Neutron Slowing Down Distances and Times in Hydrogenous Materials

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Outline

- Background / Lecture Material
  - Neutron Slowing Down Equation
  - Flux behavior in hydrogenous medium
- Fermi treatment for calculating slowing down distances and times.
- Brief derivation of mean square distance required to moderate neutron.
- Data from paper
- Average time required to moderate neutron
- Data from paper
- Monte Carlo Simulation (Explanation)
- Overview (What was learned?)
- References

Neutron Slowing Down

- The process of reducing the energy of a neutron from the fast to thermal energy range.
- To derive the neutron slowing down equation, first begin with the Neutron Transport Equation.

$$\frac{\partial n}{\partial t} = \int_{dEd\Omega} \sum\nolimits_s (E)\phi(\bar{r},E,\bar{\Omega},t)F(E\bar{\Omega} \rightarrow E,\bar{\Omega}) + \frac{\nu f(E)}{4\pi}\int dEd\Omega \sum\nolimits_f (E)\phi(\bar{r},E,\bar{\Omega},t) - \sum\nolimits_t (E)\phi(\bar{r},E,\bar{\Omega},t) - \bar{v}\bullet\bar{\nabla}n(\bar{r},E,\bar{\Omega},t) + s(\bar{r},E,\bar{\Omega},t)$$

- By getting rid of the spatial dependence of the flux, we obtain the slowing down equation.

$$\sum\nolimits_t (E)\phi(E) = S(E) + \int dE' \sum\nolimits_s (E')\phi(E')F(E' \rightarrow E)$$

Neutron Slowing Down cont.

- The energy transfer kernel seen in the neutron slowing down equation is:

$$F(E' \to E) = \frac{1}{E'(1-\alpha)} \qquad \alpha E' \le E \le E'$$

$$0 \qquad \text{otherwise}$$

α= (A-1)^2 / (A+1)^2

Where the following assumptions are made:

i.) elastic scattering
ii.) target nucleus at rest
iii.) isotropic scattering in CMCS

This kernel applies to neutron slowing down because the neutron loses energy each time it is scattered by the nucleus.

Flux behavior in Hydrogenous Mediums

- Begin with slowing down equation
- Let A=1, monoenergetic point source at E_o, neglect absorption, apply energy transfer kernel value.

$$S(E) = S_o \delta(E - E_o)$$

- $$\Sigma_s(E)\phi(E) = \frac{S_o}{E_o} + \int_E^{E_o} \Sigma_s(E')\phi(E')\frac{dE'}{E'}$$

$$G(E) = const + \int_E^{E_o} G(E')\frac{dE'}{E'}$$

$$G(E) = \frac{c}{E} = \frac{S_o}{E}; c = S_o$$

$$\phi(E) = \frac{S_o}{\Sigma_s(E)E}$$

Fundamental Question and Issues

- How far from a point source does a neutron of initial energy E random walk before it energy is moderated to some lower energy E'? (E>E')
- Fermi addressed this problem by restricting the neutron mean free path, λ, to a constant or slowly varying with energy.
- However, the neutron scattering c.x. of ^{1}H is neither constant nor slowly varying in the energy range 0.1 MeV < E < 14 MeV.

Accounting for Energy Dependence

- To determine how far a neutron wanders before it reaches a given lower energy, the Fermi treatment is manipulated such that λ is now energy dependent.

- This method is valid as long as the dominant neutron scattering mechanism is elastic and the mean free path is known.

Net Displacement

- The neutron will traverse a sequence of linear displacements $\mathbf{r}_j$, each terminated by scattering event that sends the neutron off in a new direction. After n^{th} path the net displacement, **R**, of the neutron from the origin is given by:

$$R = \sum_{j=1}^{n} r_j$$

Image removed due to copyright considerations.

Please see:

Net Displacement figure taken from: Nellis, W. J. "Slowing-down distances and times of 0.1- to 14- MeV neutron in hydrogenous materials." *American Journal of Physics* 45, no. 5 (May 1977).

Brief Derivation

- Magnitude Squared of the Net Displacement, **R**, is R^2.

$$R^2 = \sum_{j=1}^{n} r_j \bullet \sum_{k=1}^{n} r_k$$

$$\left\langle R^2 \right\rangle = \sum_{j=1}^{n} \left\langle r_j^2 \right\rangle + 2 \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \left\langle r_j \right\rangle \left\langle r_k \right\rangle \left\langle \cos\theta_{jk} \right\rangle$$

Where θ_{jk} is the angle between the j^{th} and the k^{th} paths. Neutrons scatter with radial distribution function $e^{-r/\lambda}$, where λ is the total mean free path.

$$\langle r_j \rangle = \lambda_j \quad \text{and} \quad \langle r_j^2 \rangle = 2\lambda_j^2$$

$$\langle \cos\theta_{jk} \rangle = \langle \cos\theta \rangle^{k-j} = \mu^{k-j}$$

Derivation cont.

- Thus, if λ is constant, then

$$\langle R^2 \rangle = \frac{2n\lambda^2}{1-\mu}\left[1 - \frac{\mu}{n}\left(\frac{1-\mu^n}{1-\mu}\right)\right]$$

- But if λ varies with energy, then

$$\langle R^2 \rangle = 2\left[\sum_{j=1}^{n} \lambda_j^{\,2} + \sum_{j=1}^{n-1} \lambda_j \sum_{k=j+1}^{n} \mu^{k-j}\right]$$

The root mean squared slowing down distance is:

$$\langle R_{rms} \rangle = \langle R^2 \rangle^{1/2}$$

Data: 14 MeV Neutron Moderated in Hydrogen ($N = 8 \times 10^{22} cm^{-3}$)

Image removed due to copyright considerations.

Please see:

Neutron Moderated in Hydrogen table taken from: Nellis, W. J. “Slowing-down distances and times of 0.1- to 14- MeV neutron in hydrogenous materials.” *American Journal of Physics* 45, no. 5 (May 1977).

Slowing Down Time

- Average time, t_{ave}, required to moderate neutron from high to lower energy is:

$$t_{ave} = \sum_{j=1}^{n} \Delta t_j = \sum_{j=1}^{n} \frac{\lambda_j}{\upsilon_j}$$

Where Δt is the time spent at each energy.

In the figure, 14 MeV neutrons are being moderated to Ef.

Image removed due to copyright considerations.

Please see:

Slowing Down Time graph taken from: Nellis, W. J. “Slowing-down distances and times of 0.1- to 14- MeV neutron in hydrogenous materials.” *American Journal of Physics* 45, no. 5 (May 1977).

Monte Carlo Simulation

- The sample problem consists of:
  - 1 MeV point source
  - Neutrons emitted with delta function time distribution
  - Point source at the center of the hydrogen sphere
  - Hydrogen density N = 8 x 10^{22} cm^{-3}
  - Hydrogen sphere has an outer radius of 50 cm
  - Path lengths are chosen randomly from a distribution weighted by total c.x.
  - Scattering angle are chosen randomly from a distribution weighted by the known angular dependence of the c.x.
  - Local energy deposited in a collision is then calculated based on conservation of energy and momentum.
  - Flux tallies were placed at different radiuses in hydrogen sphere so one can monitor the moderation.
  - E_f was set to be 40 ev

Overview

- As energy increases, the hydrogen c.x. decreases; thus, the total mean free path is larger. --- Higher energy neutrons penetrate deeper than lower energy neutrons.

$$\lambda_T = \frac{1}{N\sigma_T}$$

- The higher the neutron energy, the faster the neutron loses energy.
- 1/E flux behavior is characteristic of the neutron distribution during slowing down by elastic collisions with target.

* Hydrogen is the most efficient neutron moderator since it can absorb the largest fraction of neutron energy per elastic collision.

References

- Nellis, W.J., ***Slowing-down distances and times of 0.1- to 14- MeV neutron in hydrogenous materials.*** American Journal of Physics, Vol.45, No.5, May 1977.
- J.R. Lamarsh, ***Introduction to Nuclear Reactor Theory.*** © 1966
- Neutron Interactions and Applications, Lecture 9: Neutron Slowing Down.