Radioactive Decay

Activity:	the number of atoms that decay per unit time: (disintegrations per second).
Units:	**Becquerel (Bq)** = 1 dps **Curie (Ci)** [old unit] = 3.7×10^{10} Bq exactly (originally defined as the activity of 1.0 g of radium

Exponential Decay:

Activity (A) of a radioactive nuclide decreases ***exponentially*** with time.

Let N = # atoms present $\qquad dN = -\lambda N\, dt$

The constant of proportionality, λ, has units of sec^{-1}.

$$A = \frac{-dN}{dt} = \lambda N$$

Each radioactive nuclide has a unique decay constant λ.

$$\frac{dN}{N} = \lambda dt \qquad \int \frac{dN}{N} = -\lambda \int dt$$

$\ln N = -\lambda t + c$ $\qquad$ When $t = 0$, N_0 atoms are present - implies that $\ln N_0 = c$

$$\ln N = -\lambda t + \ln N_0$$

$$\ln \frac{N}{N_0} = -\lambda t \qquad \frac{N}{N_0} = e^{-\lambda t} \qquad \textbf{or } \mathbf{N = N_0\, e^{-\lambda t}}$$

$$\textbf{or } \mathbf{A = A_0\, e^{-\lambda t}}$$

Half-Life ($t_{1/2}$ or T)

When N = $\frac{1}{2} N_o$ $$\frac{\frac{1}{2} N_0}{N_0} = e^{-\lambda t_{1/2}} \qquad \frac{1}{2} = e^{-\lambda t_{1/2}}$$

$- \ln 2 = - \lambda t½$ $0.693 = \lambda t½$

$$\lambda = \frac{0.693}{t½} \qquad t_{1/2} = \frac{0.693}{\lambda}$$

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Fig. 4.1 in Turner J. E. *Atoms, Radiation, and Radiation Protection*, 2nd ed. New York, NY: Wiley-Interscience, 1995.

Specific Activity

Specific Activity (SA) defined as ***activity per unit mass.***

Units: $\frac{\text{Bq}}{\text{g}}$ or $\frac{\text{Ci}}{\text{g}}$

$A = \lambda N$ N = # of atoms

$$\frac{N}{g} = \frac{6.02x10^{23}\,\frac{atoms}{mole}}{\text{M}\,\frac{\text{grams}}{\text{mole}}} = \frac{\text{atoms}}{\text{g}}$$

$$\text{SA} = \frac{\text{A}}{\text{g}} = \frac{\lambda\text{N}}{\text{g}} \qquad SA = \frac{6.02x10^{23}}{M}\lambda$$

Example: Specific activity of radium

$\boldsymbol{M =}\ 226\,\frac{\text{g}}{\text{mole}}$ ***t½ = 1600 y*** $\lambda = \frac{.693}{\text{t½}}$

$$\text{SA} = \frac{\left(6.02x10^{23}\,\frac{\text{atoms}}{\text{mole}}\right)}{226\,\frac{\text{g}}{\text{mole}}}\left(\frac{.693}{1600y}\right)\left(\frac{1y}{365d}\right)\left(\frac{1d}{24h}\right)\left(\frac{1h}{60\,\text{min}}\right)\left(\frac{1m}{60\,\text{sec}}\right)$$

$$\text{SA} = \frac{3.66x10^{10}\,atoms}{g\cdot\text{sec}} = \frac{3.66x10^{10}\,Bq}{g}$$

1 Ci = 3.66 x 10^{10} dps

1 Ci orig. defined as activity associated with 1 g of Radium.
Ci is now defined as 3.7 x 10^{10} dps exactly.

Count rates - vs half-life

Example: Compound A: t½ = 45 min
Compound B: t½ = 45 years

Given 10^{10} atoms of each - find the activity (A)

$$A = \lambda N \qquad \lambda = \frac{0.693}{t_{1/2}} \qquad [\lambda = 2.5 \times 10^{-4}\ \text{sec}^{-1}]$$

$$A_A = \frac{0.693}{(45\,\text{min})\left(\frac{60\,\text{sec}}{\text{min}}\right)}\ 10^{10}\ \text{atoms}$$

$A_A = 2.56 \times 10^6$ Bq

$$A_B = \frac{0.693}{(45y)(365)(24)(60)(60)}\ 10^{10} \qquad [\lambda = 4.8 \times 10^{-10}\ \text{sec}^{-1}]$$

$A_B = 4.8$ Bq

^{239}Pu t½ = 24,065 y ^{235}U t½ = 7.038×10^8 y

Serial Radioactive Decay

$N_1 \rightarrow N_2$ N_{10} = # parent atoms present at t = 0.
N_{20} = # daughter atoms present at t = 0.

General Case

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \Rightarrow\Rightarrow A_2 = A_{10}\frac{\lambda_2}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) + A_{20}e^{-\lambda_2 t}$$

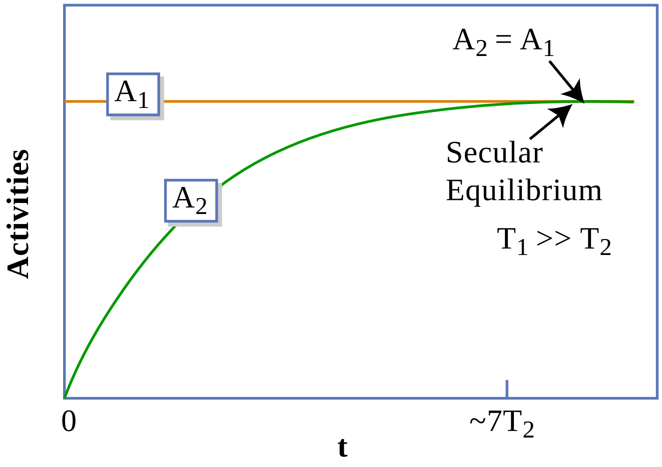
Secular equilibrium ($T_1 >> T_2$)

Simplifying assumptions: $A_{20} = 0$

T_1 is large, ∴ λ_1 is small; $\lambda_2 - \lambda_1 = \lambda_2$ $e^{-\lambda_1 t} \cong 1$

General Case simplifies to $A_2 = A_{10}(1 - e^{-\lambda_2 t})$

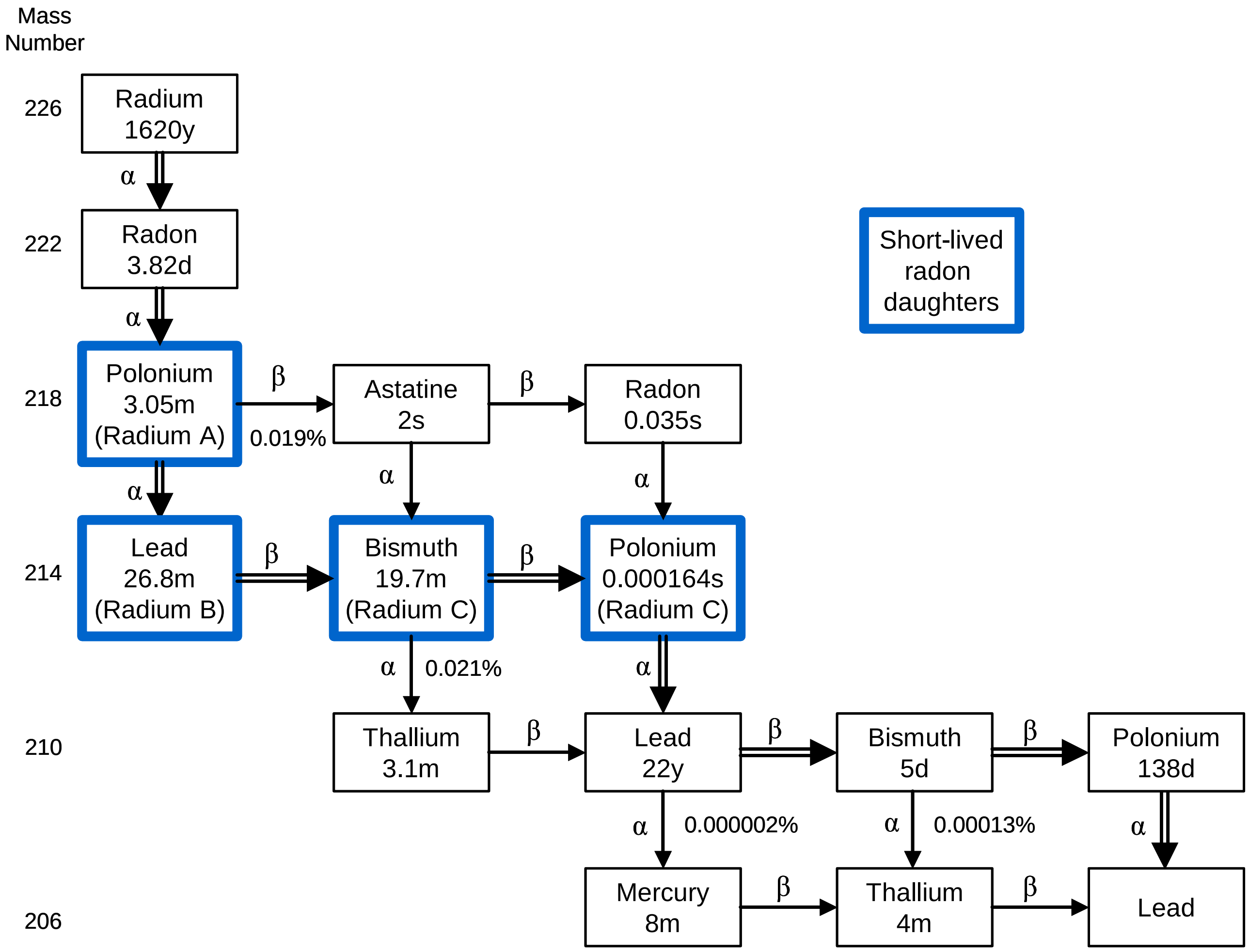
after ~ seven half-lives (of N_2 daughter), $e^{-\lambda_2 t} \approx 0$ $\mathbf{A_2 = A_{10}}$



Activity A_2 of relatively short-lived radionuclide daughter ($T_2 << T_1$) as a function of time t with initial condition $A_{20} = 0$. Activity of daughter builds up to that of the parent in above seven half-lives (~7T_2). Thereafter, daughter decays at the same rate it is produced ($A_2 = A_1$), and secular equilibrium is said to exist.

Figure by MIT OCW.

Radon Decay



- Radon itself, due to its fairly short half-life (^{222}Rn) is not a major concern.

- Radon is also an inert gas and is typically exhaled after breathing it in (although some will dissolve in the blood).

- The concern is over the daughter products of radon that are particulate (attached to aerosol particles), α-emitting, and decay within hours to ^{210}Pb ($T_{1/2}$ = 22 years).

Transient equilibrium ($T_1 \geq T_2$)

General Case

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \Rightarrow\Rightarrow A_2 = A_{10}\frac{\lambda_2}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) + A_{20}e^{-\lambda_2 t}$$

Simplifying assumptions: $A_{20} = 0$

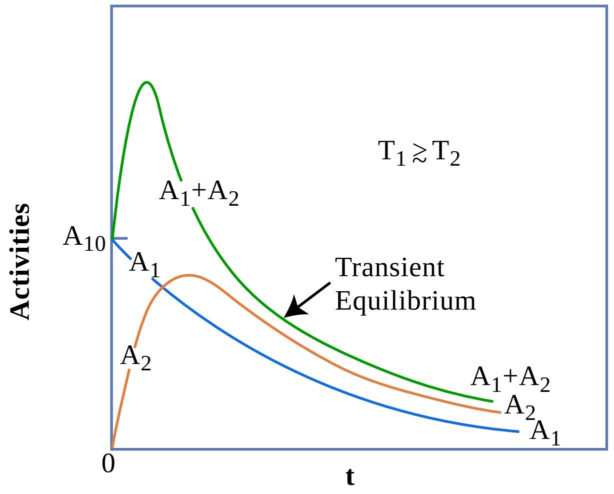
after ~ 10t½s $e^{-\lambda_2 t} << e^{-\lambda_1 t}$

$$A_2 = A_{10}\frac{\lambda_2}{\lambda_2 - \lambda_1}e^{-\lambda_1 t}$$

by definition: $A_{10}\,e^{-\lambda_1 t} = A_1$

$A_2 = A_1\dfrac{\lambda_2}{\lambda_2 - \lambda_2}$ or $\dfrac{A_2}{A_1} = \dfrac{\lambda_2}{\lambda_2 - \lambda_1}$ - at equilibrium A_1 and A_2 present in a

constant ratio



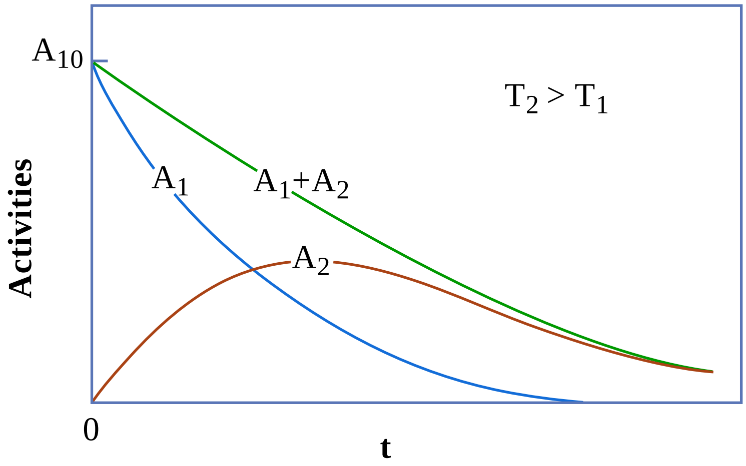
Activities as functions of time when T_1 is somewhat larger than T_2 ($T_1 \gtrsim T_2$) and $N_{20} = 0$. Transient equilibrium is eventually reached, in which all activities decay with the half-life T_1 of the parent.

Figure by MIT OCW.

No equilibrium ($T_1 < T_2$)

[no simplifying assumptions possible]

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \Rightarrow\Rightarrow \; A_2 = A_{10}\frac{\lambda_2}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) + A_{20}e^{-\lambda_2 t}$$



Activities as functions of time when $T_2 > T_1$ and $N_{20} = 0$.
No equilibrium conditions occur. Eventually, only the daughter activity remains.

Figure by MIT OCW.

The ^{99m}Tc Generator: Transient equilibrium in action

- ^{99}Mo is adsorbed on an alumina column as ammonium molybdate (NH_4MoO_4)

- ^{99}Mo (T = 67 hrs) decays (by β -decay) to ^{99m}Tc (T = 6 hrs)

- $^{99}MoO_4$ ion becomes the $^{99m}TcO_4$ (pertechnetate) ion (chemically different)

- $^{99m}TcO_4$ has a much lower binding affinity for the alumina and can be ***selectively eluted*** by passing physiological saline through the column.

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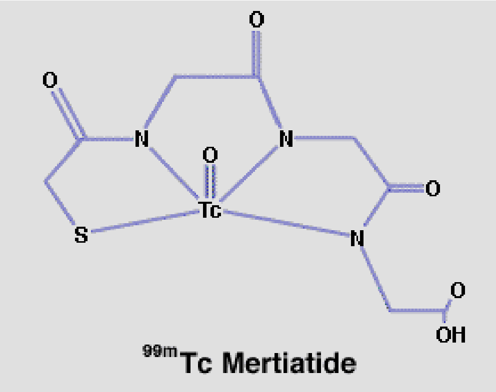
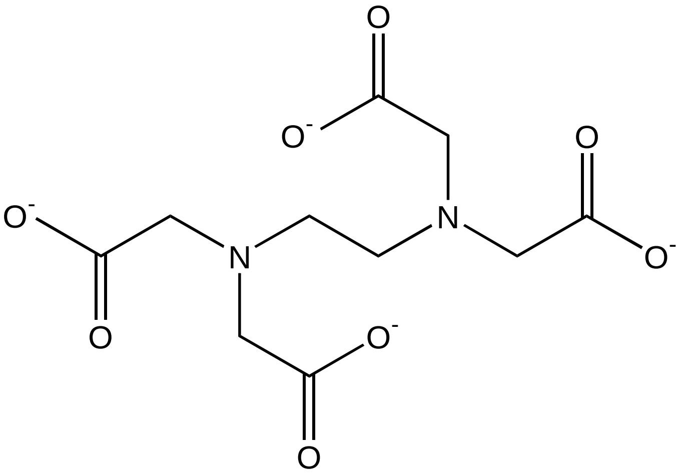
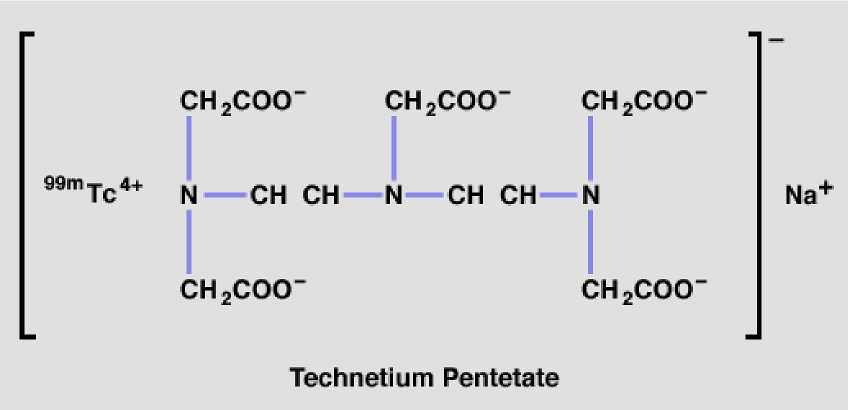


^{99m}Tc Mertiatide

EDTA
ethylenediaminetetraacetate



DTPA



Technetium Pentetate

Chelator Kits

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