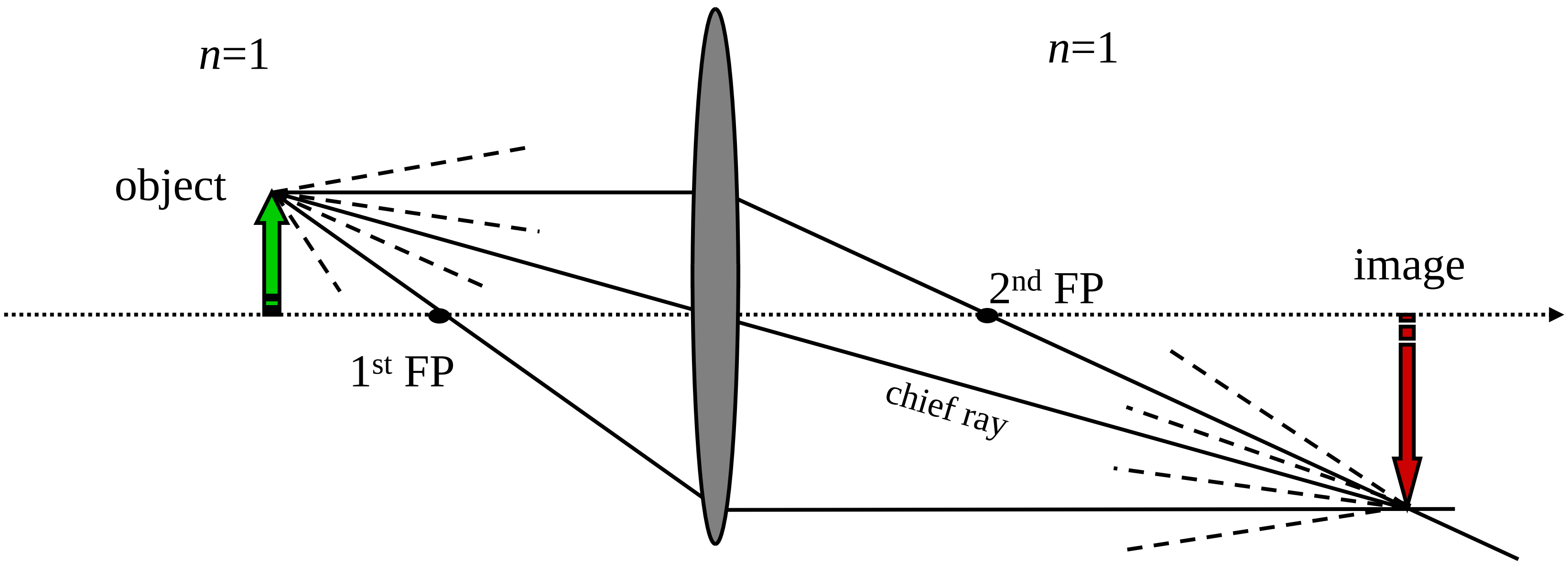


• (ABF)~(FLN) and (F′CD)~(MLF′) are pairs of similar triangles

$$\frac{(AB)}{(AF)} = \frac{(LN)}{(FL)} \qquad \frac{(LM)}{(LF')} = \frac{(CD)}{(F'C)} \qquad (AB) = (ML) \qquad (LN) = (CD)$$

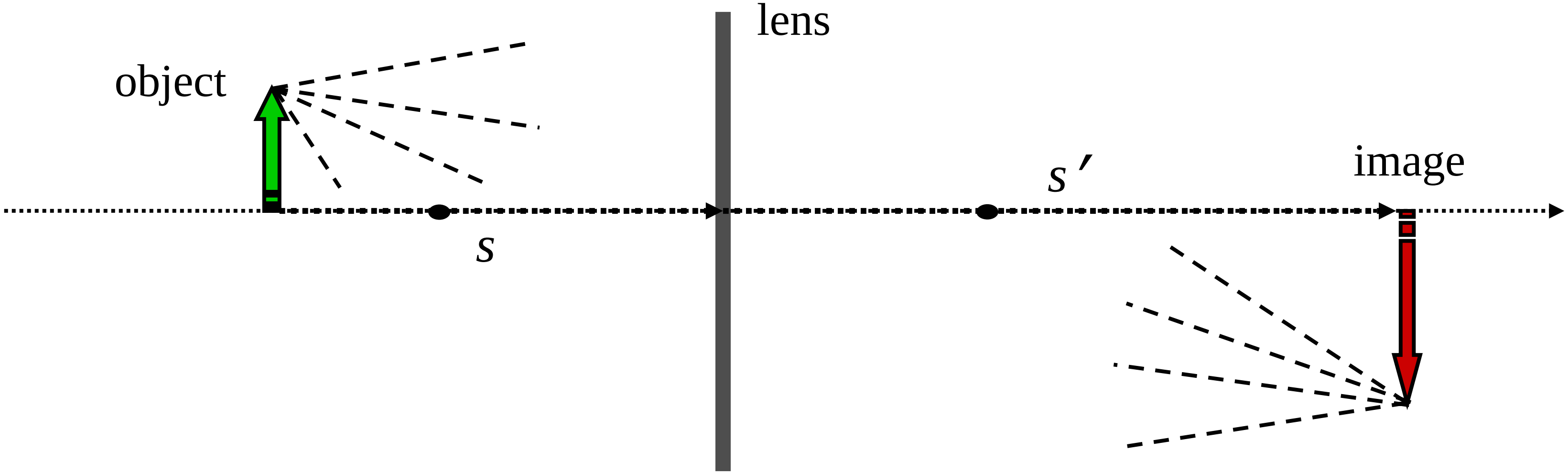
$$(AF)\,(F'C) = (FL)\,(LF') \qquad xx' = f^{\,2}$$

Imaging condition: matrix method



• Location of image point must be independent of ray departure angle at the object

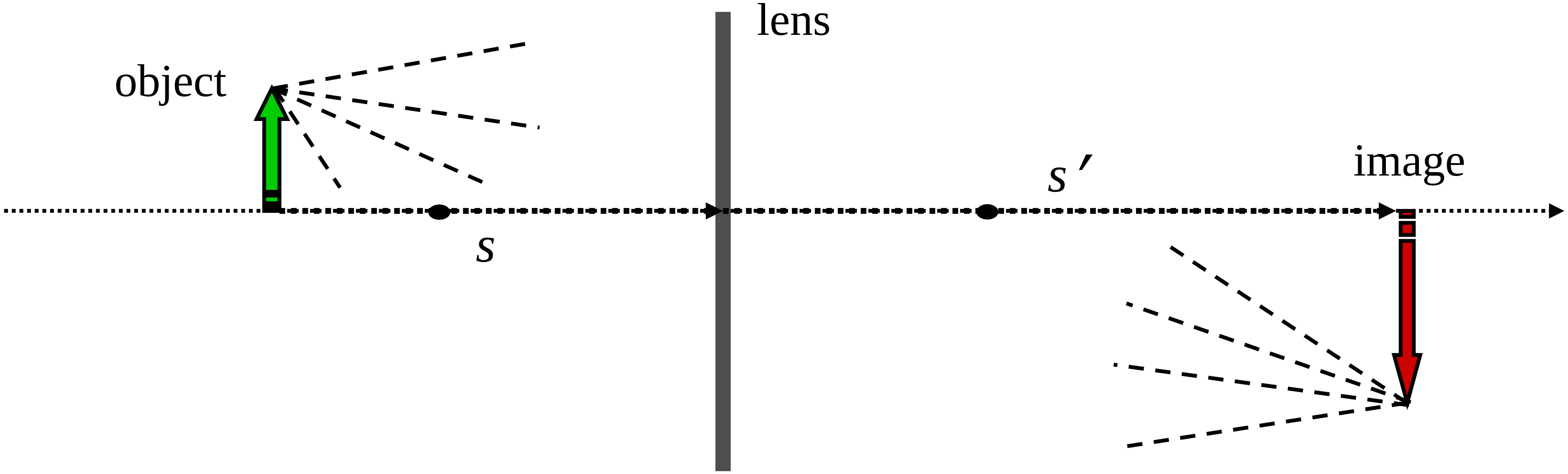
Imaging condition: matrix method



$$\begin{pmatrix}\alpha_{out}\\ x_{out}\end{pmatrix}=\begin{pmatrix}1 & 0\\ s' & 1\end{pmatrix}\begin{pmatrix}1 & -\frac{1}{f}\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ s & 1\end{pmatrix}\begin{pmatrix}\alpha_{in}\\ x_{in}\end{pmatrix}=\begin{pmatrix}1-\frac{s}{f} & -\frac{1}{f}\\ s+s'-\frac{ss'}{f} & 1-\frac{s'}{f}\end{pmatrix}\begin{pmatrix}\alpha_{in}\\ x_{in}\end{pmatrix}$$

$s+s'-\frac{ss'}{f}=0$

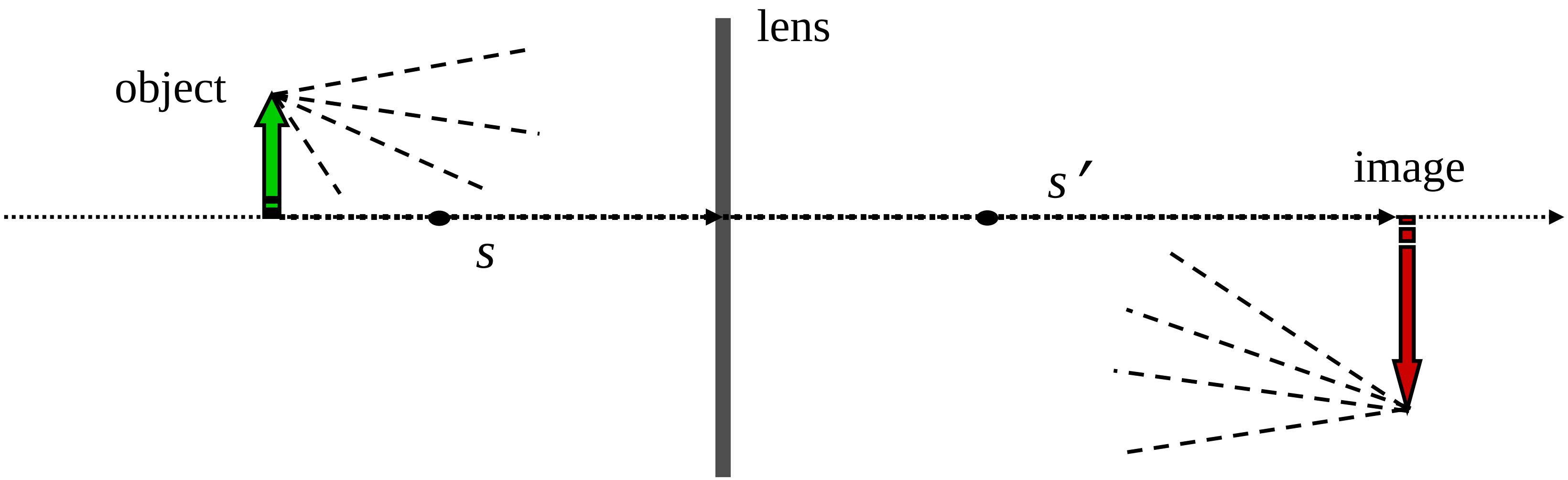
Imaging condition: matrix method



$$s + s' - \frac{ss'}{f} = 0 \quad \Leftrightarrow \quad \boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

Imaging condition
(*aka* Lens Law)

Imaging condition: matrix method



$$\begin{pmatrix}\alpha_{out}\\ x_{out}\end{pmatrix} == \begin{pmatrix}1-\frac{s}{f} & -\frac{1}{f}\\ 0 & 1-\frac{s'}{f}\end{pmatrix}\begin{pmatrix}\alpha_{in}\\ x_{in}\end{pmatrix} \Rightarrow x_{out} = \left(1-\frac{s'}{f}\right)x_{in}$$

Lateral magnification :

$$M_T = \frac{x_{out}}{x_{in}}$$

$$M_T = 1-\frac{s'}{f} = -\frac{s'}{s}$$

Real & virtual images

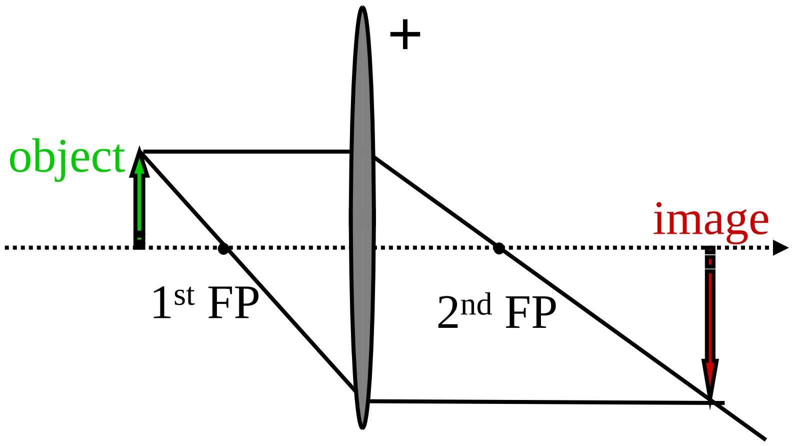


image: real & inverted; $M_T<0$

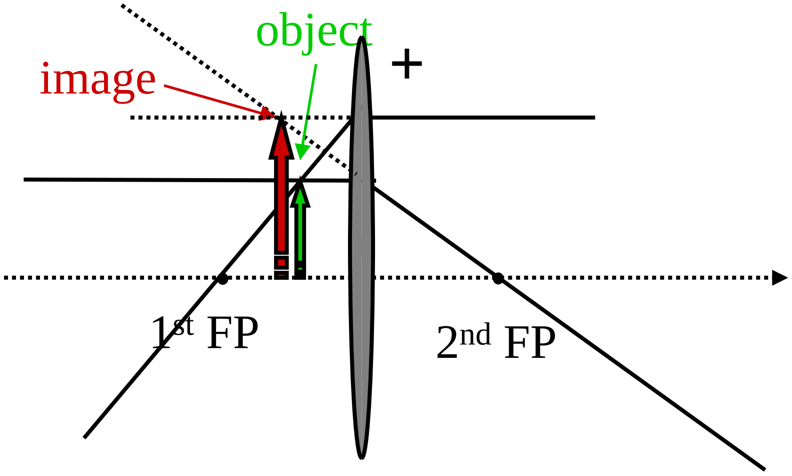


image: virtual & erect; $M_T>1$

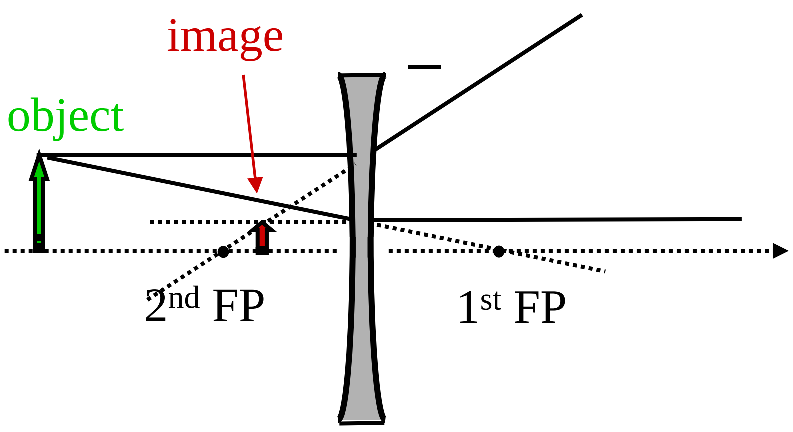


image: virtual & erect; $0<M_T<1$

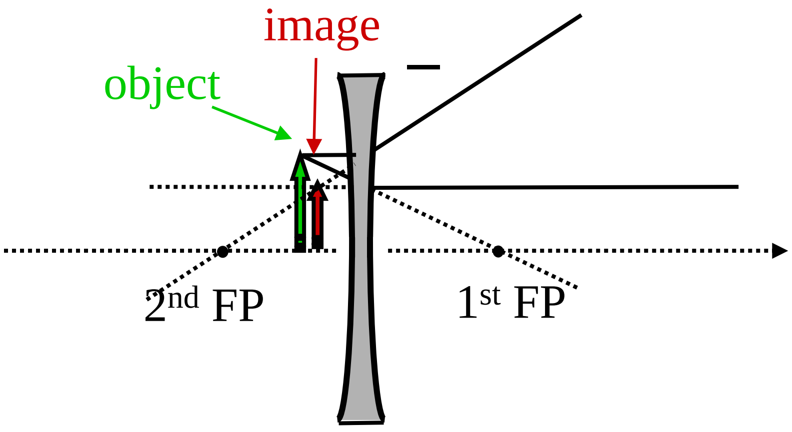
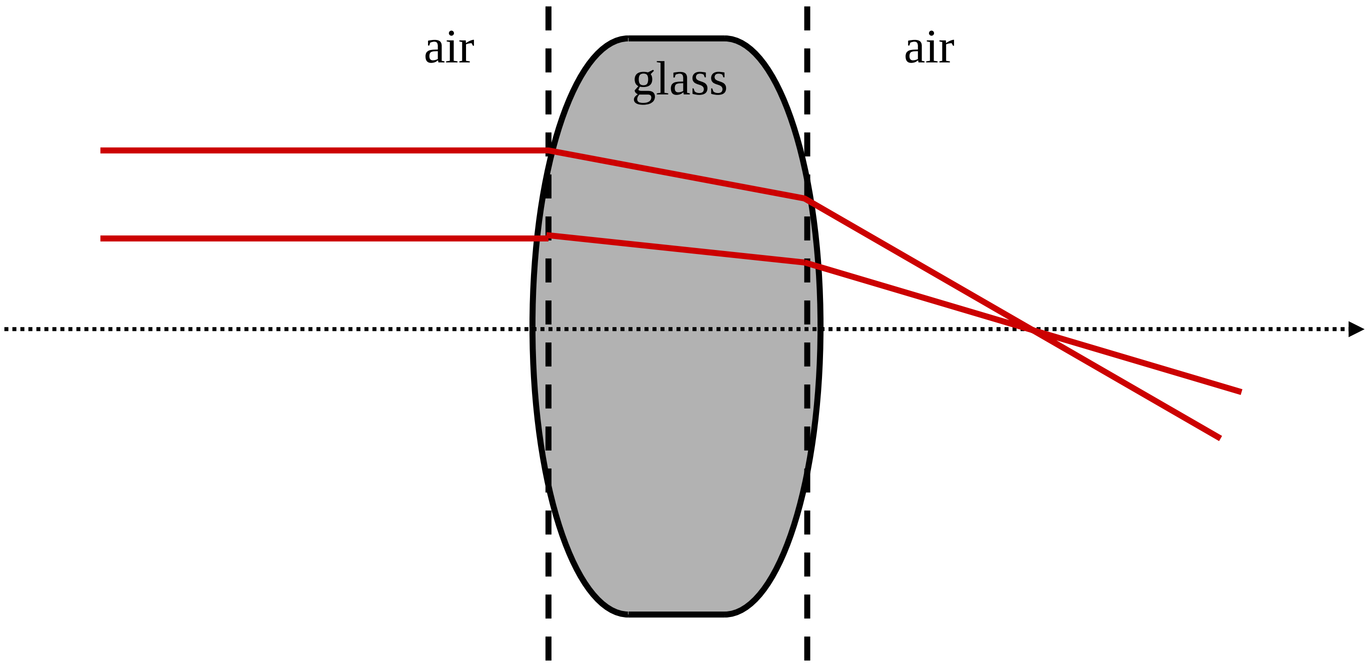


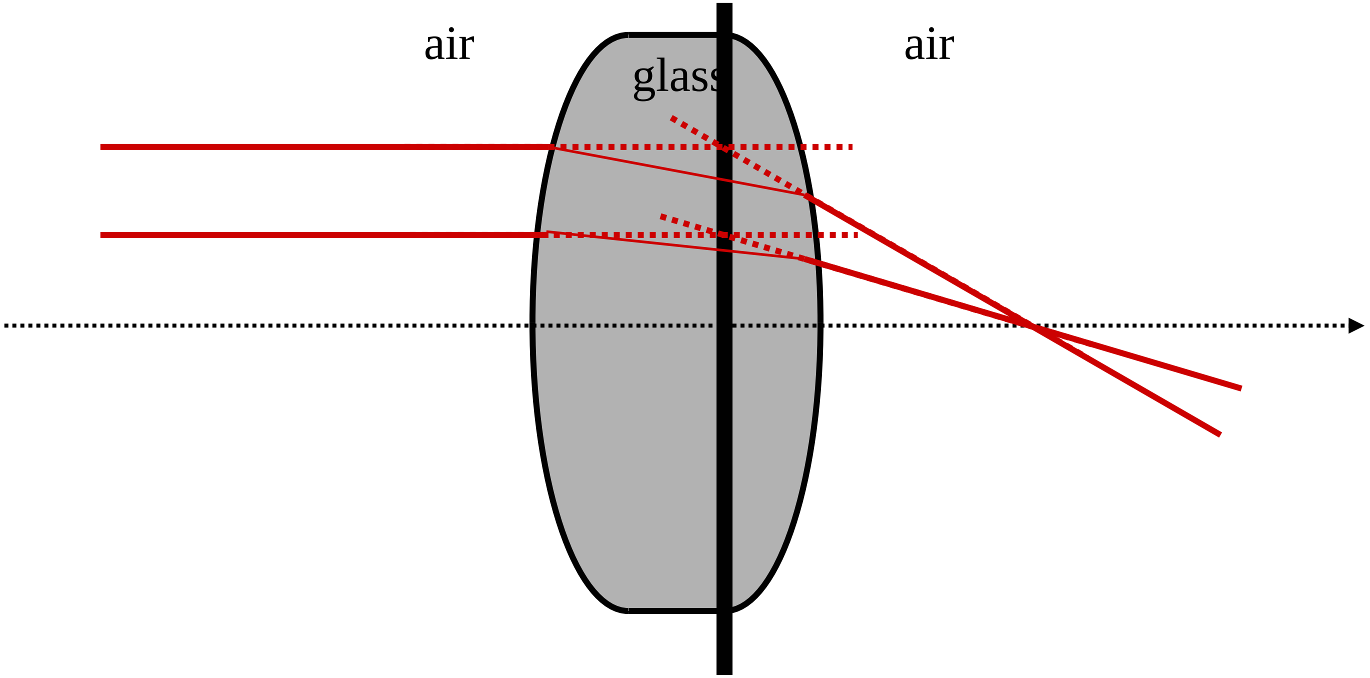
image: virtual & erect; $0<M_T<1$

The thick lens



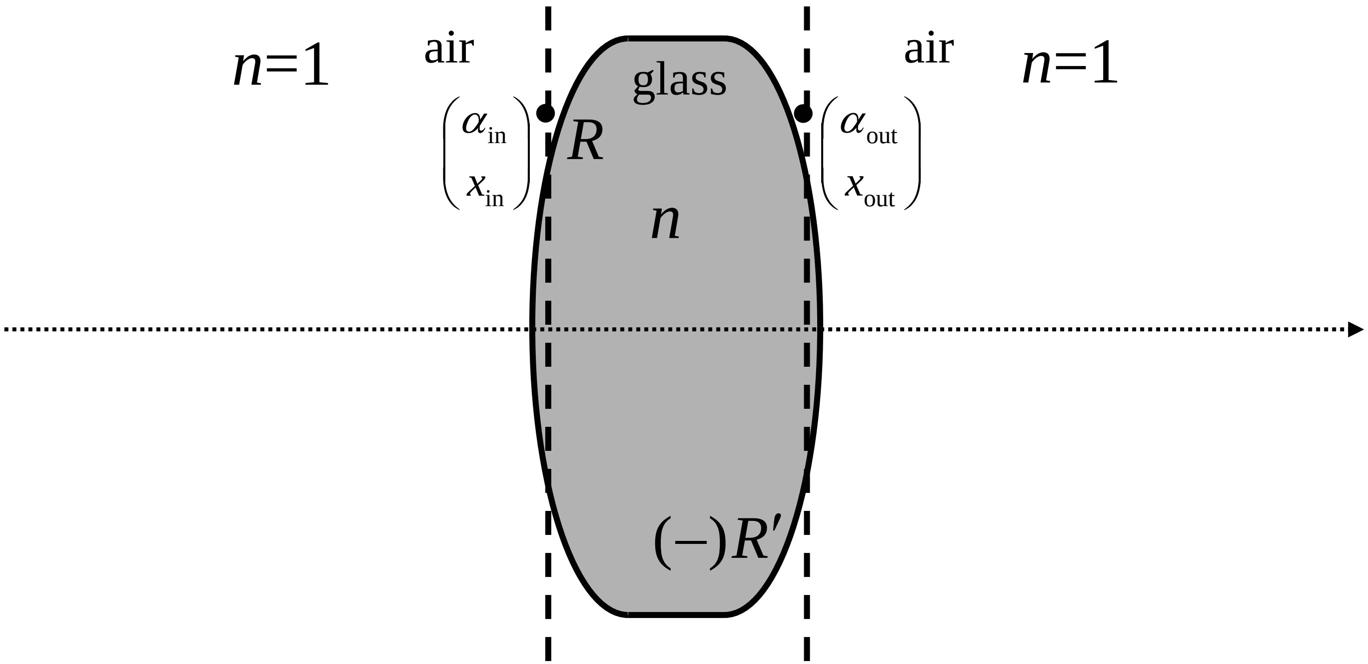
Rays bend in “two steps”

The thick lens



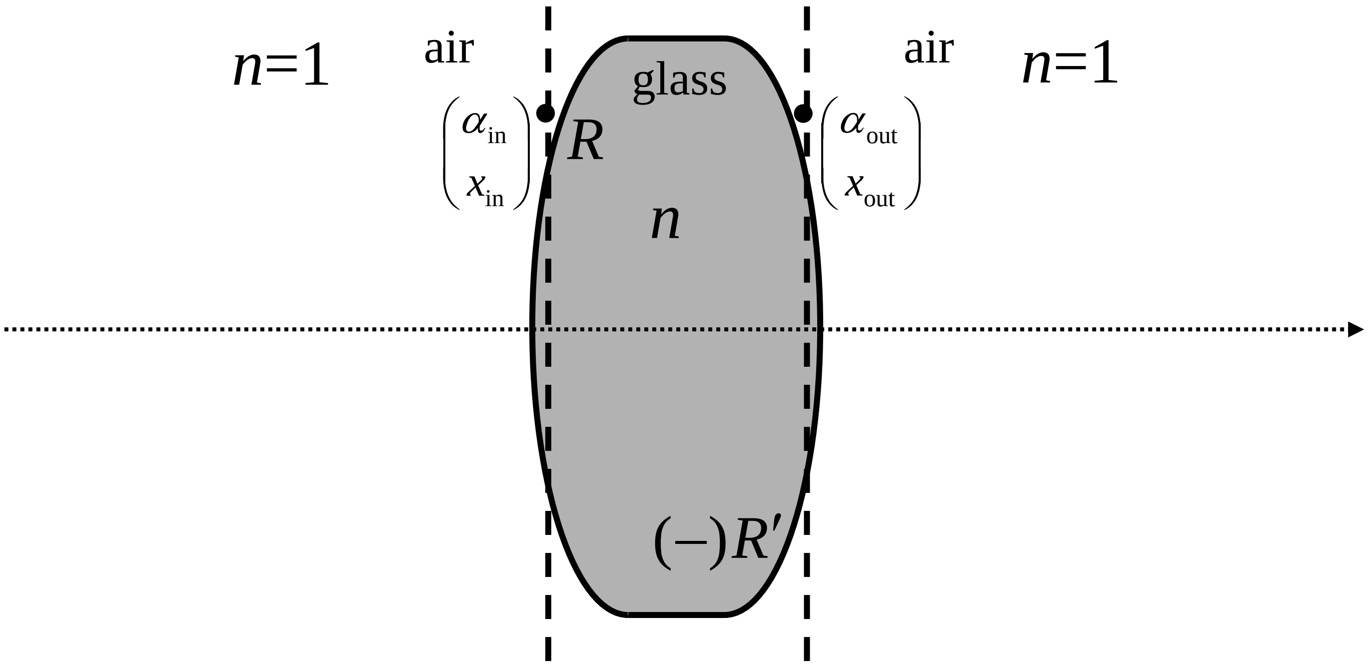
Equivalent to a thin lens placed "somewhere" within the thick element.
The location of this "equivalent thin lens" is the **Principal Plane** of the thick element

The thick lens



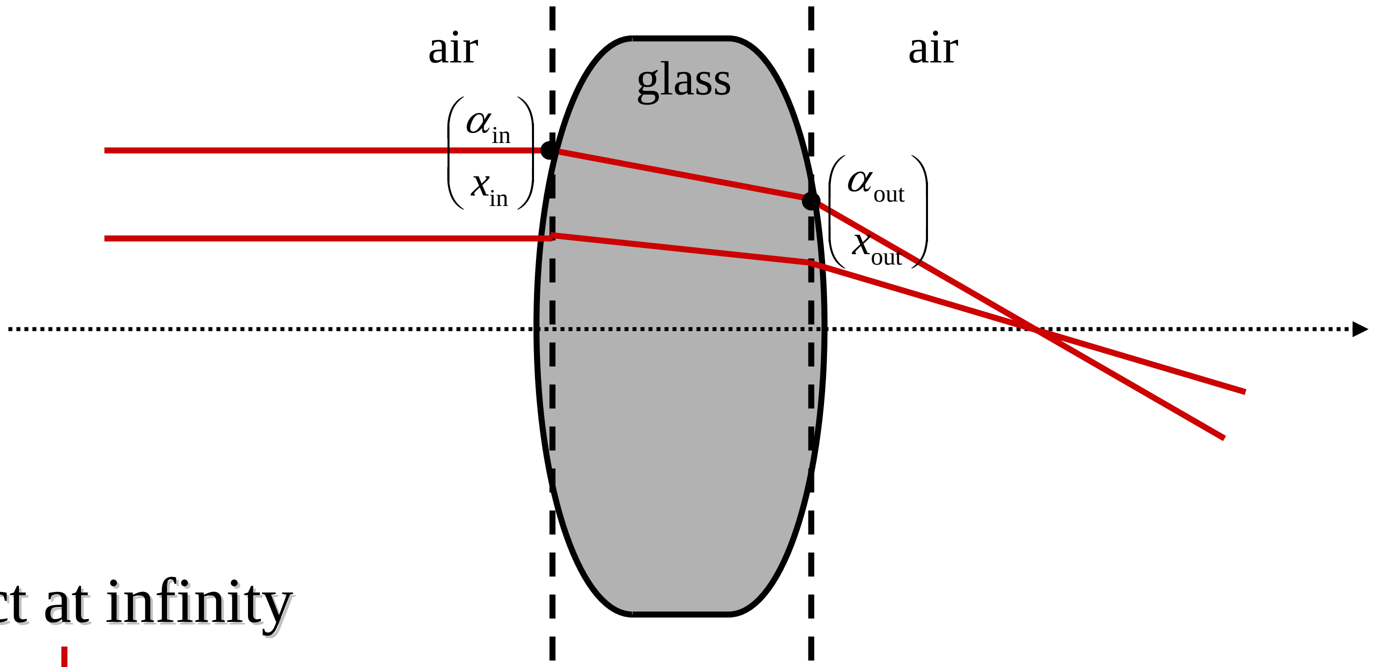
$$\begin{pmatrix}\alpha_{out}\\ x_{out}\end{pmatrix}=\begin{pmatrix}1 & -\dfrac{1-n}{R'}\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \dfrac{d}{n} & 1\end{pmatrix}\begin{pmatrix}1 & -\dfrac{n-1}{R}\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha_{in}\\ x_{in}\end{pmatrix}$$

The thick lens



$$\begin{pmatrix}\alpha_{\text{out}}\\ x_{\text{out}}\end{pmatrix}=\begin{pmatrix}1+\dfrac{n-1}{n}\dfrac{d}{R'} & -\left[(n-1)\left(\dfrac{1}{R}-\dfrac{1}{R'}\right)+\dfrac{(n-1)^2 d}{nRR'}\right]\\ \dfrac{d}{n} & 1-\dfrac{n-1}{n}\dfrac{d}{R}\end{pmatrix}\begin{pmatrix}\alpha_{\text{in}}\\ x_{\text{in}}\end{pmatrix}$$

The thick lens: power

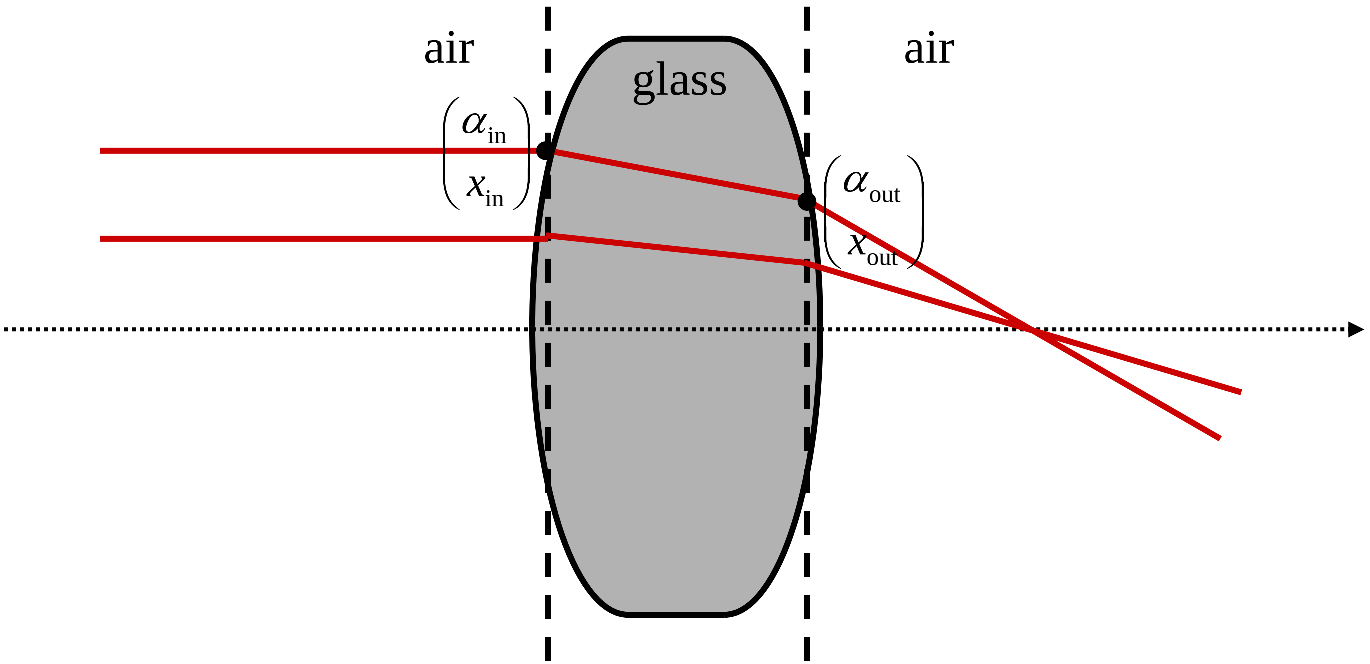


Object at infinity

$$\begin{pmatrix}\alpha_{\text{in}}\\ x_{\text{in}}\end{pmatrix}=\begin{pmatrix}0\\ x\end{pmatrix}\Rightarrow\begin{pmatrix}\alpha_{\text{out}}\\ x_{\text{out}}\end{pmatrix}=\begin{pmatrix}-\left[(n-1)\left(\dfrac{1}{R}-\dfrac{1}{R'}\right)+\dfrac{(n-1)^2 d}{nRR'}\right]x\\ \left(1-\dfrac{n-1}{n}\dfrac{d}{R}\right)x\end{pmatrix}$$

$$\Rightarrow \alpha_{\text{out}}=-Px_{\text{in}}$$

The thick lens: power



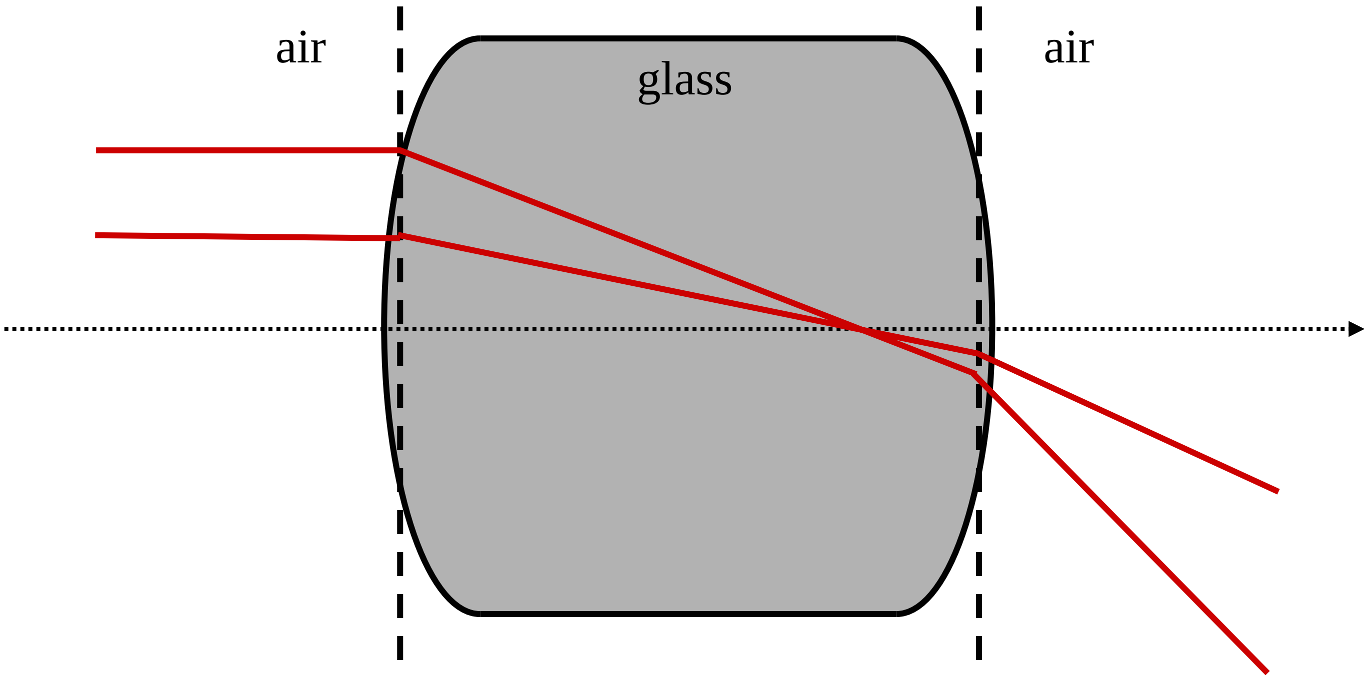
$$P = \left[(n-1)\left(\frac{1}{R} - \frac{1}{R'}\right) + \frac{(n-1)^2 d}{nRR'} \right]$$

Power

$$\frac{1}{f} = \left[(n-1)\left(\frac{1}{R} - \frac{1}{R'}\right) + \frac{(n-1)^2 d}{nRR'} \right]$$

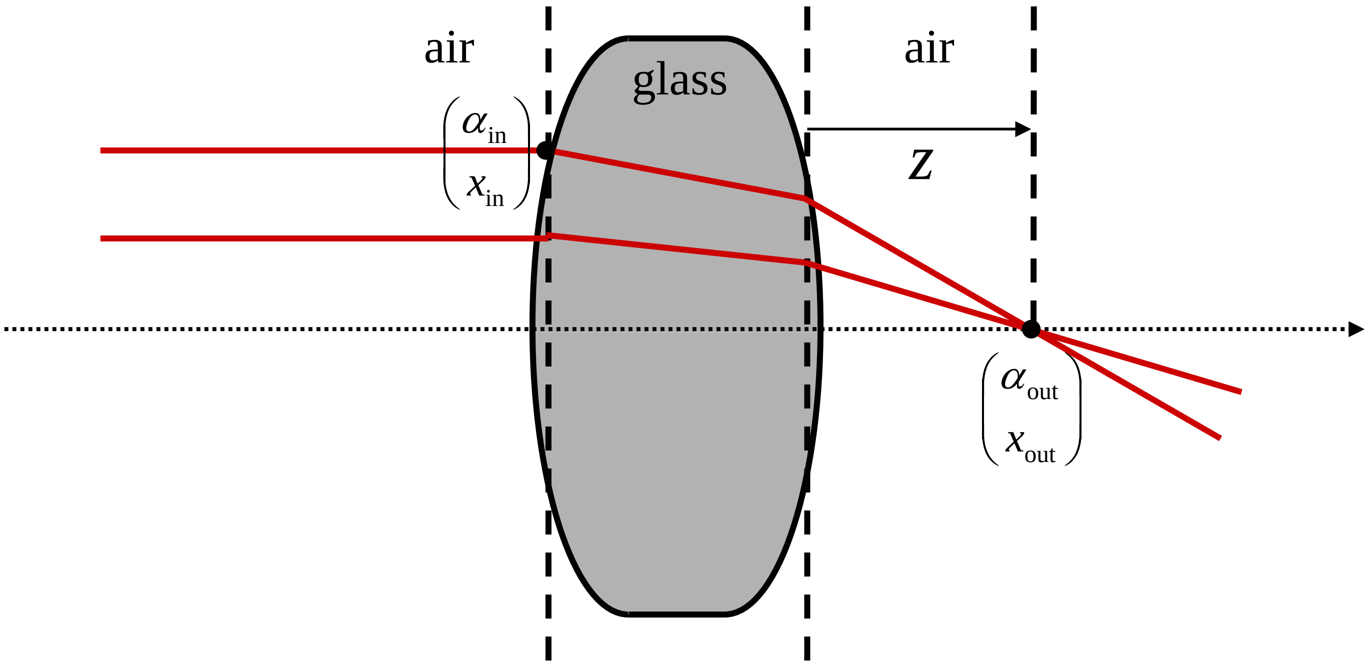
f: Effective Focal Length

The *very* thick lens



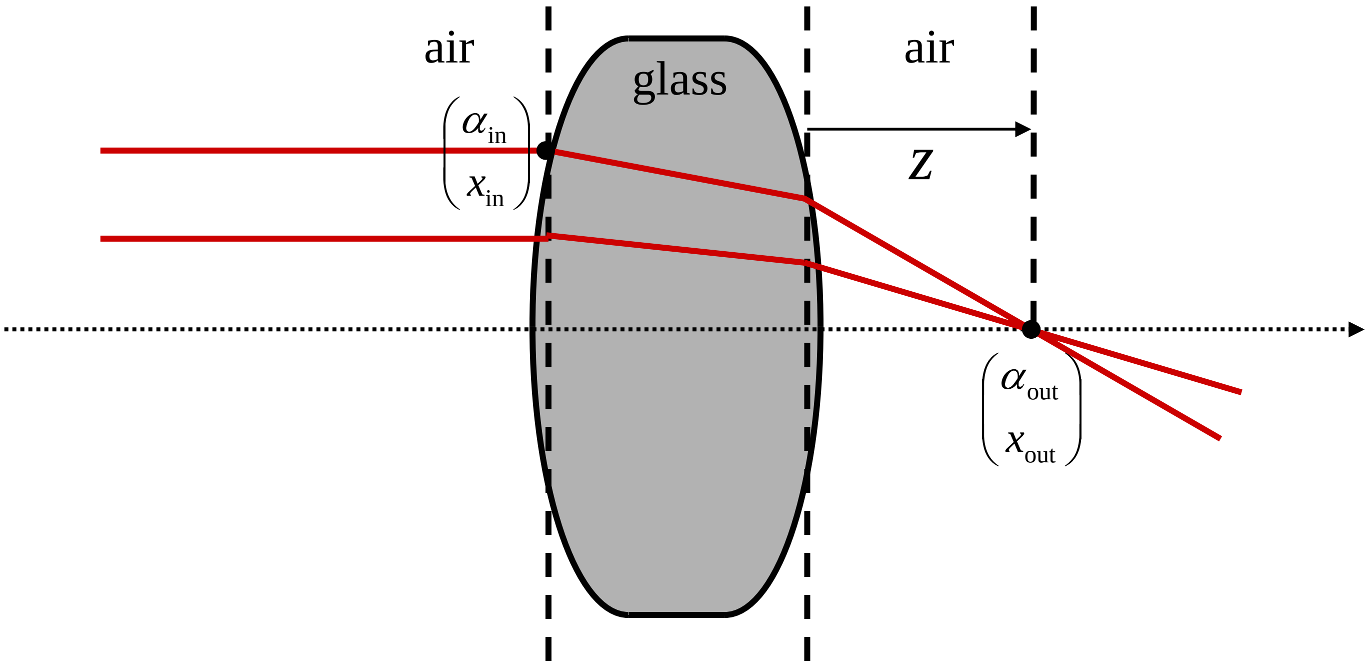
Funny things happening:
rays diverge upon exiting from the element, *i.e.*
too much positive power leading to a negative element!

The thick lens: back focal length



$$\begin{pmatrix}\alpha_{\text{out}}\\ x_{\text{out}}\end{pmatrix}=\begin{pmatrix}1 & 0\\ z & 1\end{pmatrix}\begin{pmatrix}1+\dfrac{n-1}{n}\dfrac{d}{R'} & -\dfrac{1}{f}\\ \dfrac{d}{n} & 1-\dfrac{n-1}{n}\dfrac{d}{R}\end{pmatrix}\begin{pmatrix}\alpha_{\text{in}}\\ x_{\text{in}}\end{pmatrix}$$

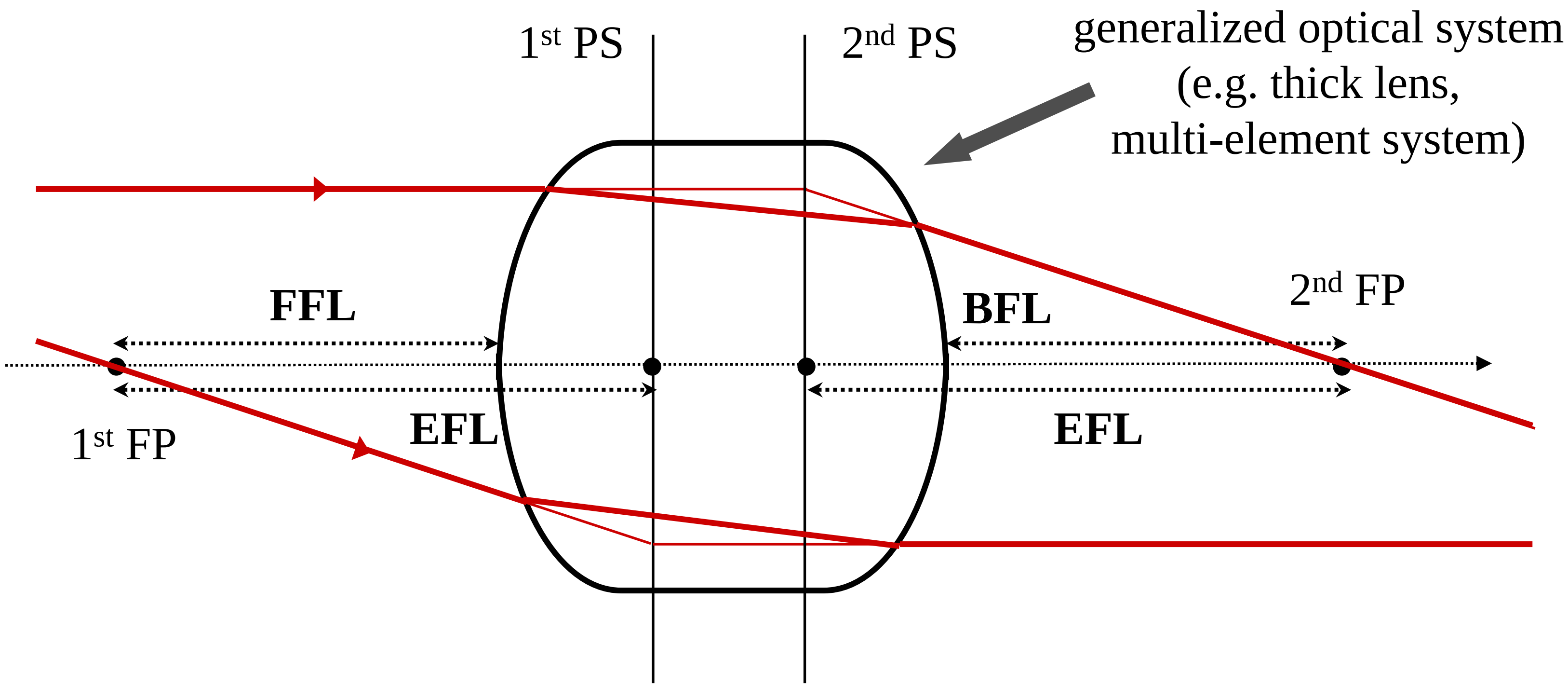
The thick lens: back focal length



$$x_{out} = 0 \Rightarrow z = f\left(1 - \frac{n-1}{n}\frac{d}{R}\right)$$

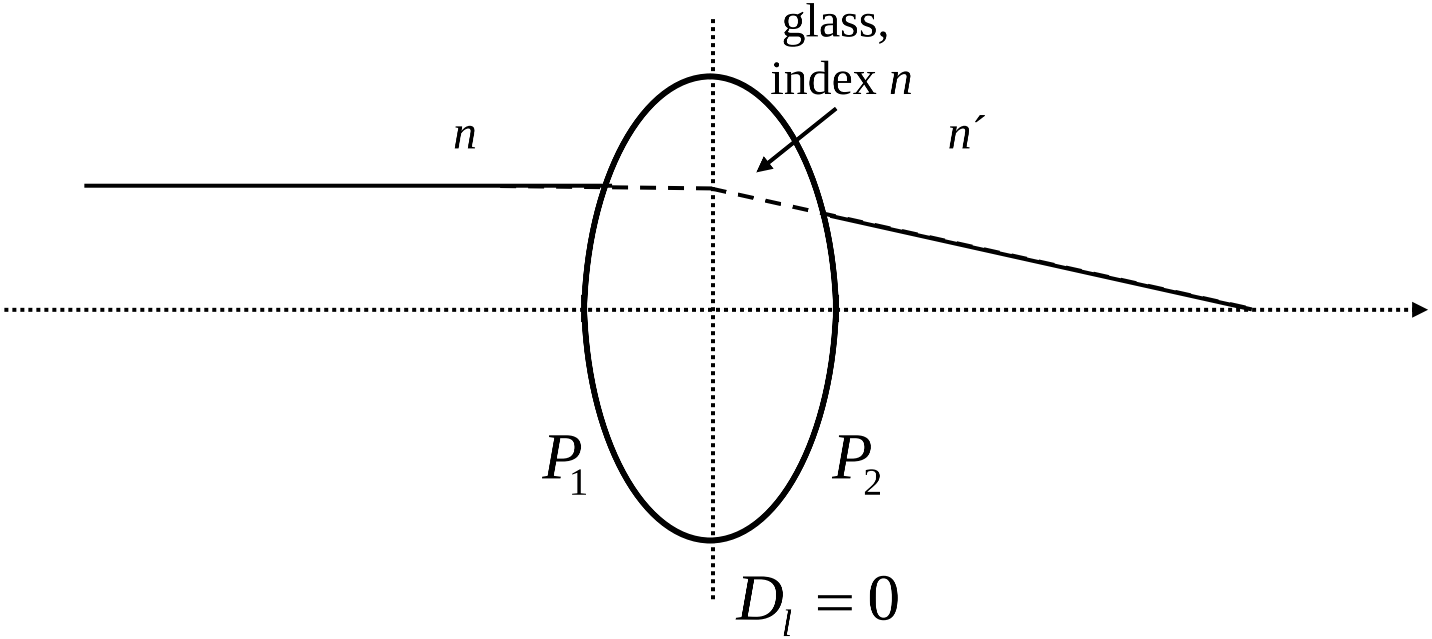
z: Back Focal Length

Focal Lengths & Principal Planes



EFL: Effective Focal Length (or simply "focal length")
FFL: Front Focal Length
BFL: Back Focal Length
FP: Focal Point/Plane **PS**: Principal Surface/Plane

PSs and FLs for thin lenses

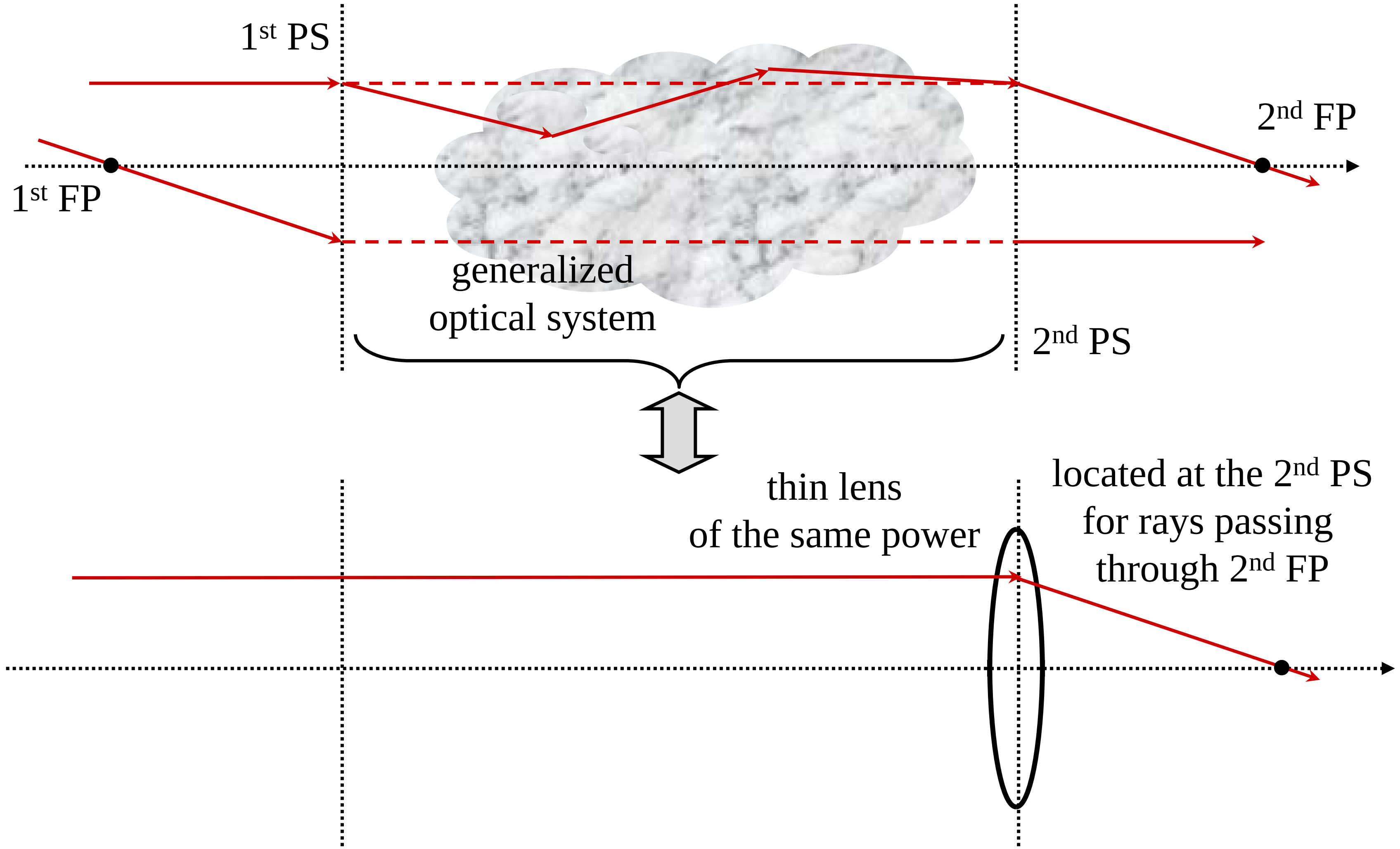


$$\frac{1}{(\text{EFL})} \equiv P = P_1 + P_2 \qquad (\text{BFL}) = (\text{EFL}) = (\text{FFL})$$

• The principal planes coincide with the (collocated) glass surfaces

• The rays bend precisely at the thin lens plane (=collocated glass surfaces & PP)

The significance of principal planes /1



The significance of principal planes /2