

## MAS 160/510 Problem Set Seven

1. **Impulse response of a z-transform** Consider a causal linear shift invariant system with system function

$$H(z) = \frac{1 - a^{-1}z^{-1}}{1 - as^{-1}}$$

where  $a$  is real

- (a) Write the difference equation that relates the input and output of this system.
- (b) For what range of values of  $a$  is the system stable?
- (c) Find the impulse response  $h[n]$  for the system.

2. **Partial Fractions in Z-transforms** Listed below are three z-transforms

- (a)  $x_1(z) = \frac{1}{1 + \frac{1}{2}z^{-1}}$  for  $|z| < \frac{1}{2}$
- (b)  $x_2(z) = \frac{1 - 2z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})}$  for  $|z| > \frac{1}{2}$
- (c)  $x_3(z) = \frac{1 - 2z^{-1} + z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$  for  $|z| > \frac{1}{2}$

For each of the z-transforms above, determine the inverse z-transform using partial fraction expansion.

- (d) Use the MATLAB function `residuez` to verify the partial fraction expansions you had obtained

3. **Inverse z-Transforms** The z-transforms below represent the systems of functions of stable linear shift invariant systems

$$H_1(z) = \frac{z-5}{z-\frac{5}{2}}$$

$$H_2(z) = \frac{z-\frac{1}{2}}{z}$$

$$H_3(z) = \frac{z-5}{5z}$$

$$H_4(z) = \frac{z^2}{(z+0.7e^{j\frac{\pi}{4}})(z+0.7e^{-j\frac{\pi}{4}})}$$

For each transform,

- (a) Sketch the pole-zero diagram
- (b) Write the difference equations that characterize the system.
- (c) Write the z-transforms of the inverse systems  $\frac{1}{H_i(z)}$  of each of the above systems, by inspection. Which of these inverse systems could be causal and stable?
- (d) Use the MATLAB function `zplane` to verify the pole and zero locations you had obtained in (a). (You will need to divide the numerator and denominator of the transforms by the same power of  $z$  to ensure that the polynomial in the z-transforms are in terms of powers of  $z^{-1}$ .)

4. **Relating impulse-response to D-to-C conversion** *DISP First 8.19*

5. **Relating pole-zero plots to frequency- and impulse-response**

- (a) *DISP First 8.13*
- (b) *DISP First 8.14*