

**Krane 3.9**

Compute the total binding energy and the binding energy per nucleon.

$$B({}_Z^A X) = [Zm({}^1\text{H}) + Nm_n - m({}_Z^A X)]c^2$$

(a)  ${}^7\text{Li} : A = 7, Z = 3, N = 4$

$$m({}^7\text{Li}) = 7.016003\text{amu}$$

$$m({}^1\text{H}) = 1.007825\text{amu}$$

$$m_n = 1.008665\text{amu}$$

$$1\text{amu} = 931.502\text{MeV}/c^2$$

$$B({}^7\text{Li}) = [3 \times 1.007825\text{amu} + 4 \times 1.008665\text{amu} - 7.016003\text{amu}] \times (931.502/c^2) \times c^2\text{MeV}$$
$$= 39.246042\text{MeV}$$

$$B({}^7\text{Li})/A = 39.246042\text{MeV}/7 = 5.606577\text{MeV}$$

(b)  ${}^{20}\text{Ne} : A = 20, Z = 10, N = 10$

$$m({}^{20}\text{Ne}) = 19.992436\text{amu}$$

$$B({}^{20}\text{Ne}) = [10 \times 1.007825\text{amu} + 10 \times 1.008665\text{amu} - 19.992436\text{amu}] \times (931.502/c^2) \times c^2\text{MeV}$$
$$= 160.650561\text{MeV}$$

$$B({}^{20}\text{Ne})/A = 160.650561\text{MeV}/20 = 8.032528\text{MeV}$$

(c)  ${}^{56}\text{Fe} : A = 56, Z = 26, N = 30$

$$m({}^{56}\text{Fe}) = 55.934939\text{amu}$$

$$B({}^{56}\text{Fe}) = [26 \times 1.007825\text{amu} + 30 \times 1.008665\text{amu} - 55.934939\text{amu}] \times (931.502/c^2) \times c^2\text{MeV}$$
$$= 492.262478\text{MeV}$$

$$B({}^{56}\text{Fe})/A = 492.262478\text{MeV}/56 = 8.790401\text{MeV}$$

(d)  ${}^{235}\text{U} : A = 235, Z = 92, N = 143$

$$m({}^{235}\text{U}) = 235.043924\text{amu}$$

$$B({}^{235}\text{U}) = [92 \times 1.007825\text{amu} + 143 \times 1.008665\text{amu} - 235.043924\text{amu}] \times (931.502/c^2) \times c^2\text{MeV}$$
$$= 1783.892467\text{MeV}$$

$$B({}^{235}\text{U})/A = 1783.892467\text{MeV}/235 = 7.591032\text{MeV}$$

**Krane 3.10**

For each of the following nuclei, use the semiempirical mass formula to compute the total binding energy and the Coulomb energy.

Semi-empirical mass formula:

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$$B(A, Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}} \frac{(A-2Z)^2}{A} + \delta \begin{cases} a_p A^{-3/4} & \text{even - even} \\ 0 & \text{even - odd / odd - even} \\ -a_p A^{3/4} & \text{odd - odd} \end{cases}$$

$$a_v = 15.5 \text{ MeV}$$

$$a_s = 16.8 \text{ MeV}$$

$$a_c = 0.72 \text{ MeV}$$

$$a_{\text{sym}} = 23 \text{ MeV}$$

$$a_p = 34 \text{ MeV}$$

$$\text{Coulomb Energy : } E_c = -a_c \frac{Z(Z-1)}{A^{1/3}}$$

(a)  $^{21}\text{Ne}$  :  $A = 21, Z = 10$

$$E_c(^{21}\text{Ne}) = -0.72 \text{ MeV} \times \frac{10 \times (10-1)}{21^{1/3}} = -23.487416 \text{ MeV}$$

$$B(^{21}\text{Ne}) = 21 \times 15.5 \text{ MeV} - 21^{2/3} \times 16.8 \text{ MeV} + E_c - \frac{(21-2 \times 10)^2}{21} \times 23 \text{ MeV} + 0 = 173.041414 \text{ MeV}$$

$$B(^{21}\text{Ne}) / A = 173.041414 \text{ MeV} / 21 = 8.240067 \text{ MeV}$$

(b)  $^{57}\text{Fe}$  :  $A = 57, Z = 26$

$$E_c(^{57}\text{Fe}) = -0.72 \text{ MeV} \times \frac{26 \times (26-1)}{57^{1/3}} = -121.605785 \text{ MeV}$$

$$B(^{57}\text{Fe}) = 57 \times 15.5 \text{ MeV} - 57^{2/3} \times 16.8 \text{ MeV} + E_c - \frac{(57-2 \times 26)^2}{57} \times 23 \text{ MeV} + 0 = 502.982352 \text{ MeV}$$

$$B(^{57}\text{Fe}) / A = 502.982352 \text{ MeV} / 57 = 8.824252 \text{ MeV}$$

(c)  $^{209}\text{Bi}$  :  $A = 209, Z = 83$

$$E_c(^{209}\text{Bi}) = -0.72 \text{ MeV} \times \frac{83 \times (83-1)}{209^{1/3}} = -825.738142 \text{ MeV}$$

$$B(^{209}\text{Bi}) = 209 \times 15.5 \text{ MeV} - 209^{2/3} \times 16.8 \text{ MeV} + E_c - \frac{(209-2 \times 83)^2}{209} \times 23 \text{ MeV} + 0 = 1618.621668 \text{ MeV}$$

$$B(^{209}\text{Bi}) / A = 1618.621668 \text{ MeV} / 209 = 7.744601 \text{ MeV}$$

(d)  $^{256}\text{Fm}$  :  $A = 256, Z = 100$

$$E_c(^{256}\text{Fm}) = -0.72\text{MeV} \times \frac{100 \times (100 - 1)}{256^{1/3}} = -1122.589655\text{MeV}$$

$$B(^{256}\text{Fm}) = 256 \times 15.5\text{MeV} - 256^{2/3} \times 16.8\text{MeV} + E_c - \frac{(256 - 2 \times 100)^2}{256} \times 23\text{MeV} + 34\text{MeV} \times 256^{-3/4}$$

$$= 1886.858038\text{MeV}$$

$$B(^{256}\text{Fm}) / A = 1886.858038\text{MeV} / 256 = 7.744601\text{MeV}$$

**Krane 3.17**

The spin-parity of  ${}^9\text{Be}$  and  ${}^9\text{B}$  are both  $\frac{3}{2}^-$ . Assuming in both cases that the spin and parity are characteristic only of the odd nucleon, show how it is possible to obtain the observed spin-parity of  ${}^{10}\text{B}$  ( $3^+$ ). What other spin-parity combinations could also appear? (These are observed as excited states of  ${}^{10}\text{B}$ .)

Solution:

For  ${}^9\text{Be}$ ,  $A=9$ ,  $Z=4$ ,  $N=5$ . The single valence neutron is at  $1P_{3/2}$ , determining the spin-parity of

$${}^9\text{Be} \text{ is } I^\pi = \frac{3}{2}^-.$$

For  ${}^9\text{B}$ ,  $A=9$ ,  $Z=5$ ,  $N=4$ . The single valence proton is at  $1P_{3/2}$ , determining the spin-parity of

$${}^9\text{B} \text{ is } I^\pi = \frac{3}{2}^-.$$

For  ${}^{10}\text{B}$ ,  $A=10$ ,  $Z=5$ ,  $N=5$ . There is an unpaired proton and an unpaired neutron, both at  $1P_{3/2}$ . The coupling of these two nucleons determines the nuclear spin-parity.

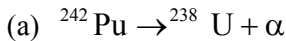
$$j_p = \frac{3}{2}, j_n = \frac{3}{2}, \vec{I} = \vec{j}_p + \vec{j}_n \Rightarrow I = 0, 1, 2, 3$$

$$l_p = 1, l_n = 1, (-1)^{l_p} (-1)^{l_n} = 1 \Rightarrow \pi = +$$

Thus possible spin-parity of  ${}^{10}\text{B}$  are  $0^+, 1^+, 2^+, 3^+$

**Krane 8.3**

From the known atomic masses, computer Q values of the decays.



$$m({}^{242}\text{Pu}) = 242.058737\text{amu}$$

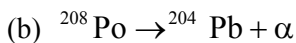
$$m({}^{238}\text{U}) = 238.050785\text{amu}$$

$$m(\alpha) = 4.001506\text{amu}$$

$$1\text{amu} = 931.502\text{MeV} / c^2$$

$$Q = [m({}^{242}\text{Pu}) - m({}^{238}\text{U}) - m(\alpha)]c^2$$

$$= 6.004462\text{MeV}$$



$$m(^{208}\text{Po}) = 207.981222\text{amu}$$

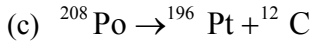
$$m(^{204}\text{Pb}) = 203.973020\text{amu}$$

$$m(\alpha) = 4.001506\text{amu}$$

$$1\text{amu} = 931.502\text{MeV}/c^2$$

$$Q = [m(^{208}\text{Po}) - m(^{204}\text{Pb}) - m(\alpha)]c^2$$

$$= 6.237337\text{MeV}$$



$$m(^{208}\text{Po}) = 207.981222\text{amu}$$

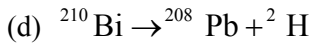
$$m(^{196}\text{Pt}) = 195.964926\text{amu}$$

$$m(^{12}\text{C}) = 12.000000\text{amu}$$

$$1\text{amu} = 931.502\text{MeV}/c^2$$

$$Q = [m(^{208}\text{Po}) - m(^{196}\text{Pt}) - m(^{12}\text{C})]c^2$$

$$= 15.179757\text{MeV}$$



$$m(^{210}\text{Bi}) = 209.984095\text{amu}$$

$$m(^{208}\text{Pb}) = 207.976627\text{amu}$$

$$m(^2\text{H}) = 2.104102\text{amu}$$

$$1\text{amu} = 931.502\text{MeV}/c^2$$

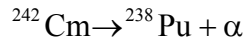
$$Q = [m(^{210}\text{Bi}) - m(^{208}\text{Pb}) - m(^2\text{H})]c^2$$

$$= -6.179584\text{MeV}$$

### **Krane 8.4**

In the decay of  $^{242}\text{Cm}$  to  $^{238}\text{Pu}$ , the maximum  $\alpha$  energy is  $6112.9 \pm 0.1\text{keV}$ . Given the mass of  $^{238}\text{Pu}$ , find the mass of  $^{242}\text{Cm}$ .

Solution:



$$m(^{238}\text{Pu}) = 238.049555\text{amu}$$

$$m(\alpha) = 4.00150618\text{amu}$$

$$1\text{amu} = 931.502\text{MeV}/c^2$$

$$\bar{Q} = 6.1129\text{MeV}, \sigma_Q = 0.1\text{KeV}$$

$$\bar{m}(^{242}\text{Cm}) = m(^{238}\text{Pu}) + m(\alpha) + \bar{Q}/c^2$$

$$= 242.05723\text{amu}$$

$$\sigma_{m(^{242}\text{Cm})} = \sigma_Q / c^2 = 0.1\text{KeV}/c^2 = 1 \times 10^{-4} \text{MeV}/c^2 = 0.093150\text{amu}$$

$$\Rightarrow m(^{242}\text{Cm}) = (242.05723 \pm 0.093150)\text{amu}$$

**Krane 8.9**

Use the semiempirical mass formula to estimate the  $\alpha$ -decay energy of  $^{242}\text{Cf}$  and compare with the measured value (see Figure 8.1).

Solution

Use semiempirical formula (8.9) in Krane:

$$Q = B(^4\text{He}) + B(Z - 2, A - 4) - B(Z, A)$$

$$\cong 28.3 - 4a_v + \frac{8}{3}a_s A^{-1/3} + 4a_c Z A^{-1/3} \left(1 - \frac{Z}{3A}\right) - 4a_{\text{sym}} \left(1 - \frac{2Z}{A}\right)^2 + 3a_p A^{-7/4}$$

$$A = 242, Z = 98 \Rightarrow Q = 9.3493\text{MeV}$$

From Figure 8.1, we get  $Q=7.6\text{MeV}$ , which is very close to the calculation result from semiempirical formula.