

$$X, Y \text{ independent} \rightarrow \text{cov}(X, Y) = 0$$

Assume random variables  $X$  and  $Y$  are *discrete*. That is, assume that there is a finite or denumerable sample space which is a set of  $\omega_i$  and a set of quantities  $x_i$  and  $y_i$  defined.

**Definition**  $X$  and  $Y$  are *independent* if

$$\text{prob}((X = x) \text{ and } (Y = y)) = \text{prob}(X = x)\text{prob}(Y = y)$$

in which  $x$  is some  $x_i$  and  $y$  is some  $y_j$  .

Then if  $X$  and  $Y$  are independent,

$$E(XY) = E(X)E(Y)$$

*Proof:*

$$\begin{aligned} E(XY) &= \sum_{i,j} x_i y_j \text{prob}(XY = x_i y_j) \\ &= \sum_{i,j} x_i y_j \text{prob}((X = x_i) \text{ and } (Y = y_j)) \\ &= \sum_{i,j} x_i y_j \text{prob}(X = x_i) \text{prob}(Y = y_j) \\ &= \sum_i x_i \text{prob}(X = x_i) \sum_j y_j \text{prob}(Y = y_j) = E(X)E(Y) \end{aligned}$$

Then if  $X$  and  $Y$  are independent,

$$\begin{aligned} \text{cov}(X, Y) &= E[(X - E(X))(Y - E(Y))] \\ &= E[XY - XE(Y) - YE(X) + E(X)E(Y)] \\ &= E[XY] - E(X)E(Y) - E(Y)E(X) + E(X)E(Y) = 0 \end{aligned}$$

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