MIT 2.852
Manufacturing Systems Analysis
Lecture 19
Long Line Optimization

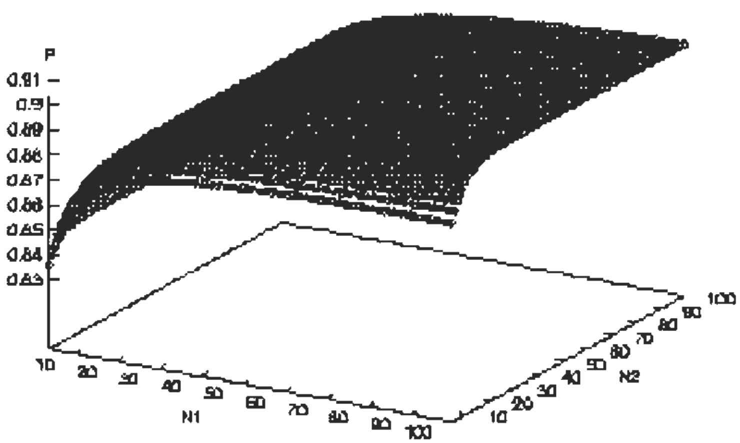
Stanley B. Gershwin

Spring, 2004

Problems

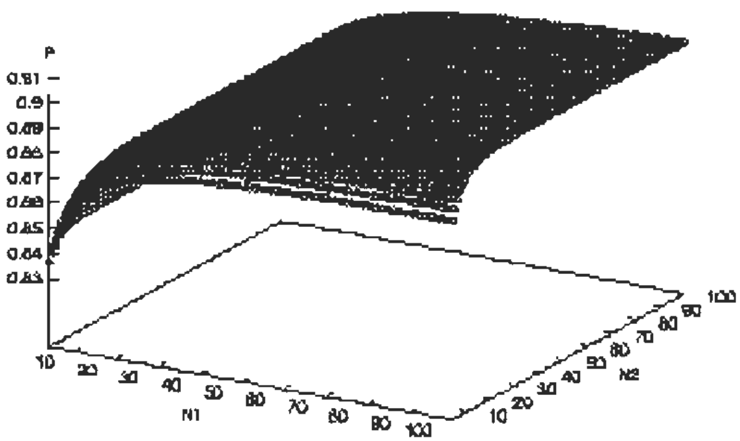
- Allocate buffer space in a production line.
- *Primal:* Minimize total buffer space required to achieve target production rate.
  - ⋆ Demand is known and limited; floor space is available but expensive.
- *Dual:* Maximize production rate subject to specified total buffer space.
  - ⋆ Demand is unlimited; floor space is limited.

P vs. (N_1, N_2)



Machine	r_i	p_i
1	0.35	0.037
2	0.15	0.015
3	0.4	0.02

P vs. (N_1, N_2)

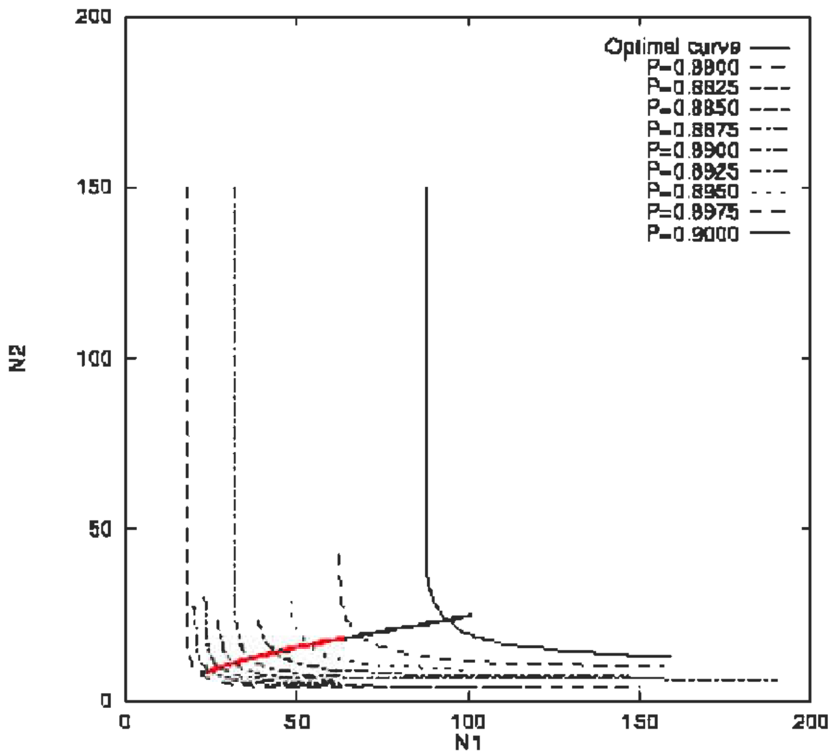


P is

- monotonically increasing,
- roughly continuous, and
- concave,

in N_1 and N_2, and P approaches a finite limit as N_1 and N_2 increase.

P vs. (N_1, N_2)



- Lines of constant P.
- Locus of optima.

Primal Problem

$$\text{Minimize } N^{\text{TOTAL}} = \sum_{i=1}^{k-1} N_i$$

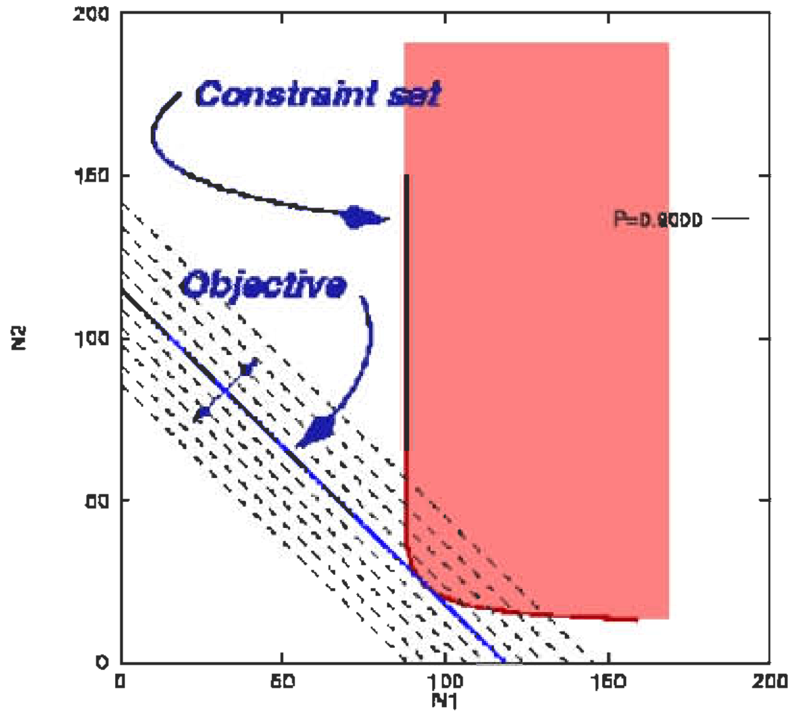
subject to

$$P(N_1, \ldots, N_{k-1}) \geq P^*; \; P^* \text{ specified}$$

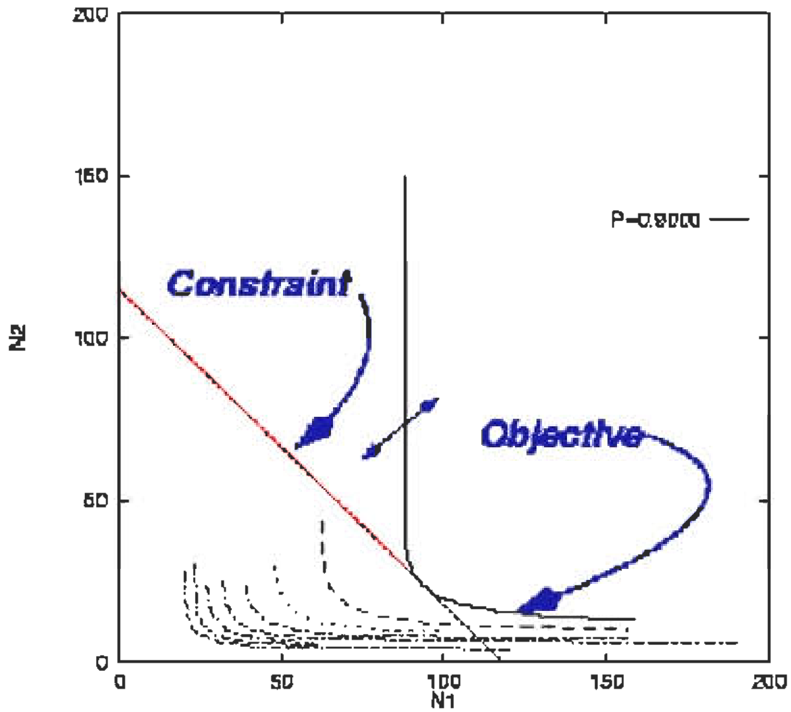
$$N_i \geq N^{\text{MIN}}, i = 1, \ldots, k-1$$

Note: Constraint set is nonlinear.

Primal Problem



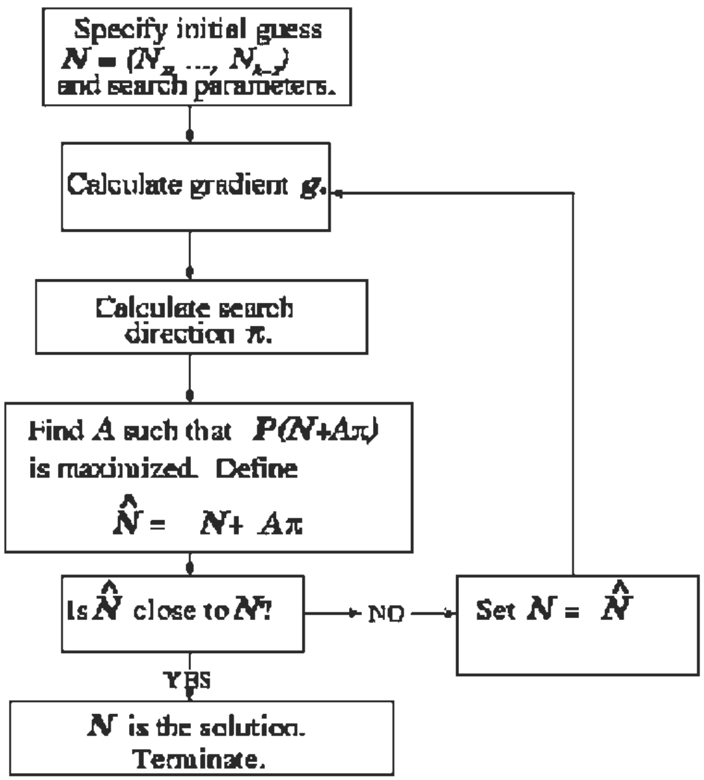
Dual Problem



- The primal and dual both have solutions at the tangent of the *same* curve to the *same* line.
- That is, every point that is a solution to *some* dual problem is also a solution *some* primal problem.
- The dual problem is easier to solve than the primal because the constraint set is linear.

Dual Problem

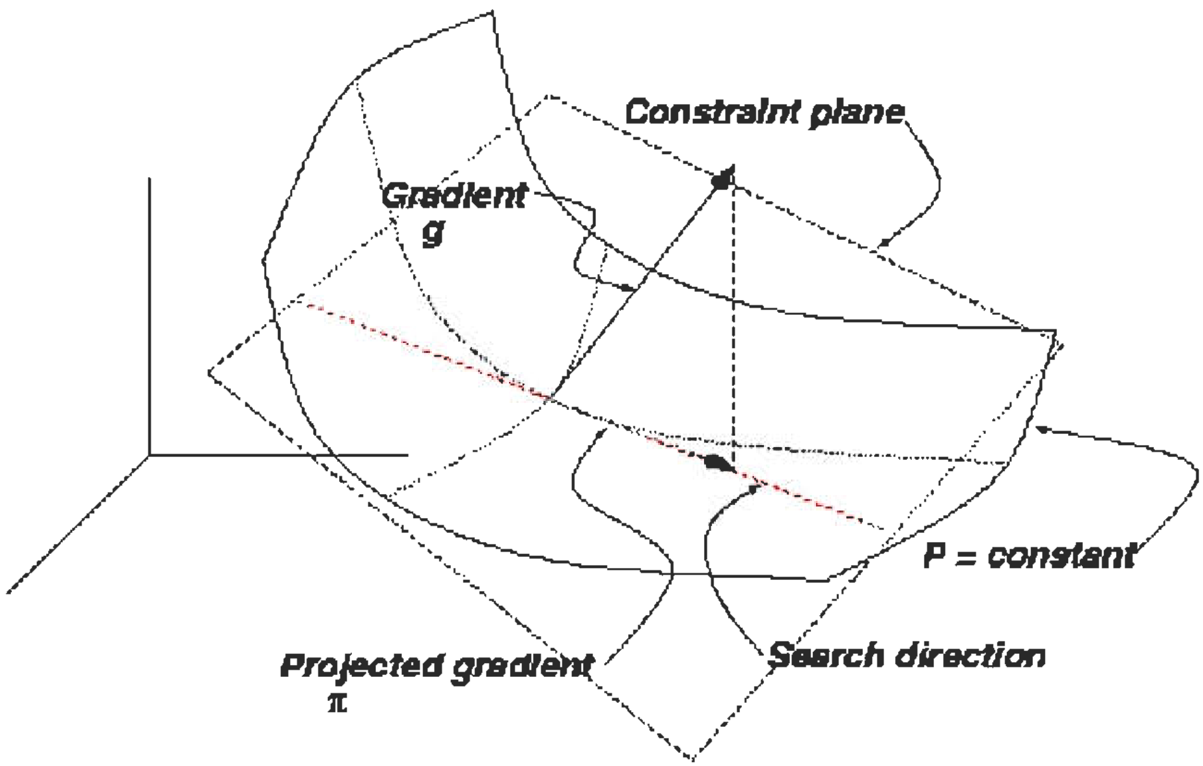
Dual algorithm



- The *search direction* lies in the constraint plane. It is the projection of the gradient.
- A is a scalar: *One-dimensional search* .

Dual Problem

Dual algorithm



Dual Problem

$$g_i = \frac{\partial P}{\partial N_i} \approx \frac{P(N_1,...,N_i+\delta N,...,N_{k-1}) - P(N_1,...,N_i,...,N_{k-1})}{\delta N}$$

$$\pi_i = g_i - \frac{1}{k-1}\sum_{j=1}^{k-1} g_j$$

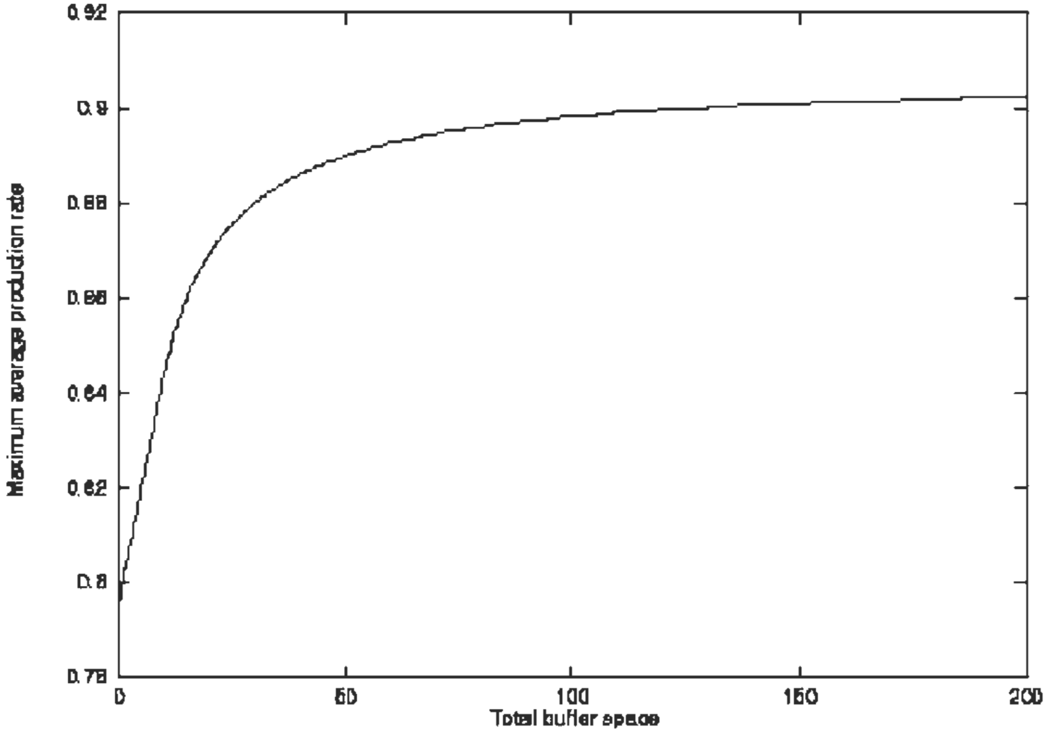
Note that

$$\sum_{i=1}^{k-1} \pi_i = 0 \quad \text{so} \quad \sum_{i=1}^{k-1}(N_i + A\pi_i) = \sum_{i=1}^{k-1} N_i = N^{\text{TOTAL}}$$

Consequently $N + A\pi$ is always on the constraint plane, for all scalar A.

Dual Problem

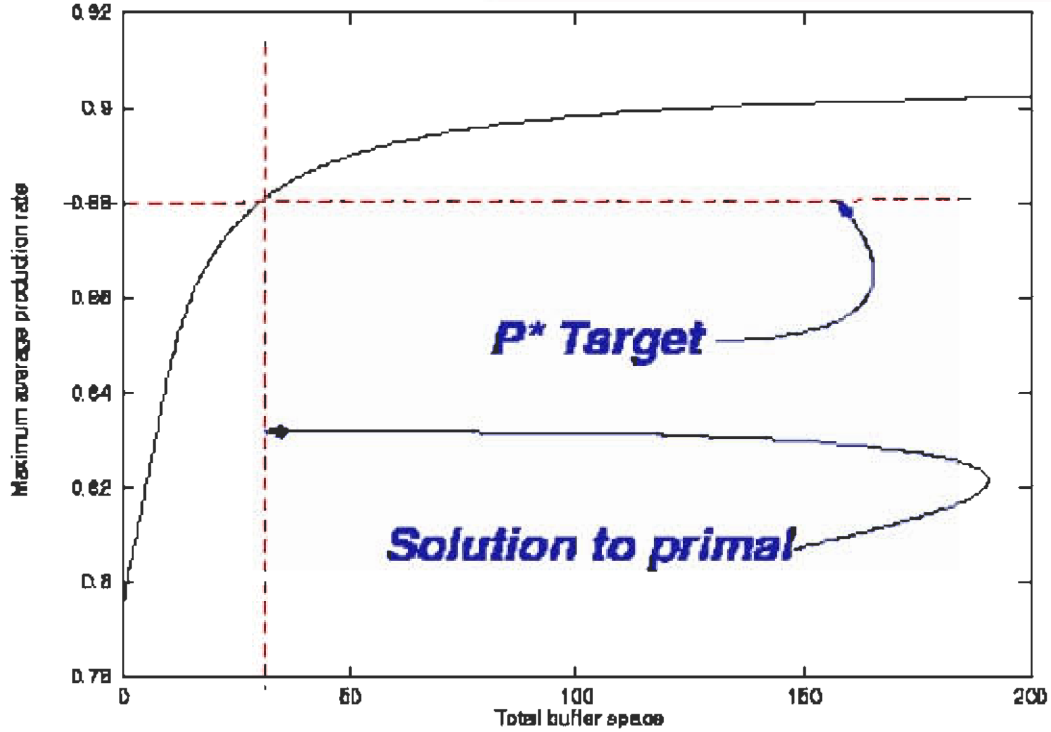
P_{MAX} vs. N^{TOTAL}



Primal

Solution

One-dimensional search on N^{TOTAL}



Primal

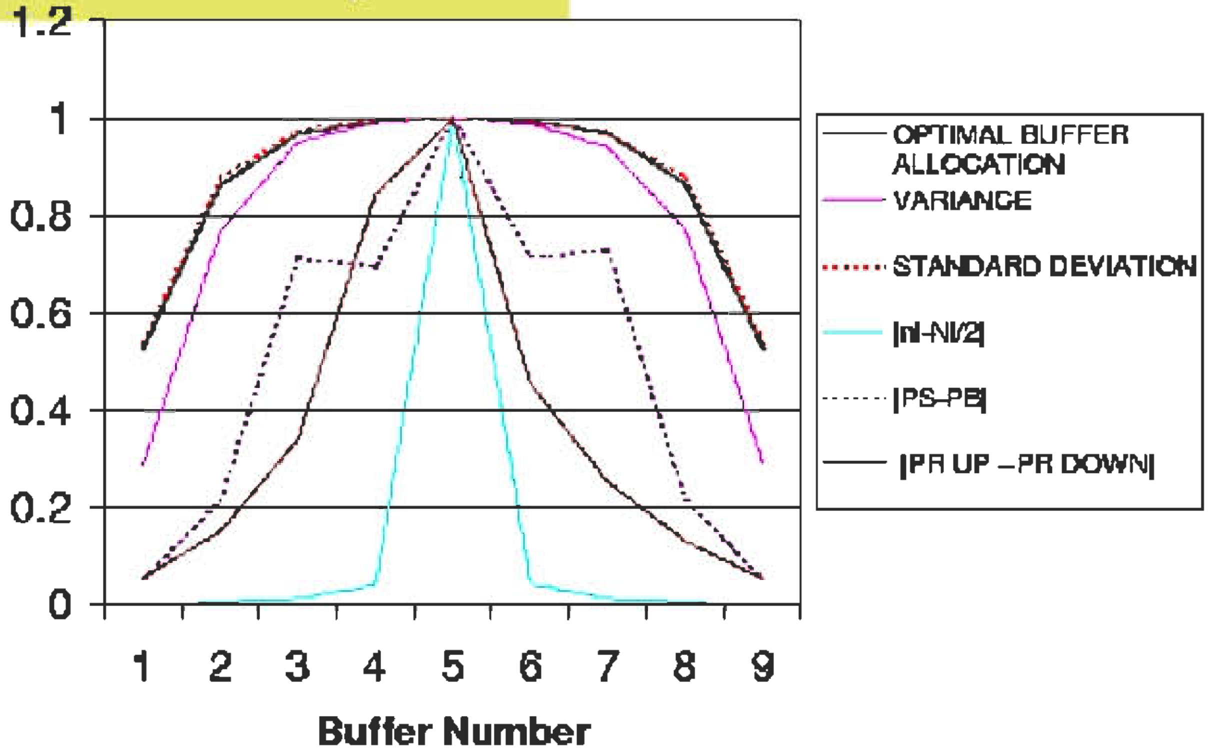
One-Dimensional Search Performance

- Algorithm produces a better optimum than traditional integer algorithms (eg, branch-and-bound), even for integer buffer sizes.
- Algorithm is also much faster.
- Algorithm works best with continuous-material model.

Optimization and Variability

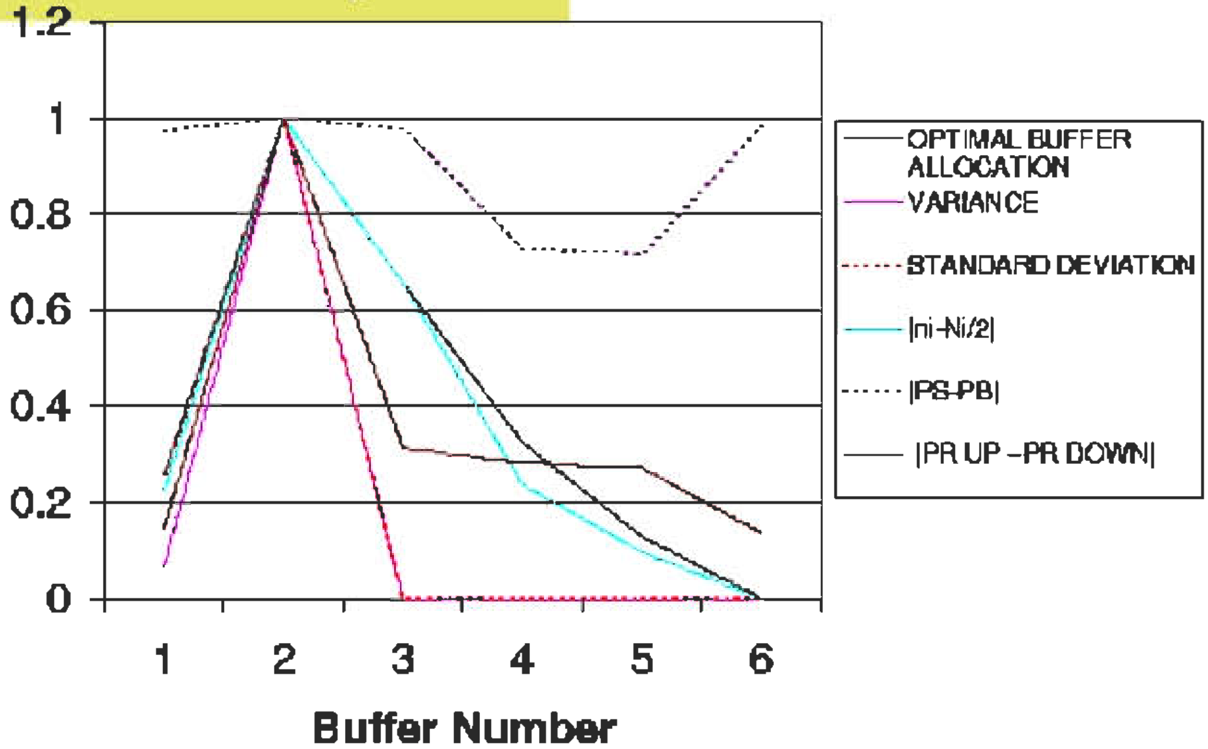
- *Hypothesis:* there is a relationship between the optimal amount of space allocated to a buffer and the variability of the amount of material in the buffer.
- We considered several measures of variability:
  - ★ variance of buffer level, s.d. of buffer level, absolute difference between average buffer level and half the buffer size, absolute difference between prob(starvation) and prob(blocking), and line imbalance around the buffer.

Optimization and Variability



10 machines, all the same.

Optimization and Variability



7 machines, all different. Machine 3 is the bottleneck.

Optimization and Variability

Observations and Conclusions

- There is a rough relationship, but not a simple proportionality between optimal allocation and any measure of variability.
- We have been investigating using one of these measures instead of the gradient to reduce computer time, but none of them make a big difference. Some optimality is sacrificed.
- Variability has been used for real-time performance improvement.