SMA 6304 / MIT 2.853 / MIT 2.854
Manufacturing Systems
Lecture 8: Inventory II

Lecturer: Stanley B. Gershwin

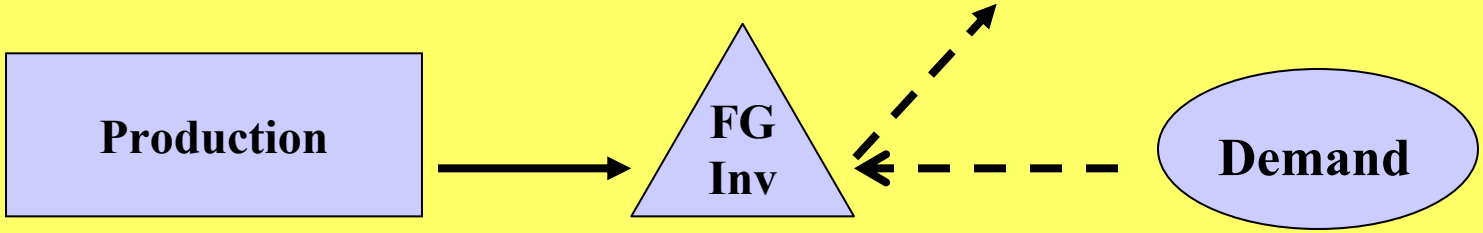
Slides by Larry Wein and Stan Gershwin

Base Stock Policy

If set up cost $S = 0$, then intuitively we set $Q = 1$

Let $S = R + 1 =$ base stock level

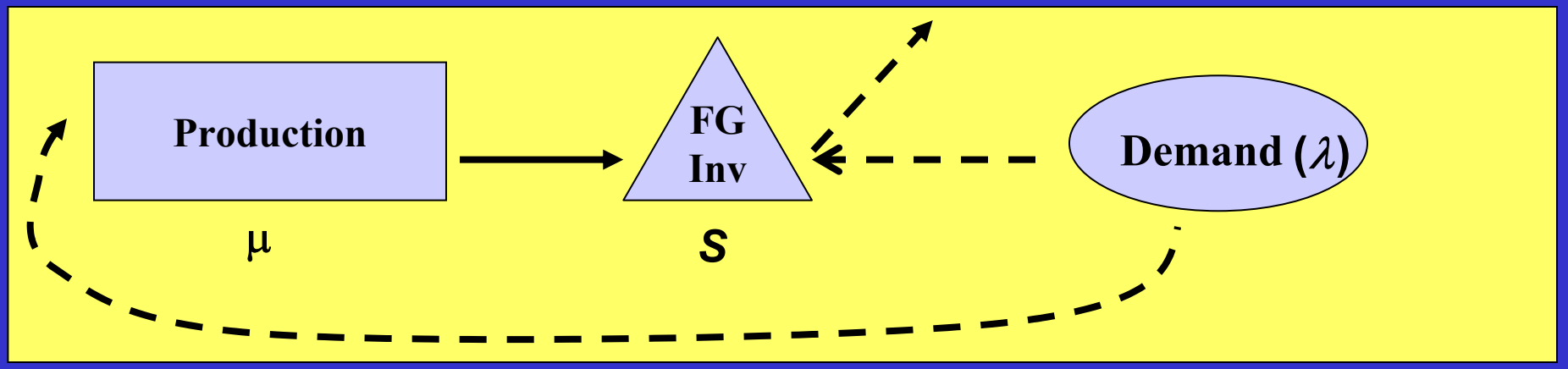
When inventory drops to $S - 1$ units, order 1 unit



- If manufacturer supplies many retailers, then lead times should be independent of retailer's ordering policy
- This is assumed in inventory theory
- Production facility is modeled as a $\bullet/M/\infty$ queue

- But if manufacturer supplies only one retailer, then lead times depend on the retailer's ordering policy
- This is assumed in queueing theory
- Production facility is modeled as a $\bullet/M/1$ queue
- Waiting time W in queueing system = lead time in inventory system

Make-to-stock Queue



$I(t)$ = *FG* inventory level

$Q(t)$ = items in production queue

$I(0) = S$

$Q(0) = 0$

$I(t) + Q(t) = S$ for all $t \geq 0$

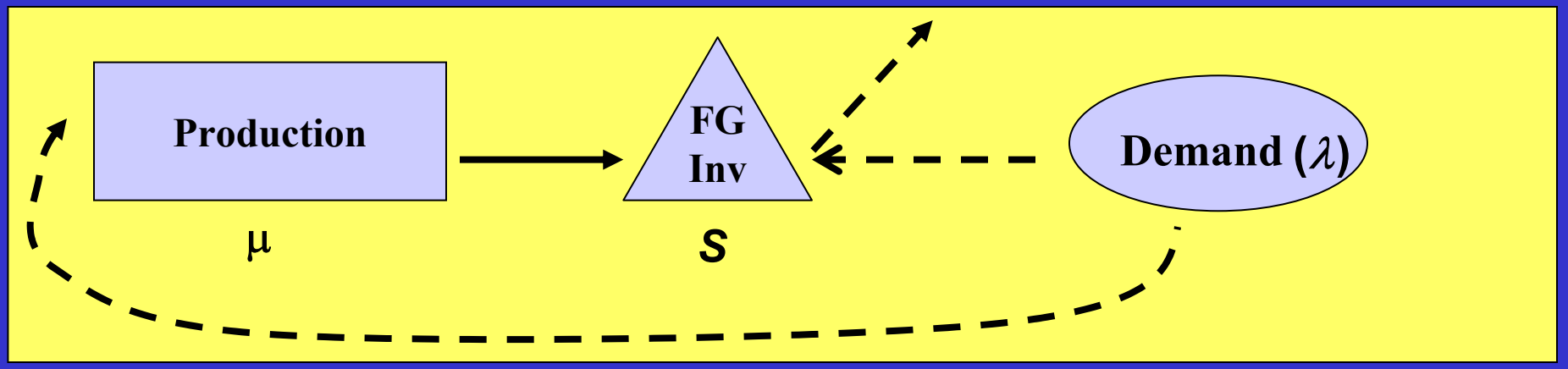
I < 0 possible!! *Backlog!!*

Poisson demand and exponential production times

$Q(t)$ = # in M/M/1 queue

$\Pr(Q(\infty) = n) = \rho^n (1 - \rho)$ geometric

Make-to-stock Queue Cont.



Instead, assume $Q(\infty) \sim$ exponential with mean $L = \frac{\rho}{1-\rho}$

Let b = backorder cost per unit inventory per unit time

h = holding cost per unit inventory per unit time

$$I(\infty) \sim f(x) = \begin{cases} L^{-1} e^{L^{-1}(x-S)} & \text{if } x \le S \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Cost} = C(S) = -\int_{-\infty}^{0} bxL^{-1}e^{-L^{-1}(x-S)}dx + \int_{0}^{S} hxL^{-1}e^{-L^{-1}(x-S)}dx$$

$$\frac{dC(S)}{dS} = 0 \Rightarrow S^* = \frac{\rho}{1-\rho}\ell n\left(1+\frac{b}{h}\right)$$

Make-to-Stock Queue

Base Stock Policy

Note that

$$\text{Prob}(q \leq Q) = F(Q) = 1 - e^{-Q/L}$$

so

$$\text{Prob}(S^* - q \geq S^* - Q) = 1 - e^{-Q/L}$$

so

$$\text{Prob}(i \geq I) = 1 - e^{-(S^* - I)/L}$$

where $I = S^* - Q, i = S^* - q$, or

$$\text{Prob}(i \leq I) = e^{-(S^* - I)/L}$$

Make-to-Stock Queue

Base Stock Policy

Probability of backlog is

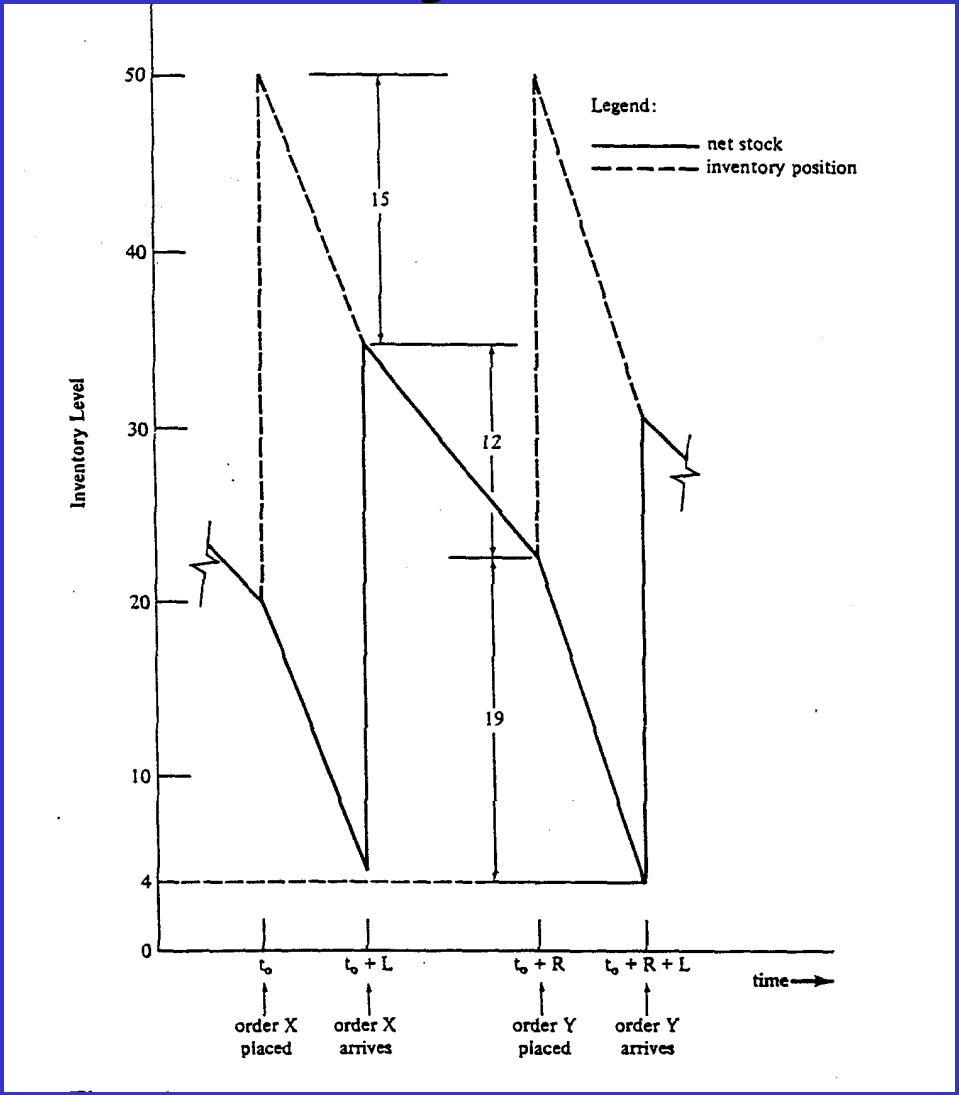
$$\text{Prob}(i \leq 0) = e^{-S^*/L} = e^{-\frac{1}{L}\{\frac{\rho}{1-\rho}\ln\{\frac{b+h}{h}\}\}}$$

$$= e^{-\{\ln\{\frac{b+h}{h}\}\}} = \frac{h}{b+h}$$

and this is *exactly* the same as the solution to the newsvendor problem when

$h = r - c$ and $b = c - s$

Inventory Position



Key insight: the current order must satisfy demand until the next order arrives

(S,T) Policy

Every T time units, replenish your inventory position to S units

Let $T = \frac{EOQ}{\lambda}$

Let *DDLTRP* = demand durning lead time plus review period

- **We need *S* units to satisfy *DDLTRP***
- **Use newsboy model**

Set *S* so that $\Pr(DDLTRP \leq S) = \frac{r - c}{r - s}$

Example

L = deterministic lead time

$L_r = L + T = LT + RP$

D_i = *iid* demand in each period with mean $E[D]$ and variance $\sigma^2[D]$

$$DDLTRP = D_1 + \ldots + D_{L_r}$$

$$E[DDLTRP] = L_r E[D]$$

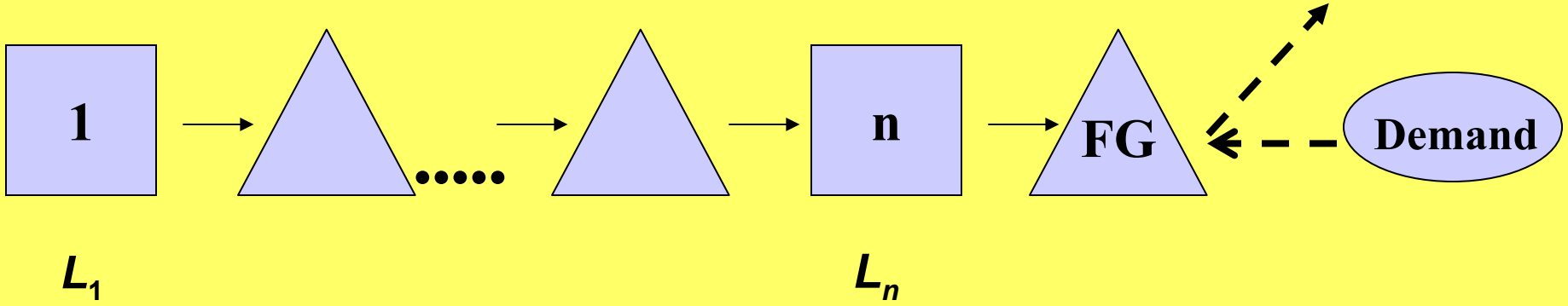
$$Var[DDLTRP] = L_r \sigma^2[D]$$

Set S so that $\Pr(DDLTRP) \leq S$

$$= \Pr\left(\frac{DDLTRP - L_r E[D]}{\sqrt{L_r}\,\sigma[D]} \leq \frac{S - L_r E[D]}{\sqrt{L_r}\,\sigma[D]} \right)$$

$$= \frac{r - c}{r - s}$$

Multi-echelon Inventory Systems



Question: How should each station replenish its inventory?

Answer: Track echelon inventory = total downstream inventory, but base stock levels are difficult to compute

Information Inaccuracy in Inventory Systems

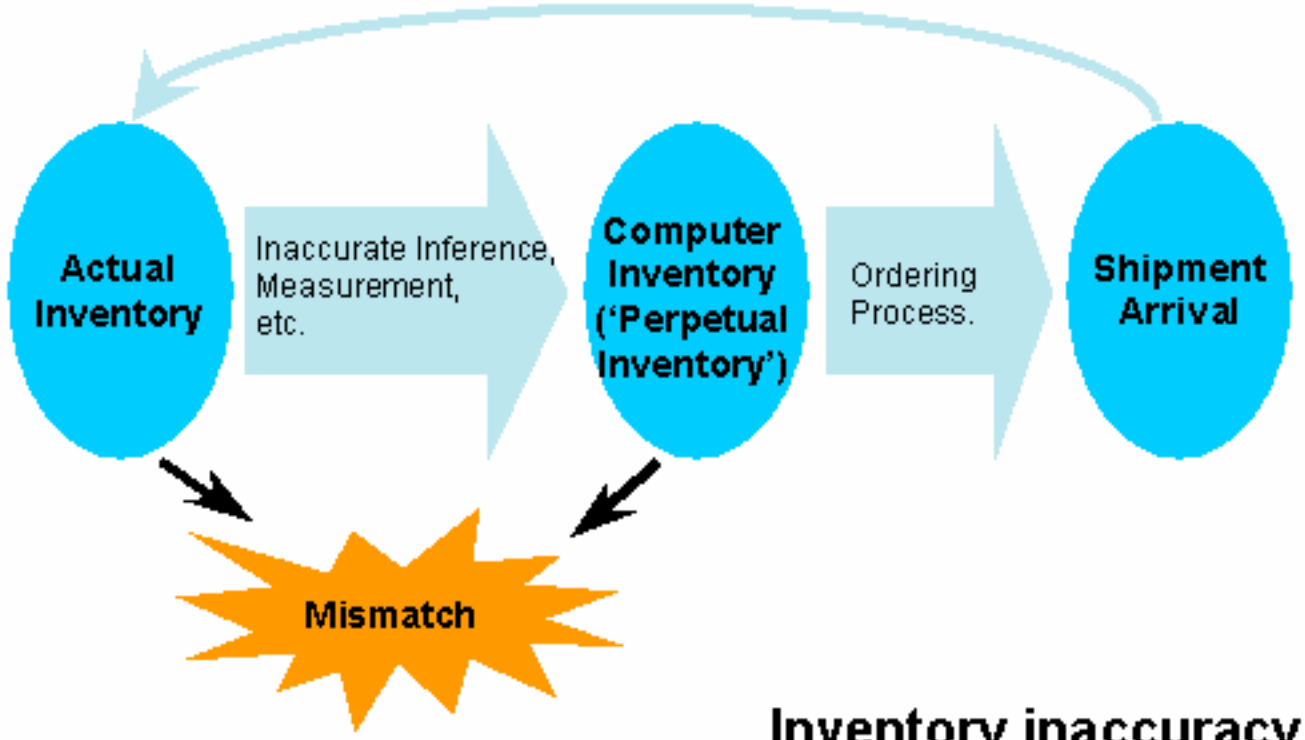
Yun Kang
Stanley B. Gershwin

Auto-ID Center Academic Alliance Meeting

June 19th, 2003

What is Inventory Inaccuracy?

Typical Ordering Cycle



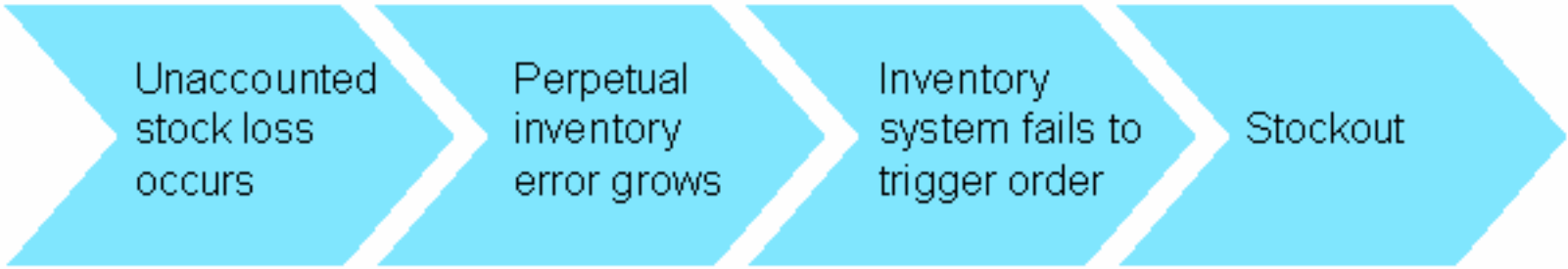
Inventory inaccuracy is the discrepancy between the actual and the estimate

Causes of Inventory Inaccuracy

- Unknown, unreported stock loss ('shrinkage')
  - Theft (by shoppers and employees)
  - Spoilage/damage & others
- Wrong shipment and receipt verification
- Checkout errors
- Mis-labeling (on items, cases, pallets, shelves, …)
- Inaccessible, misplaced inventory
- Incorrect manual counts
- …

Why is Inventory Inaccuracy Bad?

- Inaccuracy causes either higher stockout and/or higher inventory
- In case of unknown stock loss:



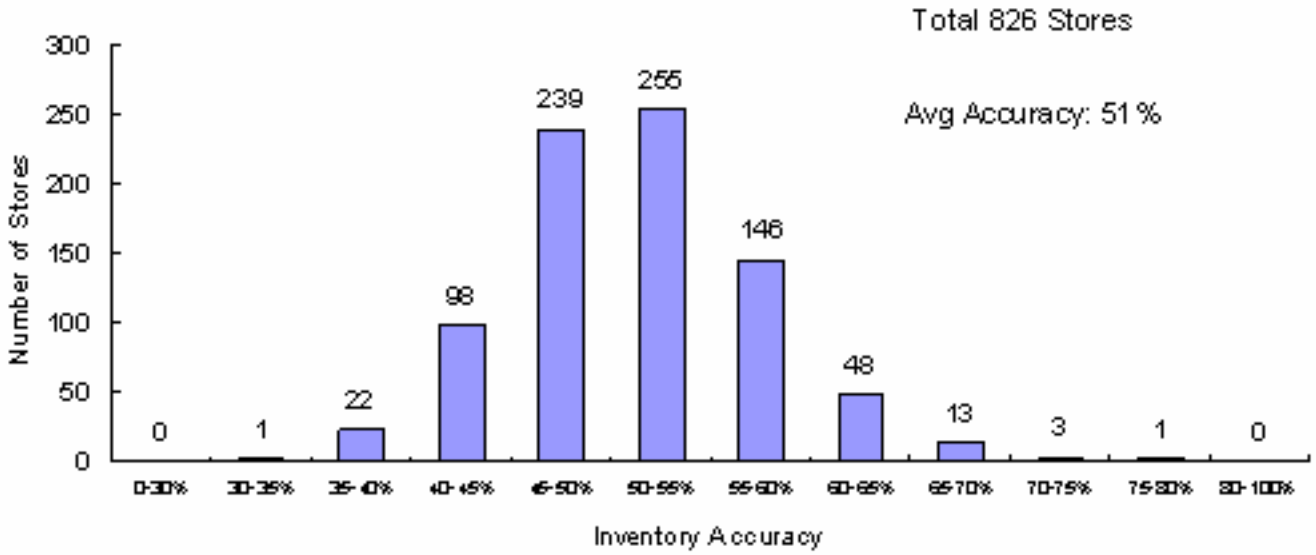
Inventory Inaccuracy in Real-Life

A North American Retailer

Definition of Inventory Accuracy:

% of SKUs in a store whose perpetual inventory matches the actual inventory perfectly

Avg Accuracy: 51%



Using alternative definition:

% of SKUs in a store whose perpetual inventory matches the actual inventory within ±5 items

Avg Accuracy: 76%

Impact of Inaccuracy in Absence of Compensation

Reorder Point Policy

- Reorder Point Policy
  - A commonly-used inventory policy
  - Proven to be optimal under perfect state information
  - Used typically for type A inventory

- Mechanisms of this policy:
  - Daily review of the perpetual inventory
  - If the perpetual inventory is below the reorder point R, order amount Q.
  - After lead time L, the shipment arrives

Impact of Inaccuracy in Absence of Compensation

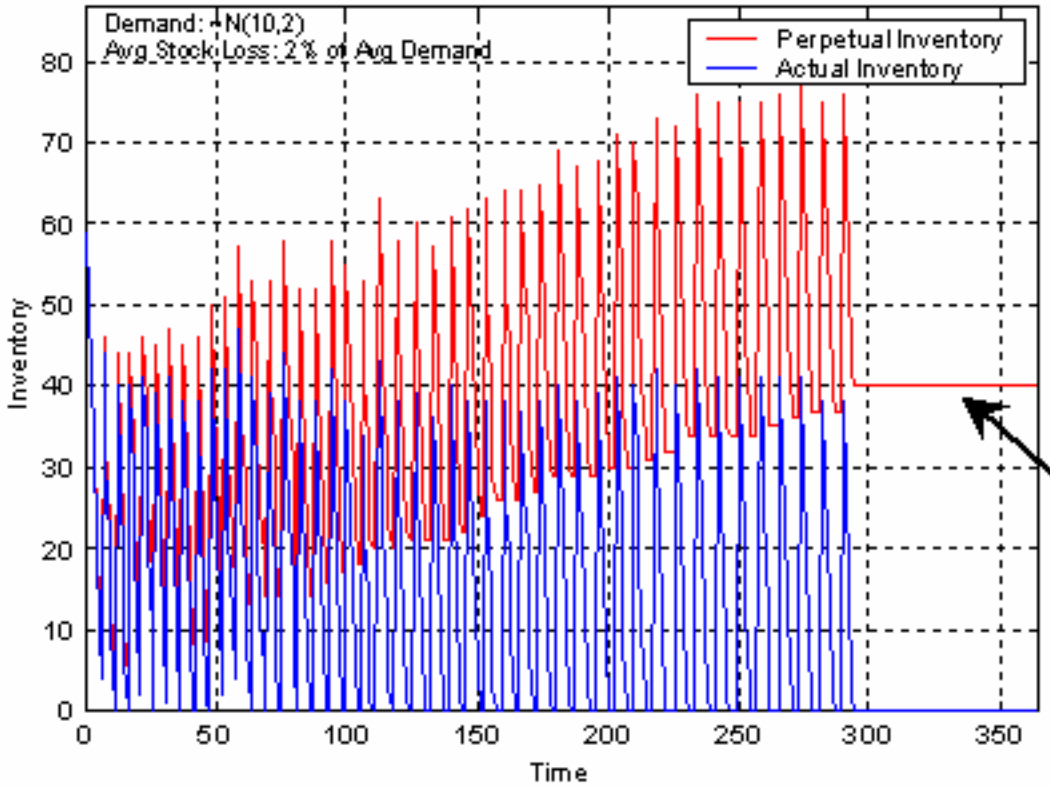
Reorder Point Policy Simulation Study

- Assumptions
  - A facility carries a single item inventory
  - It fills the daily demand which is normally distributed: ~$N(10,2)$
  - Avg stock loss is 2% of avg demand (.2 items per day) and is generated from Poisson distribution
  - Reorder point R is set at 39, which achieves 1% stockout in the absence of stock loss
  - Supplier lead time L is 3 days.
  - Order qty Q is 50 items.
  - Operating time is 365 days

Impact of Inaccuracy in Absence of Compensation

Reorder Point Policy Simulation Study

A Sample Simulation Run



Stockout: 40.2 %

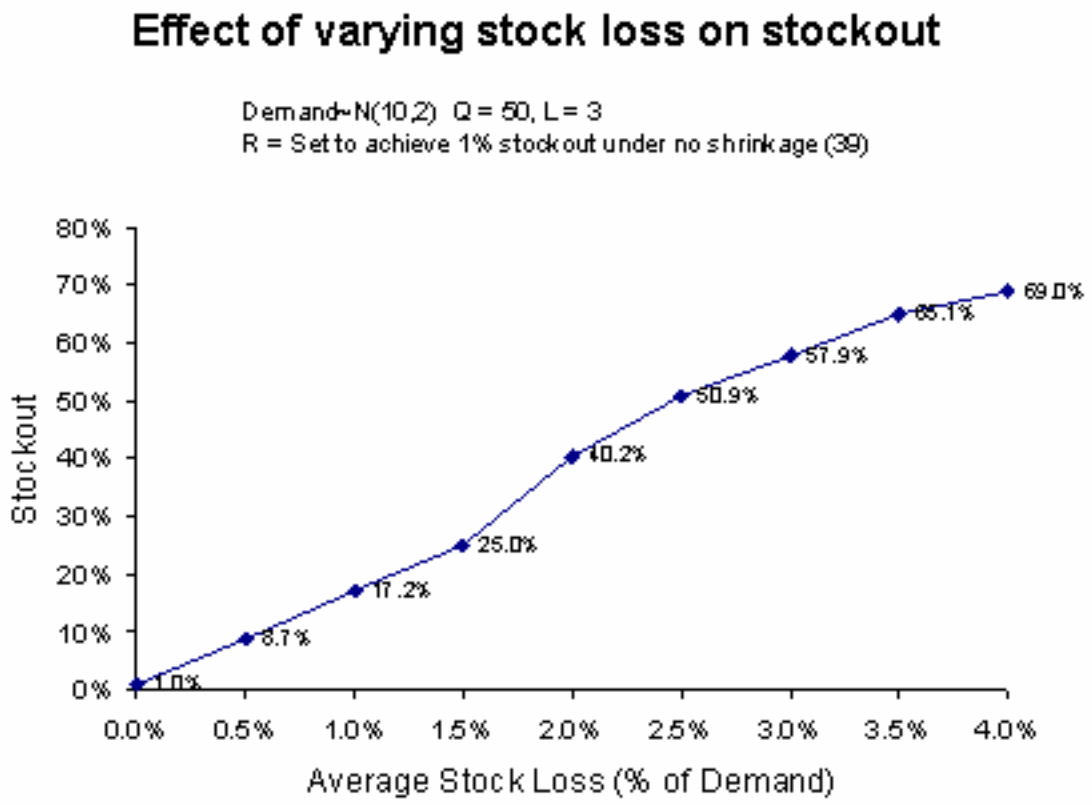
Definition:

$$\text{Stockout} = \frac{\#\text{ of lost sales items}}{\text{total demand}} \times 100$$

'Freezing' of system

Impact of Inaccuracy in Absence of Compensation

Reorder Point Policy Simulation Study

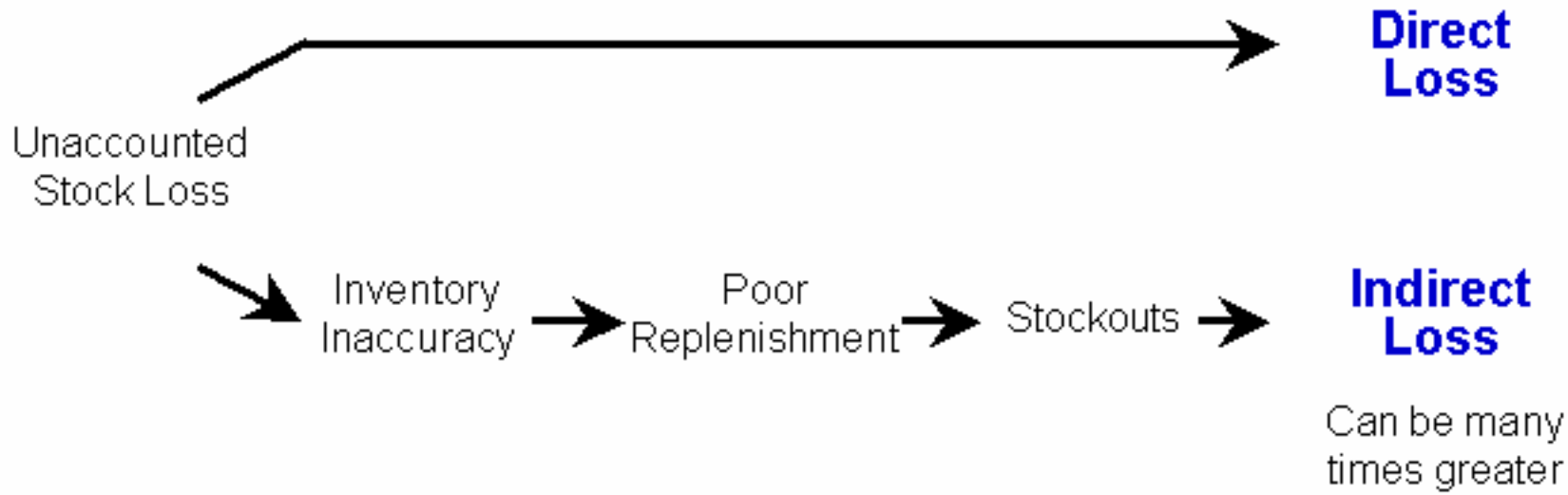


Inventory system performance is highly sensitive to the rate of stock loss

Impact of Inaccuracy in Absence of Compensation

Reorder Point Policy Simulation Study

Managerial insight



Losses due to inventory errors can be 10-20 times higher than the stock loss itself

Summary of Research Insights

Inv Inaccuracy in Absence of Compensation

- *Inventory system performance is highly sensitive to the rate of stock loss*
- *Losses due to inventory errors can be much higher than the shrinkage loss*
- *The impact of inv inaccuracy is higher in 'lean' environments*
- *Some policies are more sensitive to inventory inaccuracy than others*

Inv Inaccuracy with Compensation

- *If the stochastic behavior of stock loss is well known, performance can be significantly increased even without Auto-ID. But….at a cost.*
- *By knowing the dynamics of the system and the stochastic behavior of the stock loss, Auto-ID with measurement error can reach performance level close to the perfect state info case.*
- *It comes, however, at the price of complex ordering policy.*
- *The poorer the measurement performance, the more the performance will depend on the system-specific policy.*