

22.106 Neutron Interactions and Applications (Spring 2005) Lecture 1 (2/1/05)

Overview -- Neutron Interactions and Cross Sections

There are many references in the vast literature on nuclear physics where one can find useful introductions or comprehensive treatment of neutron interactions. The following is only a collection that this lecturer has used over the years, and by no means should be the only ones for the student to consult.

B. T. Feld, "The Neutron", in *Experimental Nuclear Physics*, E. Segre, ed. (Wiley, 1953), vol. II, p. 208;

A. M. Weinberg and E. Wigner, *The Physical Theory of Neutron Chain Reactors* (Univ. Chicago Press, 1958), Chap 2.

J. E. Lynn, *The Theory of Neutron Resonance Reactions* (Clarendon, Oxford, 1968), Chap 1.

A. Foderaro, *The Elements of Neutron Interaction Theory* (MIT Press, 1971), Chaps 1, 3.

C. G. Shull, "Neutron Interactions with Atoms", *Trans. Am. Cryst. Assoc.* **3**, 1 (1967).

Why Are Neutrons Special?

Study of 'Neutron Interactions' is a specialty of NED - *this is our niche, no other department has the expertise that we do.*

Neutrons play a central role in all three major programs of the Department.

Fission -- the 'carrier' of fission chain reaction, the 'fire' that keeps a reactor burning
Fusion -- products of fusion reactions, e.g. (D,T), causes radiation damage or activation
RST -- accelerators, therapy, imaging, materials research, etc.

Properties of a Neutron (recall 22.101) -- Discovered in 1932 by J. Chadwick*, no charge (penetrates nucleus easily), mass slightly larger than proton (momentum change significant in collision), thermal neutron wavelength comparable to x-ray but energy is much lower, neutrons of interest extend over a very wide energy range (many types of reactions), spin 1/2 (interaction with nucleus is spin-dependent), magnetic moment (couples with atomic moment in magnetic scattering), half life (free neutron is unstable).

*Other notable events - fission (1938), first chain reaction (1942); these have immense impact on our society to this date - they are made possible by neutrons.

Particle-wave duality --

particle: energy-momentum $E = mv^2/2$, $p = mv$, $E = p^2/2m$ (1.1)

wave: frequency-wavelength $E = \hbar\omega = \hbar^2k^2/2m$, $p = \hbar k$, $k = 2\pi/\lambda$ (1.2)

Compare the neutron values with those of photons (x-rays and gammas) and of a proton and an electron.

Does neutron have to be treated relativistically? (usually, no.)

Neutron energies of interest in this class span 9 orders of magnitude --

<u>E</u> (eV)	<u>wavelength</u> λ (A or 10^{-8} cm)	<u>energy designation</u>
10 ⁻³	1.44	cold
0.025	0.288	thermal
1	4.55×10^{-2}	slow
10 ⁴	4.55×10^{-4}	epithermal (≥ 0.5 eV), resonance (1-100 eV)
10 ⁶	4.55×10^{-5}	fast

Characteristics of nuclear forces that are relevant for our studies [W. E. Myerhof, *Elements of Nuclear Physics* (McGraw-Hill, 1967), p. 224; P. Marmier and E. Sheldon, *Physics of Nuclei and Particles* (Academic Press, 1969), vol. I, p. 55]:

Short-range ($\sim 2 \times 10^{-13}$ cm), strongly attractive (central, ~ 30 MeV) with repulsive core at $\sim 0.5 \times 10^{-13}$ cm

Charge symmetric, n-n = p-p, and charge independent n-p = n-n = p-p

Spin-dependent $V = V_1(r) + V_2(r)\underline{\sigma}_1 \cdot \underline{\sigma}_2 + \dots$

Other features are spin-orbit interaction, pairing (short-range, pair up in nucleus, tend to make nucleus spherical), distortion (longer-range \sim nuclear size, due to extra nucleon, tend to destroy sphericity), and exchange forces (symmetry dependent).

Nuclear interactions are **strong** interactions. Suppose they are represented by the Yukawa potential, $g^2 e^{-r/r_0} / r$ (compare with Coulomb interaction e^2 / r), where g is the "charge of the exchange field" (π -meson exchange). With $r_0 \sim 1.4 \times 10^{-13}$ cm this gives ~ 8 MeV for the nuclear binding energy. Compare this with -0.72 MeV -- electrostatic energy between 2 protons at 2×10^{-13} cm, 0.03 MeV -- magnetic potential energy, and 6×10^{-37} MeV -- gravitational energy.

Other types of interactions are **electromagnetic** and **weak** interactions. The respective coupling constants are [Marmier and Sheldon, p. 56]

$$g^2 / \hbar c \sim 0.08 \quad \text{strong}$$

$$\alpha \equiv e^2 / \hbar c \sim 7.3 \times 10^{-3} \quad \text{electronmagnetic (fine-structure constant)}$$

$$\frac{g_F^2}{(\hbar c)^2} \left(\frac{m_\pi c}{\hbar} \right)^4 \sim 5 \times 10^{-14} \quad \text{weak (e.g., } \beta \text{- decay of neutron)}$$

where $g_F \sim$ Fermi coupling constant for weak interaction, m_π is the mass of π - meson, $\sim 2 \times 10^{-39}$.

Fundamental interactions of neutron [Feld]:

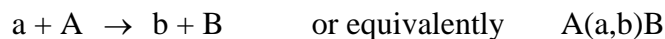
Neutron-Neutron: Not directly observable (mfp $> 10^8$ cm), usually assumed to be same n-p.

Neutron-Electron: 2 types - magnetic (coupling of magnetic moments) and direct n-e due to the "nonuniform charge structure of the neutron" ($n \leftrightarrow p + \pi^-$), latter is electromagnetic in nature - attractive and very weak, $\sigma \sim 5 \times 10^{-7}$ barns (barn = 10^{-24} cm² is unit for nuclear cross section), corresponding to a well depth of 10^3 eV (10^7 eV for n-p).

Neutron-Proton: Direct information from binding energy of deuteron - interaction is short-range, attractive and spin-dependent. In terms of a square-well representation, depth $V_0 \sim 38.5$ MeV (triplet, S=1) and range $b = 1.4 \times 10^{-13}$ cm. The bound state in this well has an energy of $E_B = 2.224$ MeV.

Two general types of nuclear reactions: Spontaneous disintegrations and reactions resulting from collisions

In this course we will study only nuclear collisions (neutron reactions) of the form



where a is the incident particle (neutron in our case), A the target nucleus, B is the product nucleus and b the outgoing particle, if any.

Types of neutron reactions --

(n,n)	elastic scattering	(shape elastic or potential, resonance - no nuclear excitation)
(n,n')	inelastic scattering	(excitation of nuclear levels)
(n, γ)	radiative capture	
(n,p), (n, α), ...	charged particle emission	
(n,f)	fission	

Fusion reactions of interest are:



The first is the largest at $E < 10^3$ keV, $\sigma \sim 5$ barns at $E \sim 100$ keV.

We will return to discuss the energy variation of the corresponding cross sections later in this class. It suffices to say that these reactions make the neutrons a very versatile radiation in applications. In fission energy production the most important reactions are fission, absorption (sum of radiative capture and fission), and scattering (for energy moderation, reflection and radiation damage). In fusion technology, the reactions are capture and charged particle emission (for activation) and scattering (for radiation damage). In therapy, the most relevant reactions are charged particle emission (for cell killing). In condensed matter and spectroscopy studies, the most important process is elastic scattering, which strictly speaking is not a proper reaction in nuclear physics.

Concepts of Cross Sections

It is instructive to review the physical meaning of a cross section σ , which is a measure of the probability of a reaction. Imagine a beam of neutrons incident on a thin sample of thickness Δx covering an area A on the sample. See Fig. 1-1. The intensity of the beam hitting the area A is I neutrons per second. The incident flux is therefore I/A .

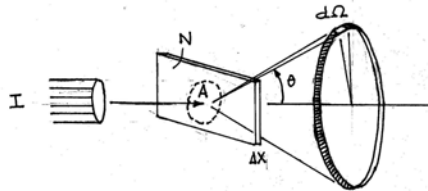


Fig. 1-1. Schematic of an incident beam striking a thin target with a particle emitted into a cone subtending an angle θ relative to the direction of incidence, the 'scattering' angle. The element of solid angle $d\Omega$ is a small piece of the cone (see also Fig. 1-2).

If the nuclear density of the sample is N nuclei/cm³, then the no. nuclei exposed is $NA\Delta x$ (assuming no shadowing effects, i.e., the nuclei do not cover each other with respect to the incoming neutrons). We now write down the probability for a collision-induced reaction as

$$\{\text{reaction probability}\} = \Theta / I = \left(\frac{NA\Delta x}{A} \right) \cdot \sigma \quad (1.3)$$

where Θ is the no. reactions occurring per sec. Notice that σ simply enters or appears in the definition of reaction probability as a **proportionality constant**, with no further justification. Sometimes this simple fact is overlooked by the students. There are other ways to introduce or motivate the meaning of the cross section; they are essentially all equivalent when you think about the physical situation of a beam of particles colliding with a target of atoms.

Rewriting (1.3) we get

$$\sigma = \{\text{reaction probability}\} / \{\text{no. exposed per unit area}\}$$

$$= \frac{\Theta}{IN\Delta x} = \frac{1}{I} \left[\frac{\Theta}{N\Delta x} \right]_{\Delta x \rightarrow 0} \quad (1.4)$$

Moreover, we define $\Sigma = N\sigma$, which is called the macroscopic cross section. Then (1.4) becomes

$$\Sigma\Delta x = \frac{\Theta}{I}, \quad (1.5)$$

or $\Sigma \equiv \{\text{probability per unit path for small path that a reaction will occur}\} \quad (1.6)$

Both the *microscopic cross section* σ , which has the dimension of an area (unit of σ is the **barn** which is 10^{-24} cm^2 as already noted above), and its counterpart, the *macroscopic cross section* Σ , which has the dimension of reciprocal length, are fundamental to our study of neutron interactions. Notice that this discussion can be applied to any radiation or particle, there is nothing that is specific to neutrons.

We can readily extend the present discussion to an **angular differential** cross section $d\sigma/d\Omega$. Now we imagine counting the reactions per second in an angular cone subtended at angle θ with respect to the direction of incidence (incoming particles), as shown in Fig. 1-1. Let $d\Omega$ be the element of solid angle, which is the small area through which the unit vector $\underline{\Omega}$ passes through (see Fig. 1-2). Thus, $d\Omega = \sin\theta d\theta d\varphi$.

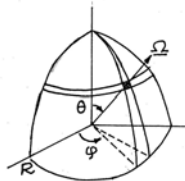


Fig. 1-2. The unit vector $\underline{\Omega}$ in spherical coordinates, with θ and φ being the polar and azimuthal angles respectively (R would be unity if the vector ends on the sphere).

We can write

$$\frac{1}{I} \left(\frac{d\Theta}{d\Omega} \right) = N\Delta x \left(\frac{d\sigma}{d\Omega} \right) \quad (1.7)$$

Notice that again $d\sigma/d\Omega$ appears as a proportionality constant between the reaction rate per unit solid angle and a product of two simple factors specifying the interacting system - the incident flux and the no. nuclei exposed (or the no. nuclei available for reaction).

Note the condition $\int d\Omega d\sigma/d\Omega = \sigma$, which makes it clear why $d\sigma/d\Omega$ is called the *angular differential cross section*.

Another extension is to consider the incoming particles to have energy E and the particles after reaction to have energy in dE' about E' . One can define in a similar way as above an *energy differential cross section*, $d\sigma/dE'$, which is a measure of the probability of an incoming with incoming energy E will have as a result of the reaction outgoing energy E' . Both $d\sigma/d\Omega$ and $d\sigma/dE'$ are distribution functions, the former is a distribution in the variable Ω , the solid angle, whereas the latter is a distribution in E' , the energy after scattering. Their dimensions are barns per steradian and barns per unit energy, respectively.

Combining the two extensions above from cross section to differential cross sections, we can further extend to a *double differential cross section* $d^2\sigma/d\Omega dE'$, which is a quantity that has been studied extensively in thermal neutron scattering. This cross section contains the most fundamental information about the structure and dynamics of the scattering sample. While $d^2\sigma/d\Omega dE'$ is a distribution in two variables, the solid angle and the energy after scattering, it is not a distribution in E , the energy before scattering.

In 22.106 we will be concerned with all three types of cross sections, σ , the two differential cross sections, and the double differential cross section for neutrons. Practically all the applications of interest involve neutron interactions with nuclei in various media. For example, in diffraction and spectroscopy we use neutrons to probe the structure and dynamics of the scattering samples. In cancer therapy we use neutrons to preferentially kill the cancerous cells. Both involve a **single collision** event between the neutron and a nucleus, for which a knowledge of the cross section is all that is required so long as the neutron is concerned. In contrast, for reactor and other nuclear systems applications one wants to know the effects of a **sequence of collisions or multiple collisions**, in which case knowing only the cross section is not sufficient. One needs to follow the neutrons as they undergo many collisions in the media. This requires the study of **neutron transport** - *the distribution of neutrons in configuration space, direction of travel, and energy*. In this course we will treat transport in two ways, theoretical discussion and direct simulation using the Monte Carlo method, and the general purpose code MCNP (Monte Carlo Neutron and Photon). Thus another name for this course is 'Neutron Interactions, Distributions, and Particle Simulations', which says more about the conceptual basis of what we will study.