Krane 3.9
Compute the total binding energy and the binding energy per nucleon.

$$B({}^{A}_{Z}X) = \left[Zm({}^{1}H) + Nm_{n} - m({}^{A}_{Z}X)\right]c^{2}$$

(a) ${}^{7}Li : A = 7, Z = 3, N = 4$

$m({}^{7}Li) = 7.016003\text{amu}$

$m({}^{1}H) = 1.007825\text{amu}$

$m_{n} = 1.008665\text{amu}$

$1\text{amu} = 931.502\text{MeV}/c^{2}$

$B({}^{7}Li) = \left[3\times1.007825\text{amu} + 4\times1.008665\text{amu} - 7.016003\text{amu}\right]\times(931.502/c^{2})\times c^{2}\text{MeV}$

$= 39.246042\text{MeV}$

$B({}^{7}Li)/A = 39.246042\text{MeV}/7 = 5.606577\text{MeV}$

(b) ${}^{20}Ne : A = 20, Z = 10, N = 10$

$m({}^{20}Ne) = 19.992436\text{amu}$

$B({}^{20}Ne) = \left[10\times1.007825\text{amu} + 10\times1.008665\text{amu} - 19.992436\text{amu}\right]\times(931.502/c^{2})\times c^{2}\text{MeV}$

$= 160.650561\text{MeV}$

$B({}^{20}Ne)/A = 160.650561\text{MeV}/20 = 8.032528\text{MeV}$

(c) ${}^{56}Fe : A = 56, Z = 26, N = 30$

$m({}^{56}Fe) = 55.934939\text{amu}$

$B({}^{56}Fe) = \left[26\times1.007825\text{amu} + 30\times1.008665\text{amu} - 55.934939\text{amu}\right]\times(931.502/c^{2})\times c^{2}\text{MeV}$

$= 492.262478\text{MeV}$

$B({}^{56}Fe)/A = 492.262478\text{MeV}/56 = 8.790401\text{MeV}$

(d) ${}^{235}U : A = 235, Z = 92, N = 143$

$m({}^{235}U) = 235.043924\text{amu}$

$B({}^{235}U) = \left[92\times1.007825\text{amu} + 143\times1.008665\text{amu} - 235.043924\text{amu}\right]\times(931.502/c^{2})\times c^{2}\text{MeV}$

$= 1783.892467\text{MeV}$

$B({}^{235}U)/A = 1783.892467\text{MeV}/235 = 7.591032\text{MeV}$

Krane 3.10
For each of the following nuclei, use the semiempirical mass formula to compute the total binding energy and the Coulomb energy.

Semi-empirical mass formula:

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \delta \begin{cases} a_p A^{-3/4} & \text{even} - \text{even} \\ 0 & \text{even} - \text{odd} / \text{odd} - \text{even} \\ - a_p A^{3/4} & \text{odd} - \text{odd} \end{cases}$$

$a_v = 15.5\text{MeV}$

$a_s = 16.8\text{MeV}$

$a_c = 0.72\text{MeV}$

$a_{sym} = 23\text{MeV}$

$a_p = 34\text{MeV}$

$$\text{CoulombEnergy}: E_c = -a_c \frac{Z(Z-1)}{A^{1/3}}$$

(a) $^{21}\text{Ne}: A = 21, Z = 10$

$$E_c(^{21}Ne) = -0.72MeV \times \frac{10 \times (10-1)}{21^{1/3}} = -23.487416MeV$$

$$B(^{21}Ne) = 21 \times 15.5MeV - 21^{2/3} \times 16.8MeV + E_c - \frac{(21 - 2 \times 10)^2}{21} \times 23MeV + 0 = 173.041414MeV$$

$$B(^{21}Ne) = 173.041414MeV / 21 = 8.240067MeV$$

(b) $^{57}\text{Fe}: A = 57, Z = 26$

$$E_c(^{57}Fe) = -0.72MeV \times \frac{26 \times (26-1)}{57^{1/3}} = -121.605785MeV$$

$$B(^{57}Fe) = 57 \times 15.5MeV - 57^{2/3} \times 16.8MeV + E_c - \frac{(57 - 2 \times 26)^2}{57} \times 23MeV + 0 = 502.982352MeV$$

$$B(^{57}Fe)/A = 502.982352MeV / 57 = 8.824252MeV$$

(c) $^{209}\text{Bi}: A = 209, Z = 83$

$$E_c(^{209}Bi) = -0.72MeV \times \frac{83 \times (83-1)}{209^{1/3}} = -825.738142MeV$$

$$B(^{209}Bi) = 209 \times 15.5MeV - 209^{2/3} \times 16.8MeV + E_c - \frac{(209 - 2 \times 83)^2}{209} \times 23MeV + 0 = 1618.621668MeV$$

$$B(^{209}Bi)/A = 1618.621668MeV / 209 = 7.744601MeV$$

(d) $^{256}\text{Fm}: A = 256, Z = 100$

$$E_c(^{256}Fm) = -0.72MeV \times \frac{100 \times (100-1)}{256^{1/3}} = -1122.589655MeV$$

$$B(^{256}Fm) = 256 \times 15.5MeV - 256^{2/3} \times 16.8MeV + E_c - \frac{(256 - 2 \times 100)^2}{256} \times 23MeV + 34MeV \times 256^{-3/4}$$

$$= 1886.858038MeV$$

$$B(^{256}Fm)/A = 1886.858038MeV / 256 = 7.744601MeV$$

Krane 3.17

The spin-parity of ^{9}Be and ^{9}B are both $\frac{3}{2}^-$. Assuming in both cases that the spin and parity are characteristic only of the odd nucleon, show how it is possible to obtain the observed spin-parity of ^{10}B (3^+). What other spin-parity combinations could also appear? (These are observed as excited states of ^{10}B.)

Solution:

For $^9\mathrm{Be}$, A=9, Z=4, N=5. The single valence neutron is at $1P_{3/2}$, determining the spin-parity of $^9\mathrm{Be}$ is $I^\pi = \frac{3}{2}^-$.

For $^9\mathrm{B}$, A=9, Z=5, N=4. The single valence proton is at $1P_{3/2}$, determining the spin-parity of $^9\mathrm{B}$ is $I^\pi = \frac{3}{2}^-$.

For $^{10}\mathrm{B}$, A=10, Z=5, N=5. There is an unpaired proton and an unpaired neutron, both at $1P_{3/2}$. The coupling of these two nucleons determines the nuclear spin-parity.

$$j_p = \frac{3}{2}, j_n = \frac{3}{2}, \vec{I} = \vec{j}_p + \vec{j}_n \Rightarrow I = 0,1,2,3$$

$$l_p = 1, l_n = 1, (-1)^{l_p}(-1)^{l_n} = 1 \Rightarrow \pi = +$$

Thus possible spin-parity of $^{10}\mathrm{B}$ are $0^+, 1^+, 2^+, 3^+$

Krane 8.3

From the known atomic masses, computer Q values of the decays.

(a) $^{242}\mathrm{Pu} \rightarrow ^{238}\mathrm{U} + \alpha$

$$m(^{242}\mathrm{Pu}) = 242.058737\mathrm{amu}$$

$$m(^{238}\mathrm{U}) = 238.050785\mathrm{amu}$$

$$m(\alpha) = 4.001506\mathrm{amu}$$

$$1\mathrm{amu} = 931.502\mathrm{MeV}/c^2$$

$$Q = \left[m(^{242}\mathrm{Pu}) - m(^{238}\mathrm{U}) - m(\alpha)\right]c^2$$

$$= 6.004462\mathrm{MeV}$$

(b) $^{208}\mathrm{Po} \rightarrow ^{204}\mathrm{Pb} + \alpha$

$$m(^{208}Po) = 207.981222amu$$
$$m(^{204}Pb) = 203.973020amu$$
$$m(\alpha) = 4.001506amu$$
$$1amu = 931.502MeV/c^2$$
$$Q = [m(^{208}Po) - m(^{204}Pb) - m(\alpha)]c^2$$
$$= 6.237337MeV$$

(c) $^{208}Po \rightarrow ^{196}Pt + ^{12}C$

$$m(^{208}Po) = 207.981222amu$$
$$m(^{196}Pt) = 195.964926amu$$
$$m(^{12}C) = 12.000000amu$$
$$1amu = 931.502MeV/c^2$$
$$Q = [m(^{208}Po) - m(^{196}Pt) - m(^{12}C)]c^2$$
$$= 15.179757MeV$$

(d) $^{210}Bi \rightarrow ^{208}Pb + ^{2}H$

$$m(^{210}Bi) = 209.984095amu$$
$$m(^{204}Pb) = 207.976627amu$$
$$m(^{2}H) = 2.104102amu$$
$$1amu = 931.502MeV/c^2$$
$$Q = [m(^{210}Bi) - m(^{208}Pb) - m(^{2}H)]c^2$$
$$= -6.179584MeV$$

Krane 8.4

In the decay of ^{242}Cm to ^{238}Pu, the maximum α energy is $6112.9 \pm 0.1keV$. Given the mass of ^{238}Pu, find the mass of ^{242}Cm.

Solution:

$$^{242}Cm \rightarrow ^{238}Pu + \alpha$$
$$m(^{238}Pu) = 238.049555amu$$
$$m(\alpha) = 4.00150618amu$$
$$1amu = 931.502MeV/c^2$$
$$\overline{Q} = 6.1129MeV, \sigma_Q = 0.1KeV$$
$$\overline{m}(^{242}Cm) = m(^{238}Pu) + m(\alpha) + \overline{Q}/c^2$$
$$= 242.05723amu$$
$$\sigma_{m(^{242}Cm)} = \sigma_Q/c^2 = 0.1KeV/c^2 = 1\times10^{-4}MeV/c^2 = 0.093150amu$$
$$\Rightarrow m(^{242}Cm) = (242.05723 \pm 0.093150)amu$$

Krane 8.9

Use the semiempirical mass formula to estimate the α-decay energy of $^{242}\mathrm{Cf}$ and compare with the measured value (see Figure 8.1).

Solution

Use semiempirical formula (8.9) in Krane:

$$Q = B(^4\mathrm{He}) + B(Z-2, A-4) - B(Z, A)$$

$$\cong 28.3 - 4a_v + \frac{8}{3}a_s A^{-1/3} + 4a_c Z A^{-1/3}(1-\frac{Z}{3A}) - 4a_{sym}(1-\frac{2Z}{A})^2 + 3a_p A^{-7/4}$$

$$A = 242, Z = 98 \Rightarrow Q = 9.3493\mathrm{MeV}$$

From Figure 8.1, we get Q=7.6MeV, which is very close to the calculation result from semiempirical formula.