

22.54 Neutron Interactions and Applications (Spring 2004)

Term Quiz (closed book)

May 4, 2004

Problem 1 (25%)

Consider the problem of s-wave neutron scattering by a carbon nucleus C^{12} where the interaction is described by a spherical well potential of height V_0 and range r_0 .

- (a) Suppose you are given the experimental value of the total (as opposed to differential) cross section is 5.5 barns, and that the sign of the scattering length is positive. What information does this give you concerning the wave function in the potential well and the phase shift δ_0 ? What difference is there between neutron scattering by C^{12} and scattering by a proton?
- (b) Sketch the angular differential cross section (in CMCS) of carbon at two incident energies, 0.5 and 14 MeV and discuss the behavior of each. Discuss briefly how your results would change when expressed in LCS.

Problem 2 (25%)

You are asked to demonstrate the similarities that exist between two problems which are usually treated separately, (a) solving the neutron diffusion equation to find the size of a critical bare reactor, and (b) solving the Schrodinger equation to find the bound states of a potential well. Write out and compare the mathematical structure of the two equations (you do not need to define the physical constants that go with each problem), the solutions and the physical meaning of the boundary conditions to be imposed in each case. Be concise and do not spend time discussing aspects of either problem unless they are common to both.

Problem 3 (25%)

You wish to do a molecular dynamics (MD) simulation without calculating the interatomic forces between particles. The idea is to use the part of the Metropolis sampling scheme for Monte Carlo (MC) simulation where one calculates the system energy after each particle move. Describe how you would implement a hybrid method where the time advancing part of the MD simulation is replaced by an energy evaluation step. Draw a flow chart for this new method, and comment on what is the difference in the physics of particle tracking between the original MD method and your hybrid method.

Problem 4 (25%)

Each of the following should be answered separately (and briefly).

- (a) Define LET and say how it is related to the interaction cross section. What is the physical meaning of RBE, what is its value for thermal neutrons?
- (b) What is the significance of the Fermi pseudopotential?

- (c) What assumptions are involved in deriving the scattering frequency $F(E \rightarrow E')$ used in describing neutron slowing down (write down this frequency first)?
- (d) What is the relation between free-atom and bound-atom cross sections? Which one is measured?
- (e) Explain the physical meaning of net current \underline{J} .

22.54 Neutron Interactions and Applications
(Spring 2003)

Written Quiz (closed book)

May 5, 2003

There are two parts, Theory and Simulation, adding up to a total of 100%. Please answer the two parts on separate solution books. Read each question carefully, be sure to answer all parts of the question in the order they are asked.

THEORY PART

Problem 1 (15%)

Consider the problem of neutron-proton scattering where you are given a spherical barrier potential, height V_0 and range r_0 .

- (a) You are asked to explain how V_0 is related to the s-wave phase shift δ_0 in three steps. You can use equations and math symbols used in the lectures, but you are NOT asked to calculate anything. Step 1: Say how V_0 is related to the wave function of the problem. Step 2: Say how δ_0 is related to the wave function of the problem. Step 3 Say how V_0 and δ_0 are related (in other words, how one can calculate the latter when given the former) after Steps 1 and 2.
- (b) Instead of phase shift method, now use the Born Approximation and the Fermi Pseudopotential, and write down the expression for the scattering amplitude.

Problem 2 (15%)

Consider neutron scattering by hydrogen (in water).

- (a) Sketch the scattering cross section $\sigma(E)$ for $E < 10$ eV and explain any feature of the curve that you can. Explain the difference between free-atom and bound-atom cross sections (say which one shows up in your sketch). Comment on the cross section behavior for $E < 1$ eV and explain what aspect of the scattering system gives rise to this behavior.
- (b) Sketch the energy transfer kernel $F(E' \rightarrow E)$ for elastic scattering by hydrogen when the proton can be taken to be at rest and the scattering is spherically symmetric in CMCS. With respect to the curve in (a) in what region of energy can you apply the kernel that you have just

sketched? Now suppose the target is heated to higher and higher temperature. Give another sketch to show how the effects of temperature affect the kernel $F(E' \rightarrow E)$.

Problem 3 (20%)

In neutron transport the balance equation describing the energy distribution of the neutron flux is

$$[\Sigma_a(E) + \Sigma_s(E)]\phi(E) = \int_0^{\infty} dE' \Sigma_s(E')\phi(E')F(E' \rightarrow E) \quad (1)$$

where $F(E' \rightarrow E)$ is a general energy transfer kernel. Now we distinguish two energy regions, the slowing region where $E > E_T$, with $E_T = 0.025$ eV, the thermal energy at room temperature, and the thermal region where $E < E_T$.

(a) Let the appropriate $F(E' \rightarrow E)$ to be used in the slowing down and thermal energy regions be called $F_1(E' \rightarrow E)$ and $F_2(E' \rightarrow E)$ respectively. What can you say about these two kernel with respect to energy transfer?

(b) Focusing on the thermal energy region, integrate equation (1) from 0 to E_T to obtain a neutron balance in an integrated sense. Show that the result does not depend on $F_2(E' \rightarrow E)$. Interpret your result. What do you know about the energy dependence of $\phi(E)$ based on the lectures and the lecture notes?

22.54 Neutron Interactions and Applications
(Spring 2002)

Mid-Term Quiz (closed book)

March 21, 2002

Read the questions carefully, be sure to answer all parts of the question in the order they are asked.

Problem 1 (12%)

Consider a neutron moving in a medium with nuclear density N and microscopic cross section σ for neutron interaction. Find the following.

- (a) Probability distribution of distance to interaction (denote it by x), $n(x)$. Check normalization.
- (b) Probability that a neutron will undergo at least one interaction while traversing a distance x , $G(x)$. Interpret your result (i.e., discuss why it is obviously correct).
- (c) Probability that a neutron will suffer one and only one interaction in a distance x , $K(x)$. Interpret your result.
- (d) What is the probability that one wants to sample in MCNP? Explain briefly.

Problem 2 (18%)

Consider neutron elastic scattering at energy E' by a target nucleus initially at rest. Assume the scattering is isotropic in CMCS. Write down the scattering frequency $F(E' \rightarrow E)$, where E is the energy after scattering. Find the energy distribution of the recoiling nucleus.

Problem 3 (20%)

For elemental crystals such as graphite the observed scattering cross section σ in the eV energy range is a constant. Explain the origin of this feature (that is, how come the cross section is a constant and how does one calculate the value of this constant) in two different ways, one based on considerations of s-wave scattering and the other based on the interaction between the neutron and the nucleus in the crystal. What is meant by the free-atom cross section and how is it related to the above σ and to the scattering length that appears in the Fermi pseudopotential?

(3/14/02)

On preparing for the midterm quiz on 3/21/02.

The quiz will be divided into two parts, one on cross sections and the theory of neutron interactions (SY) and the other on MCNP, neutron sources and detection (JCY). Denote them as Parts A and B.

The source materials for Part A consist of four sets of lecture notes.

Chap III, 22.111 Lecture Notes

Chap VII, 22.111 Lecture Notes

Chap VI. 22.113 Lecture Notes, only those parts on the Scattering Amplitude and the Born Approximation which were discussed in the class lectures

Chap. IX, 22.113 Lecture Notes, only parts of Secs. A, B, C and E which were discussed in the class lectures

For Part A the Topics which you should know are:

1. The Microscopic Cross Section σ --
basic physical definition
using σ to calculate the probability of a collision
2. Neutron-Proton Scattering --
setting up $d\sigma / d\Omega$ in terms of $f(\theta)$
introducing the s-wave phase shift δ_0
what is meant by low-energy scattering?
cross section in terms of the scattering length a
what is the meaning of the sign of the scattering length a ?
finding δ_0 for the n-p problem
numerical value of calculated $\sigma(n,p)$ as compared to experiment
how does one understand the importance of spin-dependent interaction?
3. Basic Kinematics of Elastic Neutron Scattering --
momentum and energy conservation
one-to-one relation between scattering angle and outgoing neutron energy
elastic scattering in CMCS
4. Energy and Angular Distributions in Elastic Neutron Scattering --
concept of differential cross sections $d\sigma / d\Omega$ and $d\sigma / dE$
transformation properties of distribution functions

transforming $d\sigma / d\Omega$ between CMCS and LCS

energy distribution $F(E \rightarrow E')$ and underlying assumptions

5. Thermal Neutron Scattering --

what is the Fermi Approximation, is it valid for thermal neutron scattering, and why is it useful?

explain coherent and incoherent scattering

concept of double differential scattering cross section $d^2\sigma / d\Omega dE_f$

what is the dynamic structure factor and why is it important?

energy variation of σ in a liquid or crystalline moderator

Following are some sample questions appropriate for Part A (meant to be illustrative only) -

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1. Using a simple sketch, give a physical definition of the angular differential cross section $d\sigma / d\Omega$. How would you extend this definition to the double differential cross section $d^2\sigma / d\Omega dE_f$?
2. Given the microscopic cross section σ , find the probability of a neutron having one and only one collision while traveling a distance x .
3. What is the physical reason that thermal neutron scattering can be treated as low-energy scattering?
4. Give a simple argument showing that only the s-wave contribution is important in low-energy scattering.
5. If n-p scattering can be treated quite simply, what is the reason that the theory of thermal neutron scattering is so much more complicated?
6. In a neutron diffraction experiment, what cross section, if any, is measured?
7. Which cross section is more useful for neutron diffraction, incoherent or coherent?
8. What is the difference between neutron and x-ray diffraction so far as the basic interaction with the sample is concerned?
9. Derive a relation between the scattering angles in LCS and CMCS for neutron elastic scattering by a target nucleus at rest.
10. By examining the diagram of velocities in LCS and CMCS for elastic scattering by a target nucleus at rest, decide which of the following holds for the angular distribution in LCS, if the

scattering is isotropic in CMCS: Isotropic, forward biased, or backward biased. What would be the result if the target nucleus is infinitely heavy?

11. What changes do you expect in $F(E' \rightarrow E)$ for elastic scattering at low energy by a target nucleus which is not at rest?

12. In the Born approximation for the scattering amplitude $f(\theta)$, where θ is the scattering angle, where does θ appear in the approximation?